Planar defects and the fate of the Bragg glass phase of type-II superconductors

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It is shown that the Bragg glass phase can become unstable with respect to planar defects. A single defect plane that is oriented parallel to the magnetic field as well as to one of the main axis of the Abrikosov flux line lattice is always relevant, whereas we argue that a plane with higher Miller index is irrelevant, even at large defect potentials. A finite density of parallel defects with random separations can be relevant even for larger Miller indices. Defects that are aligned with the applied field restore locally the flux density oscillations which decay algebraically with distance from the defect. The current voltage relation is changed to $\ln V(J) \sim -J^{-1}$. The theory exhibits some similarities to the physics of Luttinger liquids with impurities.

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Type-II superconductors have to contain a certain amount of disorder to sustain superconductivity: the disorder pins magnetic flux lines, hence preventing dissipation due to their motion [1, 2]. For some time it was believed that disorder due to randomly distributed impurities destroys the long range translational order (LRO) of the Abrikosov flux line lattice [3]. More recently it was shown that the effect of impurities is much weaker resulting in a phase with quasi-LRO, the so-called Bragg glass [4, 5, 6, 7]. In this phase the averaged flux line density is constant but a remnant of its periodic order is seen in the correlations of the oscillating part of the density which decay as a power law. Experimental signatures of this phase have been observed [8]. An important feature of the Bragg glass is the highly non-linear current-voltage relation related to the flux creep which is of the form $\ln V(J) \sim -J^{-1/2}$ so that the linear resistance vanishes.

Although much of the original transport data on flux creep in high-T_c superconductors was discussed in terms of point disorder (see e.g. [9]) it was realized later that many samples included planar defects like twin planes or grain boundaries which masked the Bragg glass behavior [10]. Indeed, in clean samples planar defects lead to much more pronounced pinning phenomena than point disorder because of stronger spatial correlations [2, 11]. However, the generic experimental situation is a mixture of point disorder and planar defects, a case which has not been studied theoretically in the context of the Bragg glass yet; see however [12, 13].

It is the aim of the present paper to consider exactly this case, i.e. the question of the influence of planar defects in the Bragg glass phase. Our key results are as follows: a necessary condition for a planar defect to become a relevant perturbation is that it is oriented parallel to the magnetic flux. In this case its influence on the Bragg glass phase can be characterized by the value of a *single* parameter $g \equiv \frac{3}{8}\eta(a/\ell)^2$ which depends both on the exponent η describing the decay of the density correlations in the Bragg glass phase *and* on the orientation of the defect with respect to the flux line lattice. a and ℓ are the mean spacing of the flux lines in the bulk and the distance between lattice planes of the Abrikosov lattice *parallel* to the defect, respectively. The defect is relevant for an infinitesimal weak defect potential if $g < 1$ which is realized if and only if the defect plane is parallel to one of the main crystallographic planes of the flux line lattice. In the vicinity of the (relevant) defect the density profile shows periodic order with an amplitude decaying algebraically (with exponent g) with the normal distance from the defect. The current voltage relation for voltage drops perpendicular to a defect plane is of the form $\ln V_D(J) \sim -J^{-1}$. Correlations are destroyed across (relevant) defects. For $g > 1$ a weak (and presumably even a strong) defect is irrelevant and the density profile decays with a larger exponent $2g-1 > 1$. All defects which are tilted against the magnetic flux are irrelevant and the flux density oscillations decay exponentially. For a finite density of parallel defect planes (with random distances) the Bragg glass is destroyed for $g < \tilde{g}_c$ with $\tilde{g}_c \geq 3/2$.

It is worthwhile to mention that some aspects of our results are similar to other theories at their critical dimension like 2-dimensional classical or $(1 + 1)$-dimensional quantum models. Adding a planar defect in the Bragg glass resembles the presence of a line defect in the classical or a frozen impurity in the quantum case [14, 15, 16, 17] when the coupling constant g is identified with temperature or the Luttinger liquid parameter, respectively. The periodic order seen in the vicinity of the defect plane has its counterpart in the Friedel oscillations in Luttinger liquids close to an isolated impurity. Whereas in those cases the relevance of a defect (i.e. an impurity) can be changed by tuning the interaction strength or the temperature, respectively, in the present case a change of g can be accomplished by changing the *orientation* of the defect with respect to the flux line lattice. The current voltage relations are however

different from the 2-dimensional cases. Experimentally micro-twinned crystals with one direction of twin planes have been studied, e.g., in Ref. [18].

First, we consider the effect of a single planar defect in a system of interacting flux lines which are pinned by randomly distributed impurities. Since we are interested in the behavior on large length scales, it is appropriate to describe the interacting flux lines in terms of a continuum elastic approximation with a displacement field $\mathbf{u}(\mathbf{r})$. The Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_D$ includes then the elastic energy and the impurity interaction,

$$\begin{aligned}\mathcal{H}_0 &= \frac{1}{2}\int dz d^2\mathbf{x}\Big\{c_{11}\left(\boldsymbol{\nabla}_\mathbf{x}\cdot\mathbf{u}\right)^2 + c_{66}\left(\boldsymbol{\nabla}_\mathbf{x}\times\mathbf{u}\right)^2 + \\ &+ c_{44}\left(\partial_z\mathbf{u}\right)^2 + E_{\rm pin}(\mathbf{u},\mathbf{r})\Big\}\,, \end{aligned} \tag{1}$$

and the coupling energy $\mathcal{H}_D$ to the defect, see below. The pinning energy can be written as $E_{\rm pin} = U(\mathbf{r})\rho(\mathbf{r},\mathbf{u})$, where $U(\mathbf{r})$ is a potential arising from randomly distributed impurities and $\mathbf{r} \equiv (\mathbf{x}, z)$. The local flux line density can be expressed as $\rho(\mathbf{r},\mathbf{u}) = \rho_0\left\{-\boldsymbol{\nabla}_\mathbf{x}\mathbf{u}(\mathbf{r}) + \sum_\mathbf{G} e^{i\mathbf{G}[\mathbf{x}-\mathbf{u}(\mathbf{r})]}\right\}$ [7]. Here $\rho_0 = 2/\sqrt{3}a^2$, and $\mathbf{G} \equiv \mathbf{G}_{mn} = m\mathbf{b}_1 + n\mathbf{b}_2$ is a vector of the reciprocal lattice with integer m, n [23]. The set of the six smallest reciprocal lattice vectors will be denoted by $\{\mathbf{G}_0\}$.

The energy of a planar defect has the same form as $E_{\rm pin}$ but with $U(\mathbf{r})$ replaced by $-v\delta(\mathbf{r}\cdot\mathbf{n}_D - \delta)$ where v is the strength of the defect potential, $\mathbf{n}_D$ and δ denote the normal vector of the defect plane and its distance (along $\mathbf{n}_D$) from the origin of the coordinate system, respectively. This gives

$$\mathcal{H}_D = v\rho_0\int d^3\mathbf{r}\,\delta(\mathbf{r}\cdot\mathbf{n}_D - \delta)\Big\{\boldsymbol{\nabla}_\mathbf{x}\mathbf{u}(\mathbf{r}) - \sum_{\mathbf{G}\neq\mathbf{0}} e^{i\mathbf{G}[\mathbf{x}-\mathbf{u}(\mathbf{r})]}\Big\}\,. \tag{2}$$

Without $\mathcal{H}_D$ this model has been studied in detail using different approaches [4, 5, 6, 7]. It was shown that thermal fluctuations are irrelevant (zero temperature fixed point) and that the pinned flux line lattice exhibits a power law decay of the translational order parameter $\Psi_\mathbf{G}(\mathbf{x}, z) = e^{i\mathbf{G}\mathbf{u}(\mathbf{x},z)}$, similar to pure 2D crystals at finite temperatures. In particular, $\langle\Psi_\mathbf{G}\rangle = 0$ but $\langle\Psi_\mathbf{G}(\mathbf{x},0)\Psi_{-\mathbf{G}}(\mathbf{0},0)\rangle \sim |\mathbf{x}|^{-\eta_\mathbf{G}}$, where $\langle ...\rangle$ denotes both the thermal and disorder average and $\eta_\mathbf{G} = \eta(G/G_0)^2$. From a Gaussian variational treatment in $d = 3$ dimensions follows $\eta = 1$ [6] whereas a functional renormalization group analysis in $d = 4 - \epsilon$ dimensions yields a non-universal exponent η that varies with the elastic constants of the vortex lattice [7]. Extrapolating to $d = 3$ [24], one finds only a very weak variation with $1.143 < \eta < 1.159$ [7]. Since (despite of the glassy nature of the phase) the structure factor shows still Bragg peaks the notation Bragg-glass was coined [6]. Next we discuss the influence of $\mathcal{H}_D$. In order to integrate over the delta function in Eq. (2), it is convenient to introduce an explicit parameterization for the position vector $\mathbf{r}_D$ of the defect plane which obeys $\mathbf{r}_D\cdot\mathbf{n}_D = \delta$. With

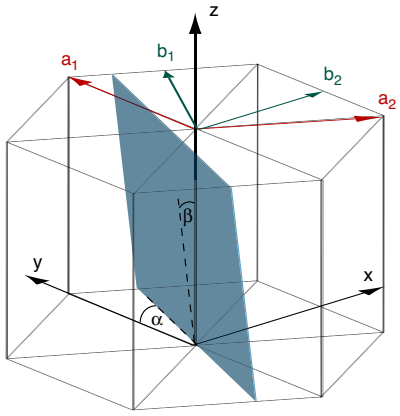

FIG. 1: Triangular flux line lattice with vectors of the direct ($\mathbf{a}_1$ and $\mathbf{a}_2$) and the reciprocal lattice ($\mathbf{b}_1$ and $\mathbf{b}_2$) and the orientation of the defect plane.

$$\begin{aligned}\mathbf{r}_D &= (\mathbf{x}_D, z_D) + \delta\mathbf{n}_D,\;\; z_D = t\cos\beta \\ \mathbf{x}_D &= (s\sin\alpha - t\cos\alpha\sin\beta,\, s\cos\alpha + t\sin\alpha\sin\beta)\end{aligned} \tag{3}$$

we introduce in-plane coordinates s, t, and two angles α and β which determine the rotation of the plane with respect to the y- and z-axis, respectively (see Figure 1). The defect energy reads then

$$\mathcal{H}_D = v\rho_0\int dt ds\Big\{\boldsymbol{\nabla}_\mathbf{x}\mathbf{u}(\mathbf{r}_D) - \sum_{\mathbf{G}\neq\mathbf{0}} e^{i\mathbf{G}[\delta\mathbf{n}_D + \mathbf{x}_D - \mathbf{u}(\mathbf{r}_D)]}\Big\}\,. \tag{4}$$

Since the displacement field $\mathbf{u}(\mathbf{r}_D)$ varies slowly on the scale of the flux line lattice constant a, the integrals over s and t vanish for all $\mathbf{G}$ except those for which the oscillatory factor $e^{i\mathbf{G}\mathbf{x}_D}$ becomes one (for all s, t). This condition can be satisfied only if $\sin\beta = 0$, i.e., if the defect plane is *parallel* to the applied magnetic field. There remains a second condition for the angle α which results from the constraint that $\mathbf{G}$ has to be perpendicular to $\mathbf{x}_D$. Expressing the defect plane (for $\sin\beta = 0$) as $\mathbf{x}_D = (c_1\mathbf{a}_1 - c_2\mathbf{a}_2)s/a$ where $\mathbf{a}_i\mathbf{b}_j = 2\pi\delta_{ij}$, this results in the condition $m/n = c_2/c_1$. Hence if c_1/c_2 is irrational the effect of the defect plane is always averaged to zero. On the other hand, for rational c_2/c_1 we may choose m_D, n_D to be the smallest coprime pair with $c_2/c_1 = m_D/n_D$. Then m_D, n_D are the Miller indices of the defect plane and only those $\mathbf{G}$ which are integer multiples of $\mathbf{G}_D \equiv \mathbf{G}_{m_D n_D}$ contribute in Eq. (4). In the following, we will concentrate on the contribution from these $\mathbf{G}$-vectors only. The flux line lattice planes (of the ideal lattice) parallel to a defect plane with Miller indices m_D, n_D have a separation of $\ell = \frac{\sqrt{3}}{2}a/\sqrt{m_D^2 + m_D n_D + n_D^2}$ and hence $G_D = 2\pi/\ell$.

Using the result for the average of $\Psi_\mathbf{G}(\mathbf{r})$ in the Bragg glass phase [7], one finds for the disorder averaged defect

energy $\mathcal{H}_D$ on scale L

$$\langle \mathcal{H}_D \rangle_0 \sim \sum_{k=1}^{\infty} v_k \cos(k\delta G_D) \left(\frac{L}{L_a}\right)^{2-k^2 g}, \; g \equiv \frac{3}{8}\frac{\eta a^2}{\ell^2}, \quad (5)$$

where $\langle \ldots \rangle_0$ denotes the average with $\mathcal{H}_0$. The linear gradient term in (2) vanishes after averaging [25]. The coefficients $\cos(k\delta G_D)$ reflect the periodicity of the defect energy under translations by ℓ normal to the plane. The Gaussian approximation used here is believed to be correct to order ϵ [19]. It is important to remark that the L-dependence of Eq. (5) holds only on length scales larger than the positional correlation length $L_a \approx L_\xi (a/\xi)^{1/\zeta_{rm}}$ where ξ is the maximum of the coherence and the disorder correlation length and L_ξ denotes the Larkin length on which the typical flux line displacement is of the order ξ. $\zeta_{rm} \approx 0.175$ is the roughness exponent of the elastic distortions on scales smaller than L_a (in the so-called random manifold regime). There the correlations of $\Psi_{\mathbf{G}}(\mathbf{r})$ decay as a stretched exponential and the effect of the defect plane is reduced by disorder fluctuations. Due to these fluctuations on intermediate length scales the initial value of the defect strength is reduced to $v_k \approx v(L_a/a)^2 e^{-c(G_D k a)^2}$ where c is a constant.

To linear order in v_k, the RG flow equation of the v_k is obtained by comparison of the defect energy scaling in in Eq. (5) with the scaling of $\mathcal{H}_0$ at the Bragg glass fixed point, yielding

$$dv_k/d\ln L = \left(1 - k^2 g\right) v_k \, . \quad (6)$$

Hence v_1 is a relevant perturbation for $g < 1$, i.e., if

$$\eta(m_D^2 + m_D n_D + n_D^2) < 2 \text{ or } \quad \ell > \sqrt{\frac{3\eta}{8}}\, a \approx 0.66\, a, \quad (7)$$

which is compatible only with $\ell = \sqrt{3}a/2 \approx 0.87a$. Hence the defect plane must be oriented parallel to one of the three main crystallographic planes of the flux line lattice (i.e. $\cos 2\beta = \cos 6\alpha = 1$).

The transition described by Eq. (6) occurs not in the bulk but on the defect plane. Hence one can develop an effective theory on the defect which could be used to describe also stronger defect potentials. Since the defect couples only to the normal displacement $u_\perp(\mathbf{r}_D) = \mathbf{n}_D \mathbf{u}(\mathbf{r}_D)$ on the defect plane, we would like to integrate out $u_\perp$ outside the defect and $\mathbf{u} - u_\perp \mathbf{n}_D$ across the entire sample. This integration is facilitated by the reasonable assumption of an effective Gaussian theory for the defect-free system at the Bragg glass fixed point [7]. We find that $u_\perp$ on the defect has long-ranged elasticity and is described by the effective Hamiltonian (compare [16, 17] for a corresponding procedure in the clean case)

$$\begin{aligned} \mathcal{H}_{2\mathrm{D}} &= \frac{K}{2}\int d^2\mathbf{q}\, |\mathbf{q}|\, |\varphi_{\mathbf{q}}|^2 \quad (8) \\ &+ \int d^2 r_D \left\{ \frac{2\sqrt{\pi g} K}{\xi} \cos(\varphi - \alpha) + \frac{v_1}{L_a^2}\cos(\varphi) \right\}, \end{aligned}$$

where $\varphi(\mathbf{r}_D) \equiv 2\pi u_\perp(\mathbf{r}_D)/\ell$ and $\mathbf{q}$ is the in-plane momentum. α is a random phase which is uncorrelated and uniformly distributed. The amplitude of the random potential has been chosen here as to reproduce the proper Gaussian replica theory of the Bragg glass for $v_1 = 0$. The elastic constant K depends on the bulk elastic moduli, the angle α and the direction of $\mathbf{q}$. The model of Eq. (8) shows a transition at $g = g_c(v_1)$ with $g_c(0) = 1$ in agreement with our previous considerations. In the present case $g = \eta(m_D^2 + m_D n_D + n_D^2)/2$ can only be changed in finite steps by changing the orientation of the defect plane. Thus at small v_1 only the defects parallel to the three main crystallographic planes are relevant (with $m_D^2 + m_D n_D + n_D^2 = 1$). Due to the long-ranged elasticity we expect from our analysis of a similar model [20] that even at large v_1 one has $g_c(v_1) = 1$.

Next we study the order of the flux lines in the vicinity of the defect. Since the superconducting order is reduced in the defect plane it is plausible to assume that $v > 0$. The potential of a relevant defect growth under renormalization and effectively decouples the two half-spaces. On large scales, we can then impose Dirichlet boundary conditions for the normal flux line displacement $u_\perp$ on the defect plane. This allows us to determine the boundary induced modifications of the flux density via the method of images. We find that $\langle u_\perp^2(\mathbf{r})\rangle = \frac{1}{2}\langle [u_\perp(\mathbf{r}) - u_\perp(\mathbf{r}_m)]^2\rangle_0$, where $\mathbf{r}_m$ is the mirror image of $\mathbf{r}$ with respect to the defect plane. From this we obtain immediately that with increasing distance $L_\perp = |\mathbf{n}_D \mathbf{x} - \delta|$ from the defect, the slowest oscillations of the flux line density decay for a *relevant* defect plane ($g < 1$) as

$$\langle \rho(\mathbf{x}, z, \mathbf{u})\rangle \sim \left(\frac{L_a}{L_\perp}\right)^g \cos(G_0(L_\perp \pm \delta)). \quad (9)$$

Hence, a single relevant defect plane yields a long-ranged restoration of the order parameter. The oscillations of the density resemble Friedel oscillations observed in Luttinger liquids close to an isolated impurity [17]. This similarity is substantiated by considering the decay of the density oscillations if the defect plane is *irrelevant*, i.e., for $g > 1$. Then the defect potential decreases under renormalization and lowest order perturbation theory can be applied. Such an approach takes into account that the defect strength decreases as $v(L) \sim L^{1-g}$ and hence we obtain Eq. (9) with g replaced by $2g - 1 > 1$, in close analogy to the $(1+1)$-dimensional counterpart [14, 15, 16].

If the defect plane is *not* parallel to the applied magnetic field, i.e. $\sin\beta \neq 0$, then the irrelevance of the defect should be also observable in the Friedel oscillations of the flux line density which should become then short-ranged. If $g > 1$ we can use perturbation theory to obtain the density oscillations since the defect strength deceases for large length scales, i.e., for sufficiently small $\sin\beta$. From this analysis we find for the slowest oscilla-

tions normal to the defect (with $\delta = 0$)

$$\langle \rho(\mathbf{x}, z, \mathbf{u}) \rangle \sim \left(\frac{L_a^2}{\xi L_\perp} \right)^{g-\frac{1}{2}} e^{-L_\perp/\xi} \cos\left(G_D L_\perp \cos\beta\right) \tag{10}$$

for large separations $L_\perp \gg \xi$ with a characteristic length scale $\xi = 1/(G_D|\sin\beta|)$. Hence, if the tilt angle β approaches zero, ξ diverges, and the Friedel oscillations for $L_\perp \ll \xi$ are described by Eq.(10) with ξ replaced by $L_\perp$, hence in agreement with our findings for $\beta = 0$. Although our perturbative analysis is restricted to $g > 1$ we expect that for any value of g the Friedel oscillations decay exponentially beyond distances of order ξ. Since the renormalization of the defect potential is cutoff on the scale ξ, even for a relevant defect perturbation theory is expected to be justified if both the bare defect potential is sufficiently weak and $\sin\beta$ is not too small.

To find the current voltage relation for a relevant defect we consider the most interesting case where the current J is parallel to the defect and normal to the magnetic field $\mathbf{B}$. Flux creep in the presence of pinning forces arises via formation of critical droplets [2]. In the present case of a defect plane the droplet is characterized by $G_D u_\perp(\mathbf{r}_D) = 0$ and 2π outside and inside of the droplet, respectively. Since on scale L_a typical distortions in the Bragg glass phase are of the order a, the droplet volume is $\sim L^2 L_a$ and the volume energy gain in the droplet is of the order $\sim -JBL^2 L_a$ which has to be compared with the the surface energy loss of the order $\sim v_1^{1/2(1-g)} L L_a^{1/2}$ where we have included the renormalization of v_1. From the balance of the two terms follows a critical droplet size $L_J \sim v_1^{1/2(1-g)} (L_a JB)^{-1}$. Droplets of this size have a free energy $\sim v_1^{1/(1-g)} (JB)^{-1}$ which determines via the Arrhenius law the creep velocity due to thermal activation, yielding the voltage drop normal to the defect plane

$$V \sim \exp\left[-(C_0/(TJ))\right], \quad C_0 \sim v_1^{1/(1-g)} B^{-1}. \tag{11}$$

Thus the flux creep across the defect is much slower than in the Bragg glass phase. If for weak defects $g \to 1-$, C_0 becomes small, hence reducing the applicability of this formula to extremely small currents.

Crystals usually contain either two orthogonal families ("colonies") of parallel twin boundaries or a *single* family of parallel twin planes. We focus on the latter case, which for impure samples has been studied so far only in (1+1) dimensions [21]. We consider the typical situation of planes with random distances δ_j that are aligned with the applied field. Each plane contributes $\mathcal{H}_D(\delta_j)$ of Eq. (4) with δ_j replacing δ. For the averages over the δ_j one finds $\overline{\langle \sum_j \mathcal{H}_D(\delta_j) \rangle} = 0$ but $\overline{\langle \sum_j \mathcal{H}_D^2(\delta_j) \rangle}^{1/2} \sim L^{5/2-g}$ from the term with $k = 1$. Since $\mathcal{H}_0 \sim L$, a family of weak defect planes is relevant and destroys the Bragg glass for $g < \tilde{g}_c = 3/2$, i.e., the planes must be oriented with a main plane of the flux lattice. Contrary to a single defect plane, we expect that a larger defect strength yields an increased $\tilde{g}_c > 3/2$ rendering additional defect orientations relevant. However, a description of the transition at strong coupling and the localized fixed point describing relevant defect planes is not available at present [22]. Only for defect planes of equal distance we expect long range order in the direction perpendicular to the planes.

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[24] A direct computation for $d = 3$ yields a value for η that depends not only on c_{11}/c_{66} but on the two ratios c_{11}/c_{44} and c_{66}/c_{44}. However, the range of variation of η remains unchanged to order ϵ.
[25] The coupling of $\boldsymbol{\nabla}_\mathbf{x}\mathbf{u}(\mathbf{r})$ to the defect potential for a defect parallel to the z-axis can be eliminated by the transformation $\mathbf{u} \to \mathbf{u} + \boldsymbol{\nabla}_\mathbf{x}\phi(\mathbf{x})$ with $\phi(\mathbf{x})$ a scalar field which obeys $\boldsymbol{\nabla}_\mathbf{x}^2\phi = \frac{v\rho_0}{2c_{11}}\delta(\mathbf{x}\cdot\mathbf{n}_D - \delta)$. This transformation does not change the terms $\sim c_{66}$, c_{44} in Eq. (1) but shifts the flux line density on the defect.