

Persistence of Manifolds in Nonequilibrium Critical Dynamics

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We study the persistence probability $P(t)$ that, starting from a random initial condition, the magnetization of a d' -dimensional manifold of a d -dimensional spin system at its critical point does not change sign up to time t . For $d' > 0$ we find three distinct late-time decay forms for $P(t)$: exponential, stretched exponential, and power law, depending on a single parameter $\zeta = (D - 2 + \eta)/z$, where $D = d - d'$ and η, z are standard critical exponents. In particular, we predict that for a line magnetization in the critical $d = 2$ Ising model, $P(t)$ decays as a power law while, for $d = 3$, $P(t)$ decays as a power of t for a plane magnetization but as a stretched exponential for a line magnetization. Numerical results are consistent with these predictions.

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First-passage (or “persistence”) problems in spatially extended systems have attracted enormous interest over the last ten years [1]. Coarsening phenomena [2], where the persistence $P(t)$ is the fraction of the sample that has remained in the same state, or phase, throughout the coarsening process, provide many experimental examples. Interest has been sparked by nontrivial power-law decays of $P(t)$ in many systems. In this Letter we study the persistence of the magnetization of submanifolds of a ferromagnet undergoing coarsening at its critical point and show that the asymptotic form of $P(t)$ depends on the dimension of the manifold.

Following a rapid quench of a spin system from temperature $T = \infty$ to $T = 0$, domains of competing ground states form and grow [2]. For nonconserved dynamics, a given spin flips only when a domain wall passes through it, which happens rarely. As a result persistence, i.e., the probability that the spin remains unflipped up to time t , decays slowly as a power law, $P(t) \sim t^{-\theta}$, at late times [1]. By contrast, after a quench to the critical point T_c , spins fluctuate rapidly due to the finite temperature and the persistence of a single spin has an exponential tail. However, for a quench to T_c , the persistence of the total magnetization (as opposed to that of a single spin) again decays as a power law, $P(t) \sim t^{-\theta_g}$, where the exponent θ_g has been argued to be a new nonequilibrium critical exponent [3]. The global persistence has since been studied in a large number of systems [4].

In a d -dimensional sample a single spin is a zero-dimensional manifold, while the global magnetization corresponds to summing over all the spins of the full sample which is a d -dimensional manifold. To interpolate between these limits, it is natural to study the persistence of the magnetization of a d' -dimensional manifold with $0 \leq d' \leq d$. In the limits $d' = 0$ and $d' = d$, the persistence has very different asymptotic decay, respectively, exponential and power law. The question naturally arises: If one tunes the manifold dimension d' from $d' = 0$ to $d' = d$, how does the asymptotic behavior of persistence

change from an exponential ($d' = 0$) to a power-law decay ($d' = d$)? Does the change occur abruptly at an intermediate value of d' , or is there a regime of d' where the behavior is neither exponential nor power law, but something in between? In this Letter we address this interesting issue and we show analytically that indeed there is an intermediate regime of d' where the persistence has a stretched-exponential tail. Our main results can be summarized in terms of the single number $\zeta = (D - 2 + \eta)/z$, where $D = d - d'$ is the codimension of the manifold and η and z are the standard critical exponents: z is the dynamical exponent that describes the temporal growth of the correlation length $\xi(t) \sim t^{1/z}$ and η describes the power-law decay of the equilibrium spin-spin correlation function at T_c , $\langle \phi(0)\phi(r) \rangle \sim r^{-(d-2+\eta)}$ for large separation r . Depending on the value of ζ , the persistence of the magnetization of a d' -dimensional manifold (for $d' > 0$) at T_c has the asymptotic behavior,

$$\begin{aligned} P(t) &\sim t^{-\theta(d',d)}, & \zeta < 0, \\ &\sim \exp(-a_1 t^\zeta), & 0 \leq \zeta \leq 1, \\ &\sim \exp(-b_1 t), & \zeta > 1, \end{aligned} \quad (1)$$

where $\theta(d', d)$ depends on d' and d , and a_1, b_1 are constants. Strictly speaking, in the intermediate regime $0 \leq \zeta \leq 1$ we show that $\exp(-a_2 t^\zeta \ln t) \leq P(t) \leq \exp(-a_1 t^\zeta)$ for large t . We derive the results in Eq. (1) within the mean-field theory (valid for $d > 4$), in the $n \rightarrow \infty$ limit of the $O(n)$ model, followed by a general scaling theory. The results in Eq. (1) hold for all manifolds of dimension $d' > 0$. For a single spin, i.e., for $d' = 0$, our results in Eq. (1) do not hold. It is clear that the persistence of a single spin always decays exponentially at $T = T_c$ due to a finite flip rate.

Two specific applications of our general results are as follows: (i) Consider the persistence of the line magnetization ($d' = 1$) in the $d = 2$ Ising model at T_c . Using $d' = 1$, $d = 2$, $\eta = 1/4$, and $z \approx 2.172$, one gets $\zeta = (d - d' - 2 + \eta)/z \approx -0.3453 < 0$. Hence Eq. (1)

predicts a power-law decay for the persistence of the line magnetization. (ii) For the $d = 3$ Ising model, using $\eta \approx 0.032$ and $z \approx 2$, one finds that while for the plane magnetization ($d' = 2$), $\zeta \approx -0.484 < 0$, for the line magnetization ($d' = 1$), $0 < \zeta \approx 0.016 < 1$. Our Eq. (1) then predicts a power-law decay for the persistence of the plane magnetization, but a stretched-exponential decay for the line magnetization. Numerical simulations for small samples in two and three dimensions are consistent with these analytical predictions.

Our starting point is the standard Langevin equation for the vector order parameter $\vec{\phi} = (\phi_1, \dots, \phi_n)$,

$$\partial_t \phi_i = \nabla^2 \phi_i - r \phi_i - (u/n) \phi_i \sum_{j=1}^n \phi_j^2 + \eta_i, \quad (2)$$

where $\vec{\eta}(\mathbf{x}, t)$ is a Gaussian white noise with mean zero and correlator $\langle \eta_i(\mathbf{x}, t) \eta_j(\mathbf{x}', t') \rangle = 2\delta_{i,j} \delta^d(\mathbf{x} - \mathbf{x}') \delta(t - t')$. The magnetization of a d' -dimensional manifold is defined by the vector field, $\psi_i(x_{d'+1}, \dots, x_d, t) = \int \phi_i(\vec{x}, t) \prod_{i=1}^{d'} dx_i / \sqrt{L}$, obtained by integrating the order parameter over the d' directions. Here L denotes the length of the sample along any direction. For a vector order parameter, we define the persistence $P(t)$ of the manifold to be the probability that any given component of the manifold magnetization, say ψ_1 , does not change sign up to time t . Since all the components of the spin are equivalent, henceforth we drop the subscript i of ψ_i for convenience. Note that the magnetization ψ is a field over the remaining $D = d - d'$ dimensional space whose coordinates $(x_{d'+1}, \dots, x_d)$ are relabeled by the vector $\vec{r} = (r_1, r_2, \dots, r_D)$ for convenience. The observation that allows us to make analytical predictions for the persistence of the magnetization $\psi(\vec{r}, t)$ is that it is a Gaussian variable at all finite times. This follows simply from the fact that $\psi(\vec{r}, t)$ is a sum of $L^{d'}$ random variables which are correlated but only over a finite correlation length $\xi(t) \sim t^{1/z}$. Thus in the thermodynamic limit when $t^{1/z} \ll L$, the central limit theorem asserts that $\psi(\vec{r}, t)$ is a Gaussian field. Hence its persistence is determined by the autocorrelation function $C(t_1, t_2) = \langle \psi(\vec{r}, t_1) \psi(\vec{r}, t_2) \rangle$ [1]. Note that this central limit theorem holds provided the dimension of the manifold $d' > 0$, which we assume from now on.

We start with the mean-field theory, valid for $d \geq 4$, where we set $u = 0$, and also $r = 0$ (at the critical point) in Eq. (2). Next we integrate the Langevin equation over the d' space directions and then solve the resulting linear equation in the Fourier space. Defining $\tilde{\psi}(\vec{k}, t) = \int \psi(\vec{r}, t) e^{i\vec{k} \cdot \vec{r}} d^D r$, it is easy to compute the two-time correlation function,

$$\langle \tilde{\psi}(\vec{k}, t_1) \tilde{\psi}(-\vec{k}, t_2) \rangle = \Delta(\vec{k}) e^{-k^2(t_1+t_2)} + \frac{1}{k^2} (e^{-k^2|t_1-t_2|} - e^{-k^2(t_1+t_2)}), \quad (3)$$

where $\Delta(\vec{k}) = \langle \tilde{\psi}(\vec{k}, 0) \tilde{\psi}(-\vec{k}, 0) \rangle$ is taken to be a constant

(for an uncorrelated initial condition). At late times, the initial condition dependent term becomes negligible, and it is sufficient to retain only the second term on the right-hand side of Eq. (3). The autocorrelation function is then obtained by integrating over the \vec{k} space, $C(t_1, t_2) = \int d^D k e^{-k^2 a^2} \langle \tilde{\psi}(\vec{k}, t_1) \tilde{\psi}(-\vec{k}, t_2) \rangle$, where we have introduced a soft ultraviolet cutoff a . One finds

$$C(t_1, t_2) = \beta A [(t_1 + t_2 + a^2)^{2\beta} - (|t_1 - t_2| + a^2)^{2\beta}], \quad (4)$$

where $2\beta = (2 - D)/2$ and A is an unimportant constant.

Consider first the case $D < 2$, i.e., $\beta > 0$. In this case there is no need for the ultraviolet cutoff since $\langle \psi^2(\vec{r}, t) \rangle = C(t, t)$ does not diverge at any finite time even if one puts $a = 0$. Setting $a = 0$ in Eq. (4) we find that the correlation function $C(t_1, t_2)$ is still nonstationary. However, one can render it stationary by employing a well-known trick [1], where one introduces a normalized Gaussian process, $X = \psi / \sqrt{\langle \psi^2 \rangle}$ which, when observed in the logarithmic time $T = \ln t$ becomes a stationary Gaussian variable with the correlator, $C(T) = \langle X(0)X(T) \rangle = \cosh(T/2)^{2\beta} - |\sinh(T/2)|^{2\beta}$. Interestingly, the same Gaussian correlator also appears in the context of the persistence of the Edwards-Wilkinson-type rough interfaces [5]. It is well-known [1,6] that for such a stationary Gaussian correlator (decaying exponentially for large T), the persistence of the process also decays exponentially for large T , $P(T) \sim \exp(-\theta T)$. In terms of the real time $t = e^T$, this indicates a power-law decay, $P(t) \sim t^{-\theta}$, with persistence exponent θ . For this particular correlator, the exponent θ has been studied in great detail in the context of the interface problem [5], and the exponent is known to depend continuously on the roughness parameter $\beta = (2 - D)/4 > 0$.

For $D > 2$, on the other hand, one needs the ultraviolet cutoff a explicitly in order to keep $\langle \psi^2(\vec{r}, t) \rangle = C(t, t)$ finite. In that case, the appropriate scaling limit is $t_1, t_2 \rightarrow \infty$, but keeping their difference $|t_1 - t_2|$ fixed. In this limit, Eq. (4) reduces to a stationary correlator in the original time variable $C(t_1, t_2) \sim (|t_1 - t_2| + a^2)^{-(D-2)/2}$ that decays as a power law for large $|t_1 - t_2|$. To calculate the persistence of Gaussian stationary processes with an algebraically decaying correlator is nontrivial. However, there exists a powerful theorem due to Newell and Rosenblatt [7], which states that if the stationary correlator decays as $C(t) \sim t^{-\alpha}$ with $\alpha > 0$ for large time difference $t = |t_1 - t_2|$, then the persistence $P(t)$ (probability of no zero crossing between t_1 and t_2) of such a process has the following asymptotic behaviors: (i) $P(t) \sim \exp(-K_1 t)$ if $\alpha > 1$ and (ii) $\exp(-K_2 t^\alpha \ln t) \leq P(t) \leq \exp(-K_3 t^\alpha)$ if $0 < \alpha < 1$, where the K_i 's are constants. Applying this theorem to our problem, we find that $P(t) \sim \exp(-K_1 t)$ for $D > 4$ and $\exp[-K_2 t^{(D-2)/2} \ln t] \leq P(t) \leq \exp(-K_3 t^{(D-2)/2})$ for $2 < D < 4$. In the borderline case $D = 4$, there will be an additional logarithmic correction.

Combining these results for $D < 2$ and $D > 2$ and noting that $z = 2$ and $\eta = 0$ within the mean-field theory, we find that the explicit exact results for the mean-field theory derived above are just the special cases of the general result in Eq. (1) provided one uses the mean-field value $\zeta = (D - 2)/2$ in Eq. (1).

The mean-field theory is valid for $d \geq 4$. In order to access the physically relevant dimensions $d \leq 4$, we now consider another solvable limit, namely, the $n \rightarrow \infty$ limit where Eq. (2) becomes

$$\partial_t \phi_i = \nabla^2 \phi_i - [r + S(t)] \phi_i + \eta_i, \quad (5)$$

where $S(t) = u \langle \phi_i^2 \rangle$ has to be determined self-consistently. The critical point corresponds to $r + S(\infty) = 0$. This self-consistent determination of $S(t)$ can be done using standard techniques [3] and one finds that at late times, $S(t) \rightarrow S(\infty) - (4 - d)/4t$ for $2 < d \leq 4$. Substituting this result into Eq. (5), summing over the d' directions, solving the resulting equation in the Fourier space, and finally integrating over the \vec{k} space (as in the mean-field theory), we finally arrive at the following autocorrelation function for the manifold magnetization $\psi(\vec{r}, t)$ in $2 < d \leq 4$:

$$C(t_1, t_2) = A_1 (t_1 t_2)^{(4-d)/4} \int_0^{t_1} \frac{dt' t'^{(d-4)/2}}{(t_1 + t_2 - 2t' + a^2)^{D/2}}, \quad (6)$$

where $t_1 \leq t_2$, A_1 is a constant, and a represents the soft ultraviolet cutoff as before.

For $D < 2$, as in the mean-field theory, one can set the cutoff $a = 0$, and the resulting nonstationary correlator in Eq. (6) can be reduced to a stationary correlator for the normalized process $X = \psi/\sqrt{\langle \psi^2 \rangle}$ in the logarithmic time $T = \ln t$,

$$A(T) = [\cosh(T/2)]^{\mu-D/2} \frac{B[\mu, 2\beta, 2/(1+e^T)]}{B[\mu, 2\beta]}, \quad (7)$$

where $\mu = (d - 2)/2$, $2\beta = (2 - D)/2$, $B[m, n]$ is the standard Beta function, and $B[m, n, x] = \int_0^x dy y^{m-1} (1 - y)^{n-1}$. Since the stationary correlator in Eq. (7) decays exponentially for large T , $A(T) \sim \exp[-(d + D - 2)T/4]$, one concludes [1,6] that the corresponding persistence also decays exponentially for large T , $P(T) \sim \exp(-\theta T)$, and hence as a power law in the original $t = e^T$ variable, $P(t) \sim t^{-\theta}$. Determining the exponent θ analytically is still a challenging task. However, one can make progress in the limit of small codimension $D \rightarrow 0$. Note that for $D = 0$, i.e., for the global persistence, Eq. (7) becomes a pure exponential $A(T) = \exp[-(d - 2)T/4]$ for all T , indicating that the process is Markovian [3]. One then finds $P(T) \sim \exp(-\theta_0 T)$ with $\theta_0 = (d - 2)/4$ [3]. For D nonzero but small, one can expand the correlator in Eq. (7) around the Markov process ($D = 0$) and then use a perturbation theory result [8] to calculate θ to first order in D . We get $\theta = \theta_0 + D\theta_0^2 I_d/\pi + O(D^2)$ for

all $2 < d \leq 4$, where I_d is a complicated d -dependent integral. For special values of d , this integral simplifies [9]. For example, for $d = 4$, we get $\theta = 1/2 + (2\sqrt{2} - 1)D/4 + O(D^2)$ and for $d = 3$, $\theta = 1/4 + 0.183615 \dots D + O(D^2)$.

For $D > 2$, it is evident from Eq. (6) that one needs to keep a nonzero cutoff a in order that the integral does not diverge at $t_1 = t_2$. In this case, for large t_1 , the dominant contribution to the integral comes from the regime $t' \rightarrow t_1$. It is easy to see that in the limit when t_1, t_2 are both large with their difference $|t_1 - t_2|$ fixed, the autocorrelator in Eq. (6) reduces to a stationary one, $C(t_1, t_2) \approx B_1 (|t_1 - t_2| + a^2)^{-(D-2)/2}$, where B_1 is an unimportant constant. One can then invoke the Newell-Rosenblatt theorem [7] once again to conclude that for large t , the persistence $P(t) \sim \exp(-\kappa_1 t)$ if $D > 4$ and $\exp(-\kappa_2 t^{(D-2)/2} \ln t) \leq P(t) \leq \exp(-\kappa_3 t^{(D-2)/2})$ for $2 < D < 4$, where the κ_i 's are constants. Combining these results we thus find that the $n \rightarrow \infty$ results for the persistence of manifold magnetization in $2 < d \leq 4$ are also compatible with our general results in Eq. (1) on noting that $\zeta = (D - 2)/2$ since $\eta = 0$ and $z = 2$ within the large n limit.

Taking hints from the two solvable cases above we now construct a general scaling theory valid for all $d \geq 2$. The two-point correlation function of the order parameter, at the critical point, has the generic scaling form $\langle \phi(\mathbf{0}, t_1) \phi(\mathbf{x}, t_2) \rangle \sim x^{-(d-2+\eta)} F(x t_1^{-1/z}, t_2/t_1)$ for large distance x and large times t_1, t_2 , where η and z are the standard critical exponents defined in the introduction. This indicates that in the Fourier space, $\langle \tilde{\phi}(\mathbf{K}, t_1) \tilde{\phi}(-\mathbf{K}, t_2) \rangle \sim K^{-(2-\eta)} G(K t_1^{1/z}, t_2/t_1)$, where \mathbf{K} is a d -dimensional vector conjugate to \mathbf{x} . The manifold magnetization ψ is obtained by summing the order parameter ϕ over d' directions. This is equivalent to putting $K_i = 0$ along the $i = 1, \dots, d'$ directions. One then obtains the scaling behavior of the two-point correlator of the manifold magnetization, $\langle \tilde{\psi}(\vec{k}, t_1) \tilde{\psi}(-\vec{k}, t_2) \rangle \sim k^{-(2-\eta)} g(k t_1^{1/z}, t_2/t_1)$, where \vec{k} is now a $D = d - d'$ -dimensional vector. For example, within the mean-field theory, the scaling function $g(x, y) = \exp[-x^2 |1 - y|] - \exp[-x^2 (1 + y)]$, as evident from Eq. (3) after dropping the Δ dependent term at late times. The autocorrelation function $C(t_1, t_2) = \langle \psi(\vec{r}, t_1) \psi(\vec{r}, t_2) \rangle$ is then obtained by integrating over \vec{k} ,

$$C(t_1, t_2) = \int \frac{d^D k}{k^{2-\eta}} g(k t_1^{1/z}, t_2/t_1) e^{-k^2 a^2}, \quad (8)$$

where a is the soft ultraviolet cutoff as before.

Consider first the case when $D - 2 + \eta < 0$. One can then set the cutoff $a = 0$ (since the integral in Eq. (8) is convergent at the upper limit), and one obtains $C(t_1, t_2) \sim t_1^{-(D-2+\eta)/z} f(t_2/t_1)$ in the limit $t_1, t_2 \rightarrow \infty$ with t_2/t_1 arbitrary. Note that the function $f(x) \sim x^{-\lambda_c/z}$ for large x such that $C(t_1, t_2) \sim t_2^{-\lambda_c/z}$ for

$t_2 \gg t_1$, where λ_c is the standard autocorrelation exponent [2,10,11]. This nonstationary Gaussian correlator can then be reduced, as before, to a stationary one for the normalized variable $X = \psi/\sqrt{\langle\psi^2\rangle}$ in the logarithmic time, $T = \ln t$, and one gets $A(T) = \langle X(0)X(T) \rangle = \exp[(D-2+\eta)T/2z]f(e^T)/f(1)$. Since $A(T) \sim \exp\{-[\lambda_c - (D-2+\eta)/2]T/z\}$ for large T , it follows, as before, that the persistence $P(T) \sim \exp(-\theta T)$ for large T . This means that the persistence decays as a power law in the original time variable $t = e^T$, $P(t) \sim t^{-\theta}$ for large t .

In the complementary case $D-2+\eta > 0$, the integral in Eq. (8) is, for $t_1 = t_2$, divergent near the upper limit without the cutoff. Hence one needs to keep a non-zero and then the appropriate scaling limit is obtained by taking t_1, t_2 both large keeping their difference $|t_1 - t_2|$ fixed but arbitrary. Then one can replace the scaling function $g(kt_1^{1/z}, t_2/t_1)$ in Eq. (8) by another function $g_1(k|t_1 - t_2|^{1/z})$ of a single scaling variable, as in the two previous solvable cases. Performing the integration, one then finds $C(t_1, t_2) \sim |t_1 - t_2|^{-(D-2+\eta)/z}$ for $|t_1 - t_2| \gg a^2$. This correlator is stationary and decays as a power law. Invoking the Newell-Rosenblatt theorem once more, we find that $P(t)$ decays exponentially for $(D-2+\eta)/z > 1$ and as a stretched exponential for $0 < (D-2+\eta)/z < 1$. Combining this with the result for $D-2+\eta < 0$ outlined in the previous paragraph gives our general result in Eq. (1) on defining $\zeta = (D-2+\eta)/z$.

To test the analytical predictions (i) and (ii), we have done preliminary Monte Carlo simulations for $d=2$ and $d=3$ Ising models for small size lattices with periodic boundary conditions. The results are summarized in Fig. 1. The systems were evolved using heat-bath Monte Carlo dynamics at their bulk critical couplings: $K_c = [\ln(1+\sqrt{2})]/2$ in $d=2$ and $K_c \approx 0.221656$ in $d=3$. For $d=2$, the data for the persistence of the line magnetization for system sizes $L=63, 95$, and 127 (in each case the data was averaged over 1000 samples) shows a power-law decay $P(t) \sim t^{-\theta}$ for $t \ll L^z$ and crosses over to a faster decay for $t \gg L^z$. A fit to the linear part of the data on the log-log plot gives an estimate $\theta \approx 0.72$. Similarly the persistence for the plane magnetization in $d=3$ shows a power-law decay $P(t) \sim t^{-\theta}$ with $\theta \approx 0.88$, estimated from small lattice sizes $L=15$ and 31 . The estimates of these exponents are only rough and may shift a little with larger lattices. The persistence of the line magnetization in $d=3$, in contrast, has a much faster decay. Note that our theory predicts a decay, $P(t) \sim \exp(-a_1 t^\zeta)$, where $\zeta \approx 0.016$. Such a small stretching exponent is difficult to determine from the small size lattices due to the strong finite-size effects at late times. All we can say is that the present data for the line magnetization in $d=3$ are consistent with a decay of persistence that is faster than a power law but slower than an exponential. More extensive simulations and a somewhat more sophisticated finite-size scaling analysis are

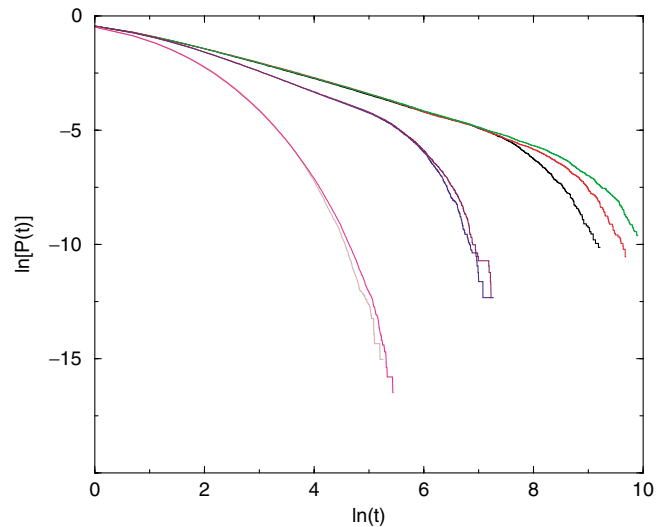


FIG. 1 (color online). The three outer curves represent the persistence of the line magnetization in the $d=2$ critical Ising model as a function of time for system sizes $L=63, 95$, and 127 . The two curves in the middle represent the persistence of the plane magnetization in the $d=3$ critical Ising model for system sizes $L=15$ and 31 . For the same system sizes, the two leftmost curves represent the persistence of the line magnetization in $d=3$.

required to pin down the precise value of the stretching exponent [9].

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