Applications of Random matrix theory in wireless communications

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There is a growing interest in the statistical properties of wireless communications via noisy channels. In the simplest single user channel setting, a signal sent by a transmitter with N antennas is received by a receiver with M antennas. The output $(M \times 1)$ vector y is linearly related to the input $(N \times 1)$ vector x via: $y = Hx + w$ where H is an $(M \times N)$ rectangular matrix that specifies the communication channel and w is an $(M \times 1)$ vector that includes the additive noise in the channel. In many situations, one can model the channel matrix H as a random matrix with independent entries. The efficiency of the communication is measured by the channel capacity C (a measure of the maximal mutual information between the transmitted and received signals) which can be expressed as a functional of the eigenvalues of $H^\dagger H$. Thus, the knowledge of the spectral properties of a random matrix H plays a very useful role in understanding the statistical properties of the channel capacity C. The simplest model studied in the literature corresponds to choosing the entries of H as Gaussian random variables.

The thesis project would be to study, analytically and numerically, several generalisations of the basic Gaussian model for the single user case. One of the generalisations is to non-Gaussian matrix H that is of much current interest. Another direction would be to compute the statistics of C for the case of multiple users. This multiuser problem leads also to an interesting optimization problem. The Ph.D candidate is expected to learn the basic analytical techniques used in random matrix theory (orthogonal polynomial method, Coulomb gas method, saddle-point analysis etc.), as well as several associated numerical algorithms. The candidate is then expected to apply these techniques to study several of the communication channel models mentioned above.