Topological Confinement of Spins and Charges:
Spinons as $\pi-$ junctions.

S. Brazovskii [1,2]

[1] *Laboratoire de Physique Théorique et de s Modèles Statistiques, CNRS;*
Bât.100, Université Paris-Sud, 91405 Orsay cedex, France;
http://ipnweb.in2p3.fr~lptms/membres/brazov/, e-mail: brazov@ipno.in2p3.fr
[2] *L.D. Landau Institute, Moscow, Russia.*

Topologically nontrivial states, the solitons, emerge as elementary excitations in $1D$ electronic systems. In a quasi $1D$ material the topological requirements originate the spin- or charge- roton like excitations with charge- or spin- kinks localized in the core. They result from the spin-charge recombination due to confinement and the combined symmetry. The rotons possess semi-integer winding numbers which may be relevant to configurations discussed in connection to quantum computing schemes. Practically important is the case of the spinon functioning as the single electronic $\pi-$ junction in a quasi $1D$ superconducting material. (Published in [1].)

I. INTRODUCTION TO SOLITONS.

Topological defects: solitons, vortices, anyons, etc., are discussed currently, see [2], in connection to new trends in physics of quantum devices, see [3]. Closest to applications and particularly addressed at this conference [4] are the $\pi-$ junctions which, linking two superconductors, provide degeneracy of their states with phase differences equal to 0 and 2π. The final goal of this publication is to show that in quasi one-dimensional ($1D$) superconductors the $\pi-$ junctions are produced already at the single electronic level extendible to a finite spin polarization. The effect results from reconciliation of the spin and the charge which have been separated at the single chain level. The charge and the spin of the single electron reconfine as soon as $2D$ or $3D$ long range correlations are established due to interchain coupling. The phenomenon is much more general taking place, in other respects, also in such a common system as the Charge Density Wave -CDW and in such a popular system as the doped antiferromagnet or the Spin Density Wave as its quasi $1D$ version [5]. Actually in this article we shall consider firstly and in greater details the CDW which is an object a bit distant to the mesoscopic community. The applications to superconductors will become apparent afterwards. We shall concentrate on effects of interchain coupling $D > 1$: confinement, topological constraints, combined symmetry, spin-charge recombination. A short review and basic references on history of solitons and related topics in correlated electronic systems (like holes moving within the antiferromagnetic media) can be found in [5].

Solitons in superconducting wires were considered very early [6], within the macroscopic regime of the Ginzburg - Landau theory, for the phase slips problem. Closer to our goals is the microscopic solution for solitonic lattice in quasi $1D$ superconductors [7] at the Zeeman magnetic field. This successful application of results from theory of CDWs, see [8], to superconductors provides also a link of pair breaking effects in these different systems. The solitonic structures in qasi $1D$ superconductors appear as a $1D$ version of the well known FFLO (Fulde, Ferrel, Larkin, Ovchinnikov, see [7]) inhomogeneous state near the pair breaking limit. Being very weak in $3D$, this effect becomes quite pronounced in systems with nested Fermi surfaces which is the case of the $1D$ limit.

To extend physics of solitons to the higher D world, the most important problem one faces is the effect of *confinement* (S.B. 1980): as topological objects connecting degenerate vacuums, the solitons at $D > 1$ acquire an infinite energy unless they reduce or compensate their topological charges. The problem is generic to solitons but it becomes particularly interesting at the electronic level where the spin-charge reconfinement appears as the result of topological constraints. The topological effects of $D > 1$ ordering reconfines the charge and the spin *locally* while still with essentially different distributions. Nevertheless *integrally* one of the two is screened again, being transferred to the collective mode, so that in transport the massive particles carry only either charge or spin as in $1D$, see reviews [8,9].

II. CONFINEMENT AND COMBINED EXCITATIONS.

A. The classical commensurate CDW: confinement of phase solitons and of kinks.

The CDWs were always considered as the most natural electronic systems to observe solitons. We shall devote to them some more attention because the CDWs also became the subject of studies in mesoscopics [10]. Being a case of spontaneous symmetry breaking, the CDW order parameter $O_{cdw} \sim \Delta \cos[\mathbf{Q}\mathbf{r} + \varphi]$ possesses a manifold of degenerate ground states. For the $M-$ fold commensurate CDW the energy $\sim \cos[M\varphi]$ reduces the allowed positions to multiples

of $2\pi/M$, $M > 1$. Connecting trajectories $\varphi \rightarrow \varphi \pm 2\pi/M$ are phase solitons, or "$\varphi-$ particles" after Bishop *et al.* Particularly important is the case $M = 2$ for which solitons are clearly observed e.g. in polyacethylene [11] or in organic Mott insulators [12].

Above the $3D$ or $2D$ transition temperature T_c, the symmetry is not broken and solitons are allowed to exist as elementary particles. But in the symmetry broken phase at $T < T_c$, any local deformation must return the configuration to the same (modulo 2π for the phase) state. Otherwise the interchain interaction energy (with the linear density $F \sim \langle \Delta_0 \Delta_n \cos[\varphi_0 - \varphi_n] \rangle$ is lost when the effective phase $\varphi_0 + \pi sign(\Delta_0)$ at the soliton bearing chain $n = 0$ acquires a finite (and $\neq 2\pi$) increment with respect to the neighboring chain values φ_n. The $1D$ allowed solitons do not satisfy this condition which originates a constant *confinement force* F between them, hence the infinitely growing confinement energy $F|x|$. E.g. for $M = 2$ the kinks should be bound in pairs or aggregate into macroscopic complexes with a particular role plaid by Coulomb interactions [13].

Especially interesting is the more complicated case of *coexisting discrete and continuous symmetries*. As a result of their interference the topolological charge of solitons originated by the discrete symmetry can be compensated by gapless degrees of freedom originated by the continuous one. This scenario we shall discuss through the rest of the article.

B. The incommensurate CDW: confinement of Amplitude Solitons with phase wings.

Difference of ground states with even and odd numbers of particles is a common issue in mesoscopics. In CDWs it also shows up in a spectacular way (S.B. 1980, see [8,9]. Thus any pair of electrons or holes is accommodated to the extended ground states for which the overall phase difference becomes $\pm 2\pi$. Phase increments are produced by phase slips which provide the spectral flow [14] from the upper $+\Delta_0$ to the lower $-\Delta_0$ rims of the single particle gap. The phase slip requires for the amplitude $\Delta(x, t)$ to pass through zero, at which moment the complex order parameter has a shape of the amplitude soliton (AS, the kink $\Delta(x = -\infty) \leftrightarrow -\Delta(x = \infty)$. Curiously, this instantaneous configuration becomes the stationary ground state for the case when only one electron is added to the system or when the total spin polarization is controlled to be nonzero, see Figure 1. The AS carries the singly occupied mid-gap state, thus having a spin $1/2$ but its charge is compensated to zero by local dilatation of singlet vacuum states [8,9].

As a nontrivial topological object (O_{cdw} does not map onto itself) the pure AS is prohibited in $D > 1$ environment. Nevertheless it becomes allowed even their if it acquires phase tails with the total increment $\delta\varphi = \pi$, see Figure 2. The length of these tails ξ_φ is determined by the weak interchain coupling, thus $\xi_\varphi \gg \xi_0$. As in $1D$, the sign of Δ changes within the scale ξ_0 but further on, at the scale ξ_φ, the factor $\cos[Qx + \varphi]$ also changes the sign thus leaving the product in O_{cdw} to be invariant. As a result the 3D allowed particle is formed with the AS core ξ_0 carrying the spin and the two phase $\pi/2$ twisting wings stretched over ξ_φ, each carrying the charge $e/2$.
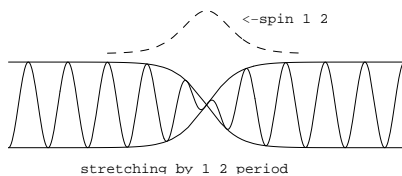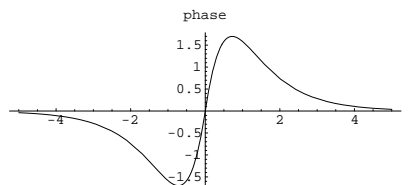

FIG. 1. Amplitude soliton in the IC CDW

FIG. 2. Phase tails adapting the AS.

C. Spin-Gap cases: the quantum CDW and the superconductivity.

We shall omit from consideration the case of repulsion which is relevant to the case of the incommensurate Spin Density Wave or to a hole within the antiferromagnetic media which is important for doped Mott insulators. These cases were emphasized in previous publications [5]. Here we shall concentrate upon systems with attraction which originate the gap and the discrete degeneracy in the spin channel. Firstly we shall generalize the description of the CDW solitons to the quantum model. Secondly we shall use the accumulated experience to arrive at our final goal: the spin carrier in the SC media.

$1D$ electronic systems are efficiently treated within the boson representation, see [16] for a review. The variables can be chosen as φ which is the analog of the CDW phase and θ which is the angle of the $SU2$ spin rotation. These phases are normalized in such a way that their increments divided by π count the electronic charge and spin.

For the incommensurate electronic systems the Lagrangian can be written as

$$\mathcal{L}_{atr} \sim \{C_1(\partial\theta)^2 + V\cos(2\theta)\} + C_2(\partial\varphi)^2 \, ; \; C_1, C_2 = cnst$$

where V is the backward exchange scattering and $(\partial f)^2 = v^{-2}(\partial_t f)^2 - (\partial_x f)^2$, $v \sim v_F$ is the velocity. Elementary excitations in $1D$ are the *spinon* as a soliton $\theta = 0 \to \theta = \pm\pi$, hence carrying the spin $\pm 1/2$, and the gapless charge sound in φ. It is important to recall the alternative description in terms of conjugated phases. We shall need only the one for the charge channel which is the standard gauge phase χ of the superconductivity. Phases φ and χ are related (Efetov and Larkin 79) since their derivatives determine the same quantity - the current j: $\partial_t\varphi/\pi = j \sim \partial_x\chi$. The term $C_2(\partial\varphi)^2$ is duel to $\tilde{C}_2(\partial\chi)^2$ with $\tilde{C}_2 \sim 1/C_2$

1. The Quantum CDW.

The CDW order parameter is $O_{cdw} \sim \exp[i(Qx + \varphi)]\cos\theta$. The spin operator $\cos\theta$ stands for what was the amplitude in the quasi-classical description and at presence of the spinon it changes the sign as it was for Δ. Hence for the CDW ordered state in a quasi $1D$ system the allowed configuration must be composed with two components: the spin soliton $\theta \to \theta + \pi$ and the phase wings $\varphi \to \varphi + \pi$ where the charge $e = 1$ is concentrated. Beyond the low dimensionality, a general view is: *the spinon as a soliton bound to the semi-integer dislocation loop.*
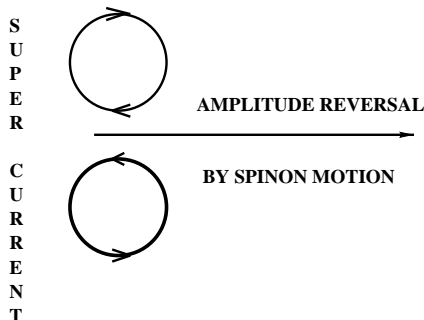

FIG. 3. Motion of the topologically combined excitation in a spin-gap media. The string of the amplitude reversal of the order parameter created by the spinon is cured by the semi-vortex pair (the loop in $3D$) of the phase circulation. For the CDW case the curls are displacements contours for the half integer dislocation pair. For the superconductivity the curls are lines of electric currents circulating through the normal core carrying the unpaired spin.

2. The Singlet Superconductivity.

For the Singlet Superconductivity the order parameter is $O_{sc} \sim \exp[i\chi]\cos\theta$. In $D > 1$ the elementary spin excitation is composed with the *spin soliton* $\theta \to \theta + \pi$ supplied with *current wings* $\chi \to \chi + \pi$. The quasi $1D$ interpretation is that the *spinon works as a Josephson $\pi-$ junction* in the superconducting wire. The $2D$ view is a pair of superconducting $\pi-$ *vortices sharing the common core where the unpaired spin is localized.* The $3D$ view is a *half flux vortex loop which collapse is prevented by the spin confined in its center.*

The solitonic nature of the spinon in the quasi $1D$ picture corresponds to the string of reversed sign of the order parameter left behind in the course of the spinon motion. The spin soliton becomes an elementary fragment of the stripe pattern near the pair breaking limit (FFLO phase). In a wire the π wings of the spinon motion become the persistent current [14].

For this combined particle the electronic quantum numbers are reconfined (while with different scales of localization). But integrally over the cross-section the local electric current induced by the spinon is compensated exactly by the back-flows at distant chains. This is a general property of the vortex dipole configuration constructed above. Finally, the soliton as a state of the coherent media will not carry a current and itself will not be driven by a homogeneous electric field.

III. CONCLUSIONS.

Our conclusions have been derived for weakly interacting SC or CDW chains. Since the results are symmetrically and topologically characterized, they can be extrapolated to isotropic systems with strong coupling where a clear microscopical derivation is not available. Here the hypothesis is that instead of normal carries excited or injected

above the gap, the lowest states are the symmetry broken ones described above as semiroton - spinon complex. This construction can be processed from another side considering a vortex configuration bound to an unpaired electron. Without assistance of the quasi one-dimensionality, a short coherence length is required to leave only a small number of intragap levels in the vortex core.

The existence of complex spin excitations in superconductors is ultimately related to robustness of the FFLO phase at finite spin polarization. It must withstand a fragmentation due to quantum or thermal melting at small spin polarization. Then any termination point of one stripe within the regular pattern (the dislocation) will be accompanied by the phase semiroton in accordance with the quasi $1D$ picture.

[1] Proceedings of the Moriond meeting *Electronic Correlations: From meso- to nano-physics*, T. Martin and G. Montambaux eds., EDP Sciences (2001) 315 .

[2] A.Yu Kitaev, **quant-ph**/9707021; D.A. Ivanov, in [1] and **cond-mat**/0005069

[3] D. Esteve in [1].

[4] V.V. Ryazanov, et al in [1] and **cond-mat**/0008364,0103240.

[5] S. Brazovskii, **cond-mat**/0006355; N. Kirova, **cond-mat**/0004313

[6] J.S. Langer and V. Ambegaokar, Phys. Rev. **164**,498 (1967).

[7] A. Buzdin et al , Sov. Phys. JETP **66, 422** (1987); Physics Letters, **A225**, 341 (1997); Physica C, **316**, 89 (1999).

[8] S. Brazovskii and N. Kirova, *Sov. Sci. Reviews.*, I.M. Khalatnikov ed., **A5**, Harwood Ac. Publ., 1984.

[9] S. Brazovskii, in *Modern Problems in Condensed Matter Science*, v. **25**, p. 425 (Elsevier Sci. Publ., Amsterdam,1989).

[10] H.S.J. van der Zant *et al*, J. de Physique IV, **9** (1999) Pr10 -157.

[11] Physics Today, **53**, #12, 19 (2000).

[12] P. Monceau, F. Ya. Nad, S. Brazovski, Phys. Rev. Lett. **86**, 4080 (2001)

[13] S. Teber *et al*, J. Phys.: Condens. Matter **13**, 4015 (2001).

[14] V. Yakovenko, private communication.

[15] S. Brazovskii and S. Matveenko, Sov. Phys.: JETP, **72**, 492, 860 (1991).

[16] V.J. Emery in *Highly Conducting One-dimensional Solids*, Plenum Press, NY