Spatial Persistence of Fluctuating Interfaces

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We show that the probability, $P_0(l)$, that the height of a fluctuating $(d+1)$-dimensional interface in its steady state stays above its initial value up to a distance l, along any linear cut in the d-dimensional space, decays as $P_0(l) \sim l^{-\theta}$. Here θ is a "spatial" persistence exponent, and takes different values, θ_s or θ_0, depending on how the point from which l is measured is specified. These exponents are shown to map onto corresponding temporal persistence exponents for a generalized $d=1$ random-walk equation. The exponent θ_0 is nontrivial even for Gaussian interfaces.

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Fluctuating interfaces have played the role of a paradigm in nonequilibrium statistical physics for the last two decades, with applications ranging from growth problems to fluid flow [1]. Traditionally the different universality classes of the interfaces are specified by the exponents associated with the dynamic correlation functions. Recently the concept of persistence [2], i.e., the statistics of first-passage events, was used to characterize the *temporal* history of an evolving interface [3]. Starting from an initially *flat* interface, it was found that the probability that the interface at a given point in space stays above its initial height up to time t decays as a power law in time with an exponent θ_0^{tem} that is nontrivial even for simple "Gaussian" interfaces (i.e., with dynamics governed by a linear Langevin equation) [3,4]. On the other hand, the statistics of returns to an initial profile chosen from the ensemble of equilibrium configurations was shown to be governed by a different exponent θ_s^{tem}. The exponent θ_s^{tem} was shown to be identical to the first passage exponent of the fractional Brownian motion, which is known exactly [3]. In [3] these two regimes were termed the *coarsening* and *stationary* regimes, respectively.

The exponents θ_0^{tem} and θ_s^{tem} describe first-passage properties in *time*. A question that naturally arises is what about the first-passage properties in space? Is there a power-law distribution and associated nontrivial exponents for the *spatial* persistence of an interface? In this paper we address this important question and show that indeed in the *steady state* (long time limit), the probability $P_0(l)$ that an interface stays above its initial value over a distance l from a given point in *space* decays as $P_0(l) \sim l^{-\theta}$ for large l. Furthermore, analogous to its temporal counterpart, the spatial persistence exponent θ again takes different values, θ_0 and θ_s, according to the "initial" condition, in this case specified by the height and its spatial derivatives at the point (the "initial point" in space) from which l is measured. If the initial point is sampled uniformly from the ensemble of steady state configurations, the relevant exponent, θ_s, is related to the Hurst exponent of the spatial roughening. Conversely, if the initial point is such that the height and its spatial derivatives are finite, independent of the system size (we will call this a *finite* initial condition), the corresponding exponent θ_0 is a new, nontrivial exponent.

A second interesting question asks is there a morphological transition in the stationary profile of a fluctuating interface if one changes the mechanism of fluctuations by changing either the dynamical exponent z or the spatial dimension d? This question has important experimental significance. For example, recent scanning tunneling microscope (STM) measurements have shown [5] that the dynamical exponent z characterizing the fluctuations of single layer Cu(111) surface changes from $z=2$ at high temperatures to $z=4$ at low temperatures [5–8]. Thus by changing the temperature, and hence z, the STM measurements may possibly detect a morphological transition if there is one. We show here that Gaussian interfaces indeed exhibit a morphological transition across a critical line $z_c(d) = d+2$ in the $(z-d)$ plane as one changes z or d. For $z > z_c(d)$, the steady state profile is smooth with a finite density of zero crossings along any linear cut in the d-dimensional space. On the contrary, for $z < z_c(d)$, the density of zero crossings is infinite and the locations of zeros are nonuniform and form a fractal set.

A seemingly unrelated problem is the usual temporal persistence of the stochastic process [9]

$$\frac{d^n x}{dt^n} = \eta(t), \tag{1}$$

where x is the position of a particle and $\eta(t)$ is a Gaussian white noise with zero mean and delta correlation, $\langle \eta(t)\eta(t')\rangle = \delta(t-t')$. The probability, $P_0(t)$, that x does not change sign up to time t decays as $P_0(t) \sim t^{-\theta(n)}$ for large t, where the persistence exponent $\theta(n)$ depends continuously on n [2]. The exact value of the exponent is known only for $n=1$ (the usual Brownian motion), $\theta(1) = 1/2$ [10], and $n=2$ (the random acceleration problem), $\theta(2) = 1/4$ [11,12]. The latter problem has recently attracted attention in connection with the problem of inelastic collapse in granular materials [13].

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In this paper, we establish an intricate mapping between the process in Eq. (1) and the steady state profile of Gaussian interfaces evolving via the Langevin equation

$$\frac{\partial h}{\partial t} = -(-\nabla^2)^{z/2} h + \xi , \tag{2}$$

where ∇^2 is the d-dimensional Laplacian operator, z is the dynamical exponent associated with the interface, and $\xi(\vec{r}, t)$ is a Gaussian white noise with zero mean and $\langle \xi(\vec{r}, t)\xi(\vec{r}\,', t')\rangle = 2\delta(\vec{r} - \vec{r}\,')\delta(t - t')$ [14]. The continuum equation (2) is well defined only for $z > d$. For $z < d$, one needs a nonzero lattice constant in real space or an ultraviolet cutoff in momentum space. Below we will assume $z > d$.

Let us summarize our main results. We find two independent spatial persistence exponents θ_0 and θ_s depending on the initial point in space (from where measurements start). The two main results are (i): by exploiting the mapping to the stochastic process (1), we show that for a "finite" initial starting point,

$$\theta_0 = \theta(n), \quad \text{where } n = (z - d + 1)/2 . \tag{3}$$

(ii) For initial points sampled from the steady state configurations, however, we show exactly that

$$\theta_s = \begin{cases} 3/2 - n, & 1/2 < n < 3/2 , \\ 0, & n > 3/2 , \end{cases} \tag{4}$$

where $n = (z - d + 1)/2$.

Exploiting the two exact results, $\theta(1) = 1/2$ and $\theta(2) = 1/4$ in Eq. (3), we find that for any $(d + 1)$-dimensional interface with $z - d = 1$, $\theta_0 = 1/2$ and for $z - d = 3$, $\theta_0 = 1/4$ exactly. An explicit example of the former case is the (1 + 1)-dimensional Edwards-Wilkinson model with $z = 2$ and $d = 1$, giving $\theta_0 = 1/2$ [15]. An example of the latter case is the (1 + 1)-dimensional continuum version of the Golubovic–Bruinsma–Das Sarma–Tamborenea (GBDT) model [16] with $z = 4$ and $d = 1$, which gives $\theta_0 = 1/4$, a new exact result. For general $n > 3/2$, we show that the exponent $\theta(n)$ and hence θ_0 can be estimated rather accurately by using an "independent interval approximation." Thus Eq. (3), besides giving accurate, and in some cases exact, results for the new exponent θ_0, also gives a physical meaning to the stochastic process in Eq. (1) for general n. We will use this correspondence to further establish a morphological transition for Gaussian interfaces, across the critical line $z_c(d) = d + 2$, as one changes z or d.

We start by considering the nth derivative of the scalar field $h(\vec{r}, t)$ with respect to any particular direction, say x_1, in the d-dimensional space, $g(\vec{r}, t) = \partial^n h/\partial x_1^n$. From Eq. (2), it follows that the Fourier transform, $\tilde{g}(\vec{k}, t) = \int g(\vec{r}, t) e^{i\vec{k}.\vec{r}}\, d^d\vec{r}$ then evolves as $\partial \tilde{g}(\vec{k}, t)/\partial t = -|k|^z \tilde{g}(\vec{k}, t) + \xi_1(\vec{k}, t)$, where $|k|^2 = k_1^2 + k_2^2 + \cdots$, ξ_1 is a Gaussian noise with zero mean, and $\langle \xi_1(\vec{k}, t)\xi_1(\vec{k}\,', t')\rangle = 2|k_1|^{2n}\delta(\vec{k} + \vec{k}\,')\delta(t - t')$. One can easily evaluate the correlator, $\langle g(\vec{k}, t) g(\vec{k}\,', t)\rangle = G(\vec{k}, t)\delta(\vec{k} + \vec{k}\,')$, to obtain, in the steady state, $G(\vec{k}) = |k_1|^{2n}/|k|^z$. Inverting the Fourier transform gives the stationary, real-space correlator, $\langle g(x_1, x_2, \ldots) g(x_1', x_2, \ldots)\rangle = \int |k_1|^{2n}|k|^{-z} e^{ik_1(x_1 - x_1')}\, d^d\vec{k}$. It is now easy to see that, with the choice $n = (z - d + 1)/2$, the integral yields a delta function, i.e.,

$$\langle g(x_1, x_2, x_3, \ldots) g(x_1', x_2, x_3, \ldots)\rangle = D\delta(x_1 - x_1') , \tag{5}$$

where D is just a dimension-dependent number.

Since the basic Langevin equation, Eq. (2), is a linear equation and the noise ξ is Gaussian, clearly h is a Gaussian process and therefore its nth derivative, g, is also a Gaussian process. A Gaussian process is completely specified by its two-point correlator. Hence in the steady state limit $t \to \infty$, one can write the following effective equation, which gives the x_1 dependence of g at fixed $x_2, x_3, \ldots$:

$$g(x_1, x_2, \ldots) = \frac{\partial^n h}{\partial x_1^n} = \eta(x_1) , \tag{6}$$

where η is Gaussian white noise with zero mean and $\langle \eta(x_1)\eta(x_1')\rangle = \delta(x_1 - x_1')$. Note that we have rescaled the x_1 axis to absorb the constant D. This effective stochastic equation will generate the correct two-point correlator as given by Eq. (5) and also all higher order correlations as well since g is a Gaussian process. Making the identification, $x_1 \Longleftrightarrow t$ and $h \Longleftrightarrow x$, Eq. (6) immediately reduces to the process in Eq. (1). Hence the steady state profile of a Gaussian interface evolving via Eq. (2) can be effectively described by the stochastic equation (1) with $n = (z - d + 1)/2$. Therefore, the spatial persistence of the Gaussian interface also gets mapped onto the temporal persistence of the stochastic process (1).

Some aspects of the stochastic process (1) have briefly been discussed by us previously [2,9]; however, here we considerably expand and elaborate on those discussions. Consider the process (1) starting from the initial condition at $t = 0$ where x and all its time derivatives up to order $(n - 1)$ vanish. It turns out that the statistics of first-passage events of this process depends crucially on whether one starts measuring these events right from $t = 0$ or if one first waits for an infinite time and then starts measuring the events. This is similar to the "coarsening" versus "stationary" regimes studied in [3]. This can be quantified more precisely in terms of two-time correlations. Integrating Eq. (1) n times gives $x(t) = [\Gamma(n)]^{-1} \int_0^t \eta(t_1)(t - t_1)^{n-1}\, dt_1$, where $\Gamma(n)$ is the usual gamma function. From this integral, which provides a natural continuation from integer n to real n, it is easy to evaluate the correlation function

$$\begin{aligned} C(t, t', t_0) &= \langle [x(t_0 + t) - x(t_0)][x(t_0 + t') - x(t_0)]\rangle \\ &= A(t_0 + t, t_0 + t') - A(t_0 + t, t_0) \\ &\quad - A(t_0, t_0 + t') + A(t_0, t_0) , \end{aligned} \tag{7}$$

where the autocorrelation $A(t, t')$ is given by

$$A(t,t') = \frac{1}{\Gamma^2(n)} \int_0^{t_m} (t - t_1)^{n-1}(t' - t_1)^{n-1}\, dt_1 , \quad (8)$$

where $t_m = \min(t,t')$. Thus the correlation function that fully characterizes the Gaussian process, and thereby its persistence, depends explicitly on the "waiting" time t_0, the starting point of measurements. It turns out, however, that there are only two asymptotic behaviors, controlled respectively by the $t_0 = 0$ and $t_0 \to \infty$ fixed points. Any finite t_0 flows into the $t_0 = 0$ fixed point.

To see how the two exponents, θ_0 and θ_s, emerge, consider first the limit $t_0 \to \infty$. From Eq. (7), we get after some algebra,

$$C(t,t',\infty) \sim \begin{cases} \alpha_n[t^{2n-1} + t'^{2n-1} - |t - t'|^{2n-1}], & \text{(I)}, \\ \beta_n t_0^{2n-3} tt', & \text{(II)}, \end{cases} \quad (9)$$

where (I) and (II) are the regimes $1/2 < n < 3/2$ and $n > 3/2$, $\alpha_n = -\Gamma[n]\Gamma[1 - 2n]/\Gamma[1 - n] > 0$ and $\beta(n) = [n^2 - n + 1 - (n - 2)/(2n - 3)]/(2n - 1) > 0$. To calculate the persistence properties from this correlator, we note that the form of the correlator for $\frac{1}{2} < n < \frac{3}{2}$ in Eq. (9) is precisely that of fractional Brownian motion, i.e., the differences $\Delta(t,t_0) = x(t + t_0) - x(t_0)$ have stationary increments: $\langle[\Delta(t,t_0) - \Delta(t',t_0)]^2\rangle \sim 2\alpha_n|t - t'|^{2H}$, with Hurst exponent $H = n - \frac{1}{2}$. The corresponding persistence exponent is known exactly [3,17] to be $\theta_s = 1 - H = \frac{3}{2} - n = (d + 2 - z)/2$, from Eq. (3), provided $z < d + 2$. For $z > d + 2$, i.e., $n > 3/2$, Eq. (9) gives the normalized correlator $C(t,t',\infty)/\sqrt{C(t,t,\infty)C(t',t',\infty)} = 1$, implying $\theta_s = 0$.

We now turn to the limit $t_0 = 0$. From Eq. (7), we find that the process is nonstationary in time, and so are the increments of $\Delta(t,t_0)$. However, using the standard transformation [2], $X = x(t)/\sqrt{\langle x^2(t)\rangle}$ and $T = \ln t$, one finds that $X(T)$ becomes a Gaussian *stationary* process in the "new" time variable T, with a correlator $C_n(T) = \langle X(0)X(T)\rangle$, given by

$$C_n(T) = \left(2 - \frac{1}{n}\right)e^{-T/2}F(1 - n, 1; 1 + n; e^{-T}), \quad (10)$$

where $F(a,b;c;z)$ is the standard hypergeometric function. This form of the correlator suggests that the exponent θ_0 is nontrivial. To find it one is confronted with the following problem: given a Gaussian stationary process $X(T)$ with a prescribed correlator $C(T) = \langle X(0)X(T)\rangle$, what is the probability $P_0(T)$ that the process $X(T)$ does not cross zero up to time T?

For general correlator $C(T)$, $P_0(T)$ is hard to solve [18]. For $n = 1$, we have, from Eq. (10), $C_1(T) = \exp(-T/2)$, a pure exponential. This corresponds to a Markov process, for which the exact result is known: $P_0(T) \sim \exp(-T/2) \sim t^{-1/2}$ implying $\theta_0(1) = 1/2$. For higher values of n, the process $X(T)$ is non-Markovian and hence to determine $\theta_0(n)$ is harder. Fortunately for $n = 2$ where $C_2(T) = \frac{3}{2}\exp(-T/2) - \frac{1}{2}\exp(-3T/2)$, the exact result is also known: $P_0(T) \sim \exp(-T/4) \sim t^{-1/4}$ [11]. For general n, one can extract some useful information about the stochastic process by studying the short time properties of the correlator $C_n(T)$. Expanding Eq. (10) for small T we get

$$C_n(T) \simeq \begin{cases} 1 - a_n T^{2n-1}, & 1/2 < n < 3/2, \\ 1 + \frac{T^2}{4}\ln T, & n = 3/2, \\ 1 - \frac{(2n-1)}{8(2n-3)}T^2, & n > 3/2, \end{cases} \quad (11)$$

where $a_n = \Gamma(n)\Gamma(2 - 2n)/\Gamma(1 - n)$. Thus for $n > 3/2$, the process is "smooth" with a finite density of zero crossings that can be derived using Rice's formula [19], $\rho = \sqrt{-C_n''(0)}/\pi = (2\pi)^{-1}\sqrt{(2n - 1)/(2n - 3)}$, where $C_n''(0)$ is the second derivative at the origin. For $1/2 < n < 3/2$, the density is infinite and the zeros are not uniformly distributed but instead form a fractal set with fractal dimension $d_f = n - 1/2$ [18]. For the marginal case $n = 3/2$, Eq. (10) gives $C_{3/2}(T) = \cosh(T/2) + \sinh^2(T/2)\ln[\tanh(T/4)]$, with the density still divergent but only logarithmically. A physical example corresponding to this case is the 2D GBDT model with $z = 4$. The exponent $\theta(n)$ diverges as $n \to 1/2$ from above. For $n < 1/2$ or equivalently $z > d$, the continuum equation is not well defined and requires a nonzero lattice constant in space (for the interface problem) or time [for the stochastic process in Eq. (1)].

This transition at $n = n_c = 3/2$ can now be directly interpreted in the interface context using the relation (3). Gaussian interfaces with finite initial conditions undergo a morphological transition at $z_c = d + 2$. For $z > z_c(d)$ ($n > 3/2$), the surface has a finite density of zero crossings along any linear cut in the d-dimensional space. On the other hand, for $z < z_c(d)$, the surface crosses zero in a nonuniform way and the density is strictly infinite. If it crosses zero once, it crosses subsequently many times before making a long excursion.

This transition is reminiscent of the "wrinkle" transition found recently by Toroczkai *et al.* [8] in $(1 + 1)$-dimensional Gaussian interfaces. However there is an important difference. Toroczkai *et al.* studied the density of extrema of a Gaussian $(1 + 1)$-dimensional surface and found that the density of extrema is finite for $z > 5$ and infinite for $z < 5$. Their results can be understood very simply within our general framework by noting that an extremum of the surface h corresponds to $\partial_x h = 0$. Thus the density of extrema of the surface h corresponds to the density of zeros of the derivative process $\partial_x h$. One can then follow our chain of arguments mapping a Gaussian interface to the stochastic process in Eq. (1) and one finds that the derivative process also maps to Eq. (1), but with $n = (z - d - 1)/2$, i.e., n gets replaced by $(n + 1)$ in Eq. (3) due to the extra derivative. Using $n_c = 3/2$ again, we find that the transition for the derivative process occurs at $z = z_c = d + 4$. This is thus a generalization of the $d = 1$ result ($z_c = 5$) of Ref. [8].

For $n > 3/2$, where the process is smooth, one can apply the independent interval approximation which assumes that successive intervals between zero crossings of the process $X(T)$ are statistically independent. This method was used very successfully for rather accurate analytical estimates of the persistence exponent for the diffusion equation [9,20]. According to this approximation, the exponent θ for a general smooth Gaussian process with correlator $C(T)$ is given by the first positive root of the following transcendental equation [9],

$$1 + \frac{2\theta}{\pi} \int_0^\infty \sin^{-1}[C(T)] e^{\theta T} \, dT = \frac{2\rho}{\theta}, \qquad (12)$$

where $\rho = \sqrt{-C''(0)}/\pi$ is the density of zero crossings. Applying this formula to our problem, with $C_n(T)$ given by Eq. (10), we can obtain estimates for $\theta(n)$ for all $n > 3/2$. For example, putting $n = 2$ in Eqs. (10) and (12), we get $\theta(2) = 0.264\,66\cdots$ which can be compared to the exact result $\theta(2) = 1/4$. Similarly for $n = 3$, we find $\theta(3) = 0.222\,83\cdots$, to be compared to the value $\theta(3) \approx 0.231 \pm 0.01$ obtained by numerical simulation [21].

In the context of a stationary interface, the morphological transition at $n = 3/2$ (i.e., $z = d + 2$) is associated with the familiar transition from a rough to a super-rough interface [22], extracted from the mean-square height difference $C(x) = \langle [h(x) - h(0)]^2 \rangle$. Using the stationary correlator, $\langle h_{\mathbf{k}} h_{-\mathbf{k}} \rangle \sim 1/|\mathbf{k}|^z$ gives $C(x) \sim |x|^{z-d}$ for $d < z < d + 2$, and $C(x) \sim x^2 L^{z-d-2}$ for $z > d + 2$, where the system size L acts as a regulator for the small-k divergence ($k_{\min} \sim 1/L$) in the latter case. This analysis also illustrates the difference between finite and steady state initial points: if the derivatives of $h(x)$ up to order $(n - 1)$ are fixed, i.e., not averaged over the steady state distribution, one obtains, for large $|x|$, $C(x) \sim |x|^{z-d}$, independent of L, for all $z > d$. The same result follows from Eq. (1) after the replacements $x \to h$, $t \to x$.

In summary, we have shown that fluctuating interfaces exhibit a form of spatial persistence analogous to the temporal persistence exhibited by stochastic processes. For Gaussian interfaces the analogy is precise—the spatial fluctuations in the stationary state are isomorphic to the stochastic process (1), with n given by Eq. (3). A *non-Gaussian* process for which exact results are possible is the (1 + 1)-dimensional Kardar-Parisi-Zhang (KPZ) equation, since the stationary probability distribution of the interface field is given by $P(\{h(x)\}) \sim \exp[-\int (\partial_x h)^2 \, dx]$ [1]. Thus the stationary interface can be described by the effective Langevin equation, $\partial_x h = \eta(x)$, the $n = 1$ version of Eq. (1). From the exact results $\theta(1) = 1/2$, and $H = 1/2$ we immediately obtain $\theta_0 = \theta_s = 1/2$, in agreement with Ref. [15]. For the KPZ equation in higher dimensions, and other non-Gaussian interface models, the determination of θ_0, in particular [23], remains a challenge.

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