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FAST TRACK COMMUNICATION

Statistics of layered zigzags: a two-dimensional generalization of TASEP

Mikhail Tamm[1], Sergei Nechaev[2,3] and Satya N Majumdar[2]

[1] Physics Department, Moscow State University, 119992, Moscow, Russia
[2] Laboratoire de Physique Théorique et Modèles Statistiques, Université Paris-Sud, 91405 Orsay Cedex, France
[3] PN Lebedev Physical Institute of the Russian Academy of Sciences, 119991, Moscow, Russia

E-mail: tamm@polly.phys.msu.ru

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Abstract
A novel discrete growth model in 2+1 dimensions is presented in three equivalent formulations: (i) directed motion of zigzags on a cylinder, (ii) interacting interlaced TASEP layers and (iii) growing heap over 2D substrate with a restricted minimal local height gradient. We demonstrate that the coarse-grained behavior of this model is described by the two-dimensional Kardar–Parisi–Zhang equation. The coefficients of different terms in this hydrodynamic equation can be derived from the steady state flow-density curve, the so-called fundamental diagram. A conjecture concerning the analytical form of this flow-density curve is presented and is verified numerically.

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It is well established in the last two decades [1–4] that the one-dimensional Kardar–Parisi–Zhang (KPZ) equation [5] adequately describes the long-range dynamics of a collective motion of hopping particles on a line known as the 'asymmetric simple exclusion process' (ASEP) [6–9]. Apart from many fruitful theoretical advantages, this ASEP-to-KPZ mapping enables a fast and simple way of modeling the KPZ dynamics. The latter is of wide interest since it appears in various contexts (provided the symmetry of a nonequilibrium statistical system under discussion allows for the effective (1+1)-dimensional description), including, to name but a few, the models of crystal growth [10], molecular beam epitaxy [11], Burgers' turbulence [12, 13], polynuclear growth [14–18], ballistic deposition [19–22], etc.

It is therefore an appealing idea to seek for a similar simple discrete multi-particle system whose long-range dynamics would be governed by a two-dimensional KPZ-type equation. Lately, several models of the desired nature, i.e. ones which combine the discreteness with the long-range KPZ-type dynamics, were suggested [23–28]. In this communication we propose another model belonging to this class, which, in our opinion, combines the advantage of physical transparency with the flexibility of tuning the internal parameters of the model to

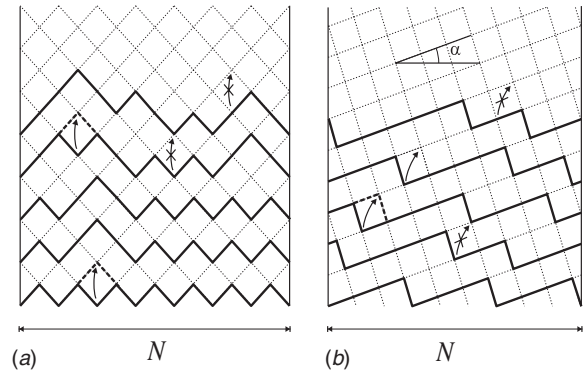

Figure 1. The zigzags on a cylinder for different tilt angles $\alpha = \pi/4$ (a) and $\alpha = \arctan(1/3)$ (b); the examples of allowed and forbidden moves are shown. The system is periodic in the horizontal direction, and the stationary flow is obtained for the periodic boundary condition in the vertical direction.

catch the different desired regimes both in 1+1 and in 2+1 dimensions. This model, which we call the 'zigzag model', has a simple geometrical formulation.

Take an infinite cylinder covered by a tilted square grid as shown in figures 1(a) and (b) and consider a directed closed path ('zigzag') around a cylinder. Any such path consists of a constant number of rises and descents, constituting 'kinks' (here and below we conventionally define rises and descents with respect to a rightward step). The density of descents, ρ, is defined by the tilt angle α—see figures 1(a) and (b) where this density equals $\rho = 1/2$ (for $\tan\alpha = 1$) and $\rho = 1/4$ (for $\tan\alpha = 1/3$), respectively. Consider evolution of a system of such nonintersecting zigzags. At each infinitesimal time step dt any elementary kink oriented downward, can turn upward with probability $p\,\mathrm{d}t$ under the condition that such a move is not blocked by the upper nearest-neighboring zigzag (i.e. all zigzags stay nonintersecting at all times). By an appropriate rescaling of time, t, we set $p \equiv 1$. The examples of elementary jumps which are allowed and those which are blocked are shown in figure 1.

For better understanding of the dynamics of the model, note that the evolution of a separated zigzag (if, for the time being, we neglect its interaction with the other ones) can be interpreted as a hopping dynamics in a standard one-dimensional totally asymmetric simple exclusion process (TASEP) [2]—see figure 2 for the corresponding mapping, which is both conventional and self-explanatory. Essentially, a descent (rise) in a zigzag is identified with a particle (hole) in the corresponding TASEP. Therefore, the set of zigzags can be viewed as a system of interacting TASEP layers.

The connection between zigzags and TASEP layers is shown in figure 2(a). Here, there are two sorts of constraints on the movement of the particles in the layer B_2. Indeed, for a jump to be possible at some point of the A_2 zigzag, two conditions should be simultaneously fulfilled: (i) it should be a downward kink, and (ii) the movement should not be blocked by the upper zigzag. The translation of the first condition into the TASEP language is convenient: there should be a particle at a given position in B_2 and a void immediately to the right of it. In turn, the second rule (i.e. the one describing the interaction of the B_1 and B_2 layers) can be formulated as follows: it is possible to label the particles in the layers B_1 and B_2 in a way such that the kth particle in the layer B_2 can never surpass the particle with the same label (i.e. the kth one) in the upper layer B_1, giving rise to an 'interlaced TASEP' picture shown in figure 2(b) [4].

[4] Note that the described system of interlaced TASEPs bears many similarities with the one proposed by Borodin and Ferrari in [25–27], with the exception that there is no 'push-TASEP' regime in our model.

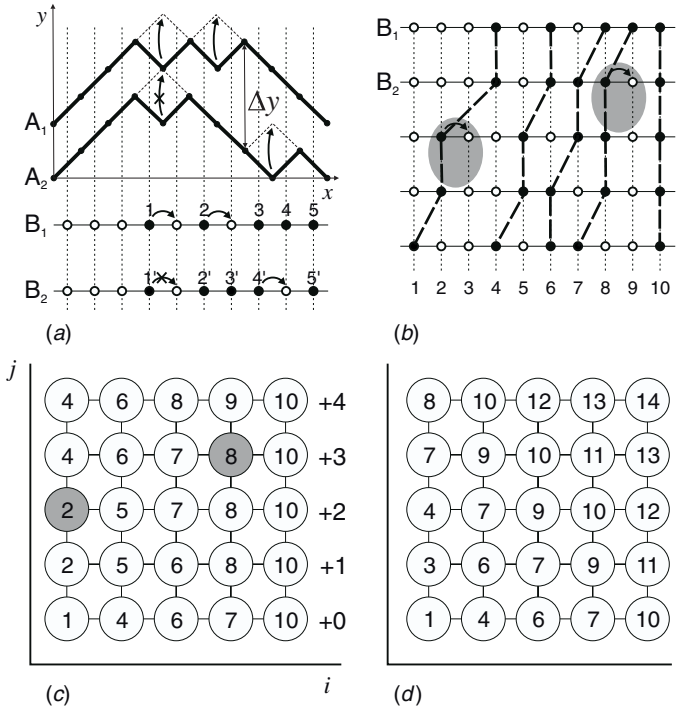

Figure 2. The interconnections between zigzag, interacting TASEP and tag diffusion representations of the model. (*a*) Interaction of two zigzags and the two corresponding TASEP layers; examples of the same allowed and forbidden moves are shown in both representations; (*b*) interacting TASEP layers with examples of allowed moves; the bold dashed lines connect the particles with the same number (see the explanation in the text); (*c*) the system of tagged particles corresponding to the TASEP layers depicted in figure (*b*), with the same allowed moves highlighted; (*d*) the result of the shift $h_{i,j} \to h_{i,j} + j$.

To prove these jumping rules, consider the distance Δy between the zigzags A_1 and A_2 (see figure 2(*a*)). At two adjacent spatial positions it satisfies

$$\Delta y(x) = \begin{cases} \Delta y(x-1)+1 & \text{there is a particle at } x-1/2 \\ & \text{on } B_2 \text{ and a void on } B_1 \\ \Delta y(x-1)-1 & \text{there is a void at } x-1/2 \\ & \text{on } B_2 \text{ and a particle on } B_1 \\ \Delta y(x-1) & \text{otherwise} \end{cases} \tag{1}$$

At the same time, $\Delta y(x) \geqslant 1$ at all positions. Therefore, for each particle on the upper line there is a 'partner' particle on the lower line that cannot surpass it, and this 'partnership' is preserved by the elementary moves. Indeed, if there is a particle on the upper line A_1 in the position $x - 1/2$ (particle positions are shifted by half-step as compared to the positions of the kinks) and the zigzag-to-zigzag distance at x is $\Delta y(x)$, then each particle on A_2 jumping from $x - 1/2$ to $x + 1/2$ will decrease Δy by 1, and, therefore, up to $(\Delta y - 1)$ particles can pass this point without being affected, while the particle number Δy, which is the desired 'partner' particle, will get stacked at $x - 1/2$. Moreover, assume now that the upper particle hops from

$x - 1/2$ to $x + 1/2$. For this move to take place, the position $x + 1/2$ should be a void before it thus, (compared to equation (1))

$$\Delta y(x+1) = \begin{cases} \Delta y(x) + 1 & \text{there is a particle at } x + 1/2 \\ & \text{on } B_2 \\ \Delta y(x) & \text{otherwise} \end{cases} \tag{2}$$

and $\Delta y(x + 1)$ does not change as a result of the move. In both cases of (2) the 'partner' particle after the move is still the same: either the distance and the order of particles do not change, or the distance increases but the $(\Delta y + 1)$th particle as viewed from the new position is the same as the Δyth particle from the old position.

In the case of a finite cylinder with periodic boundary conditions the 'partner' particle can, in principle, lag behind by a whole lap. In this case, one should ensure that the two particles do not interact even if formally they are at the same place, so one should keep track of the 'real' distances between the particles (i.e. those which correspond to the distances between the kinks on the original cylinder), not the 'apparent' modulo N distances (compare with [29]).

There is now evidence of some spatial symmetry in the model: for each given particle there are exactly two other particles, one to the right and one, with the same number, 'on top' of it (see the dashed lines connecting the particles of the same number in figure 2(*b*)), which can block its movement via excluded-volume interaction. To better exploit this symmetry, it is convenient to reformulate the model, following the logic of the so-called tagged particle diffusion introduced in [3] in order to show that on a coarse-grained level the ASEP dynamics is subject to the one-dimensional KPZ equation.

Consider a set of m TASEP layers (zigzags) of length N with n particles within each layer. Note that instead of enumerating the sites of TASEP layers and marking which particular sites are filled with particles, one can store the very same information in a different way by enumerating the particles with two indices $i \in [1, n]$, $j \in [1, m]$, and ascribing to each particle a 'height' $h_{i,j}$ equal to its position on the corresponding layer (as measured from some arbitrary chosen 'first site'). It is clear now that locally the values in the $h_{i,j}$ matrix are increasing in the i direction and non-decreasing in the j direction. To make the model completely symmetric make the transformation (compare [30]) $h_{i,j} \to h_{i,j} + j$ which (see figures 2(*c*) and (*d*)) ensures that $h_{i,j}$ is now an increasing function in both directions, i and j. Under this transformation the dynamic rules become totally symmetric:

$$h_{i,j} \to \begin{cases} h_{i,j} + 1 & \text{with probability } \mathrm{d}t, \\ h_{i,j} & \text{otherwise} \end{cases} \qquad \text{if } \begin{Bmatrix} h_{i+1,j} > h_{i,j} + 1 \\ h_{i,j+1} > h_{i,j} + 1 \end{Bmatrix}. \tag{3}$$

One can interpret these rules as the dynamics of a heap growing over a two-dimensional substrate, with an additional constraint of heap gradient being not less than 1 in each of the transverse directions.

To obtain insights about the large scale dynamics it is useful to consider a coarse-grained hydrodynamic description of the model [8]. To describe the model at a coarse-grained level we will henceforth use (x, y) as the spatial coordinates instead of (i, j) in the lattice model. The coarse-grained dynamics can be described in terms of a local 'smooth' velocity field $u(x, y, t)$ which is, in fact, nothing but the average (over spacetime element $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t$) rate of successful hops $h_{i,j} \to h_{i,j} + 1$, i.e. the average probability to find simultaneously $(h_{i+1,j} - h_{i,j}) > 1$ and $(h_{i,j+1} - h_{i,j}) > 1$. This value obviously depends on the average slope (or equivalently on the local densities in both directions in the zigzag model) of the surface in both directions. Similar to the one-dimensional case [4, 8] we assume that the velocity field $u(x, y, t)$ depends on the spacetime coordinates only through the local slopes, i.e.

$$u(x, y, t) \equiv u(\rho_x(x, y, t), \rho_y(x, y, t)) \tag{4}$$

where $\rho_x^{-1}(x, y, t) = \frac{\partial h(x,y,t)}{\partial x}$ and $\rho_y^{-1}(x, y, t) = \frac{\partial h(x,y,t)}{\partial y}$. Note that the value ρ_x introduced here has a meaning of particle density in the corresponding TASEP layer.

In order to get the continuum equation for the fluctuating interface, we proceed as was prescribed in [31, 32] for the (1+1)D case. Namely, we suppose that the slopes ρ_x and ρ_y are weakly fluctuating around some average values $\overline{\rho}_x$ and $\overline{\rho}_y$, which are determined by the boundary conditions. Now coarse grain the particle labels so that the discrete tag labels i, j become the continuous tag variables x, y, and divide x and y by corresponding particle densities. Clearly the mean height at (x, y) is given by $\overline{h}(x, y) = \frac{x}{\overline{\rho}_x} + \frac{y}{\overline{\rho}_y}$ around which the height field $h(x, y, t)$ smoothly fluctuates. We denote this fluctuation by $d(x, y, t)$ which also denotes the displacement $d(x, y, t)$ of a particle located at (x, y) from its average position at $t = 0$. Thus

$$d(x, y, t) = h(x, y, t) - \frac{x}{\overline{\rho}_x} - \frac{y}{\overline{\rho}_y}. \tag{5}$$

The goal now is to write a stochastic hydrodynamic equation of motion for this height fluctuation $d(x, y, t)$. Clearly, the rate of growth $\partial_t d(x, y, t)$ will have two components: (i) a first part that arises due to local density fluctuations and local noise in the dynamics and is completely independent of the local drift (this term will be present even, e.g., in the symmetric exclusion process) and (ii) a second deterministic part that is induced by the local asymmetric drift present in the microscopic model which is simply $u(x, y, t)$ in the continuum description. In the simplest description one assumes that the first part is simply diffusive in the presence of a white noise (in the height description this gives rise to the Edwards–Wilkinson model that corresponds to the continuum description of the symmetric exclusion process). Thus within this description,

$$\frac{\partial d}{\partial t} = D\Delta d + \eta + u(\rho_x, \rho_y) \tag{6}$$

where we have assumed isotropic diffusion with the diffusion constant D and $\eta(x, y, t)$ is the white noise

$$\langle \eta(x, y, t) \rangle = 0, \ \langle \eta(x, y, t)\eta(x', y', t') \rangle = \delta_{x,x'}\delta_{y,y'}\delta_{t,t'}. \tag{7}$$

Note that even if the diffusion is anisotropic with diffusion constants D_i $(i = 1, 2)$, one can always rescale $x \to x\sqrt{D/D_1}$ and $y \to y\sqrt{D/D_2}$ to make the diffusion term isotropic. The white noise is chosen for simplicity and is enough (and minimal) to lead to an equilibrium Gaussian distribution for the height fluctuations $P_{\rm eq}[d] \propto \exp[-\int {\rm d}x\, {\rm d}y (\nabla d)^2]$ in the absence of the drift field $u(\rho_x, \rho_y)$.

The next step is to express the drift field $u(\rho_x, \rho_y)$ in terms of the displacement field $d(x, y, t)$ to obtain a closed equation. To proceed, we note that the local values of $\rho_{x,y}$ and d are connected via

$$\frac{1}{\rho_x(x, y, t)} = \frac{1}{\overline{\rho}_x} + \frac{\partial d(x, y, t)}{\partial x}; \qquad \frac{1}{\rho_y(x, y, t)} = \frac{1}{\overline{\rho}_x} + \frac{\partial d(x, y, t)}{\partial y}. \tag{8}$$

Assuming now $\left|\frac{\partial d}{\partial x}\right| \ll 1$ and $\left|\frac{\partial d}{\partial y}\right| \ll 1$, one can expand both $\rho_{x,y}$ and $u(\rho_x, \rho_y)$ as power series in the derivatives of d, obtaining up to the second order:

$$\begin{aligned} \rho_x(x, y, t) &= \overline{\rho}_x \left(1 - \overline{\rho}_x \frac{\partial d}{\partial x} + (\overline{\rho}_x)^2 \left(\frac{\partial d}{\partial x}\right)^2 + \cdots\right) \\ \rho_y(x, y, t) &= \overline{\rho}_y \left(1 - \overline{\rho}_y \frac{\partial d}{\partial y} + (\overline{\rho}_y)^2 \left(\frac{\partial d}{\partial y}\right)^2 + \cdots\right) \end{aligned} \tag{9}$$

which when substituted into (8) gives

$$\begin{aligned} u(\rho_x, \rho_y) = u(\overline{\rho}_x, \overline{\rho}_y) &- \overline{\rho}_x^2 u_x(\overline{\rho}_x, \overline{\rho}_y)\frac{\partial d}{\partial x} - \overline{\rho}_y^2 u_y(\overline{\rho}_x, \overline{\rho}_y)\frac{\partial d}{\partial y} \\ &+ \left(\overline{\rho}_x^3 u_x(\overline{\rho}_x, \overline{\rho}_y) + \frac{1}{2}\overline{\rho}_x^4 u_{xx}(\overline{\rho}_x, \overline{\rho}_y)\right)\left(\frac{\partial d}{\partial x}\right)^2 \\ &+ \left(\overline{\rho}_y^3 u_y(\overline{\rho}_x, \overline{\rho}_y) + \frac{1}{2}\overline{\rho}_y^4 u_{yy}(\overline{\rho}_x, \overline{\rho}_y)\right)\left(\frac{\partial d}{\partial y}\right)^2 \\ &+ \overline{\rho}_x^2\overline{\rho}_y^2 u_{xy}(\overline{\rho}_x, \overline{\rho}_y)\frac{\partial d}{\partial x}\frac{\partial d}{\partial y} + \cdots \end{aligned} \tag{10}$$

where for brevity we introduced the notation u_x, u_y, etc for the partial derivatives $u_x = \frac{\partial u}{\partial \rho_x}$, $u_y = \frac{\partial u}{\partial \rho_y}$, ...

This then allows us to finally write down a time-dependent differential equation for d which belongs to the two-dimensional anisotropic KPZ class:

$$\begin{aligned} \frac{\partial d}{\partial t} &= D\Delta d + \eta + u(\rho_x, \rho_y) = D\Delta d + \eta + u(\overline{\rho}_x, \overline{\rho}_y) \\ &\quad - \sum_{\alpha=x,y} a_\alpha \frac{\partial d}{\partial \alpha} + \sum_{\alpha=x,y}\sum_{\beta=x,y} b_{\alpha\beta}\frac{\partial d}{\partial \alpha}\frac{\partial d}{\partial \beta}, \end{aligned} \tag{11}$$

where a_α, $b_{\alpha\beta}$ for $\{\alpha, \beta\} = x, y$ are

$$\begin{aligned} a_\alpha &= \overline{\rho}_\alpha^2 u_\alpha(\overline{\rho}_x, \overline{\rho}_y), \\ b_{\alpha\alpha} &= \overline{\rho}_\alpha^3 u_\alpha(\overline{\rho}_x, \overline{\rho}_y) + \tfrac{1}{2}\overline{\rho}_\alpha^4 u_{\alpha\alpha}(\overline{\rho}_x, \overline{\rho}_y), \\ b_{xy} &= \overline{\rho}_x^2\overline{\rho}_y^2 u_{xy}(\overline{\rho}_x, \overline{\rho}_y). \end{aligned} \tag{12}$$

The behavior of the system is controlled by the usual flow-density dependence $u(\rho_x, \rho_y)$, which is central in all models of traffic (see, for example, [33]) and is usually referred to as 'the fundamental diagram'. Indeed, the flow of the particles in the interlaced TASEP formulation of the zigzag model is equal to $I(\rho_x, \rho_y) = \rho_x u(\rho_x, \rho_y)$. Recall that in the 1D case the function $u(\rho) = I(\rho)/\rho$ is just $u = 1 - \rho$ and it can be obtained by the mean-field arguments. In the absence of interaction between layers one would have expected the same dependence in our model:

$$u(\rho_x, \rho_y) = 1 - \rho_x. \tag{13}$$

In the presence of interaction the corresponding 2D mean-field result would be

$$u(\rho_x, \rho_y) = (1 - \rho_x)(1 - \rho_y) \tag{14}$$

where the symmetry of the model is taken into account: a particle can hop if there is a void both in front (with probability $1 - \rho_x$) and on top (with probability $1 - \rho_y$) of it and the horizontal and vertical jumps are supposed to be independent. However, contrary to the 1D case, this result is not exact. Indeed, the connectivity of the surface dictates that the local increments $(h_{i+1,j} - h_{i,j})$ and $(h_{i,j+1} - h_{i,j})$ are positively correlated and cannot be considered as independent.

The exact analytical evaluation of the velocity $u(\rho_x, \rho_y)$ up to now is beyond our reach. We have made numerical simulations of $u(\rho_x, \rho_y)$, the results being presented in figure 3. The simulations were performed for the systems of size $N = 32$ in both directions with periodic boundary conditions ensuring that the average densities take the

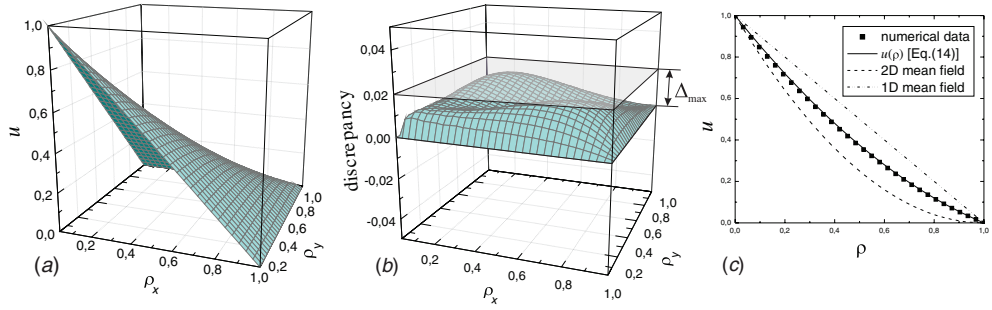

Figure 3. Comparison of the theoretical and numerical results for $u(\rho_x, \rho_y)$. (*a*) $u(\rho_x, \rho_y)$ dependence as measured by direct numerical simulations (see the main text for the details about the simulations); (*b*) the discrepancy between the numerical result for u and the one conjectured in equation (16); (*c*) the $u(\rho)$ behavior in the $\rho = \rho_x = \rho_y$ plane: the points correspond to numerical results, while the dashed line is the naive mean-field guess $u = (1-\rho)^2$ (compare equation (14)) and the bold line is the conjectured dependence $u = (1-\rho)(1-\rho/2)$ (compare equation (16)).

(This figure is in colour only in the electronic version)

values $\overline{\rho}_x = \overline{\rho}_y = 0, \frac{1}{31}, \frac{2}{31}, \ldots, 1$.[5] The results were averaged over 9×10^6 Monte Carlo steps. To make the comparison with the mean field more visually compelling, we plot in figure 3(*c*) separately the numerical data for $\rho_x = \rho_y = \rho$ together with the mean-field results (13) and (14). As expected, the first of them overestimates the flow, while the second one underestimates it. In fact, the numerical data fits perfectly the form

$$u(\rho) = (1-\rho)(1-\rho/2) \tag{15}$$

and the consideration of limiting cases $\rho \to 0$ and $\rho \to 1$ suggests that this result may be exact. In particular by developing the perturbation theory at high densities, we are able prove that $u(\rho) \to (1-\rho)/2$ as $\rho \to 1$; the details of these computations will be published separately [34].

The simplest generalization of equation (15) onto the case of $\rho_x \neq \rho_y$ which respects the boundary conditions $u(0,0) = 1$, $u(1,\xi) = u(\xi,1) = 0$ for any $\xi \in [0,1]$ reads

$$u(\rho_x, \rho_y) = (1-\rho_x)(1-\rho_y)\left(1 + \frac{2\rho_x\rho_y}{(\rho_x+\rho_y)(2-\rho_x-\rho_y)}\right). \tag{16}$$

In figure 3(*b*) we have plotted the discrepancy $\Delta u(\rho_x, \rho_y) = u_n(\rho_x, \rho_y) - u(\rho_x, \rho_y)$ between numerical, $u_n(\rho_x, \rho_y)$ and conjectured, $u(\rho_x, \rho_y)$, (see (16)) functions. One sees that $u(\rho_x, \rho_y)$ is in very good agreement with the results of numerical simulations.

The conjectured function $u(\rho_x, \rho_y)$ allows us to evaluate the coefficients in (12) for any ρ_x and ρ_y and to calculate the eigenvalues of the coefficient matrix $B\{b_{\alpha\beta}\}$ in front of the nonlinear term in equation (11). We thus show (see figure 4) that the domain $[0 < \rho_x < 1,\ 0 < \rho_y < 1]$ can be separated into two regions: (i) a region where one of the eigenvalues of the matrix B dominates ($|\lambda_1| \gg |\lambda_2|$) signaling a quasi-one-dimensional behavior, and (ii) a region where $|\lambda_1| \sim |\lambda_2|$ corresponding to the truly two-dimensional KPZ dynamics. In the latter region both eigenvalues are negative, and thus the matrix B is positive definite.

[5] In terms of the original zigzag model our choice of boundary conditions corresponds to consideration of the system on a torus so that each zigzag consists of N steps, and there is a room for at most N zigzags on the torus. The densities in the system of m zigzags each of those has n descents are defined by $\rho_y = (m-1)/(N-1)$; $\rho_x = (n-1)/(N-1)$ (i.e. ρ_x and ρ_y are the average densities in the vicinity of a particle excluding the influence of the particle itself, compare the exact solution for finite TASEP ring).

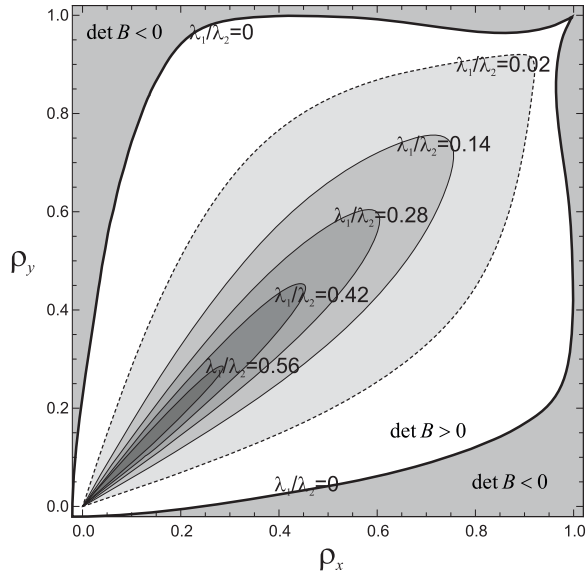

Figure 4. Density plot of the ratio λ_1/λ_2 of the eigenvalues of the matrix B. In the region det $B < 0$ the eigenvalues have opposite signs. Outside the dashed area the system can be regarded as effectively one-dimensional since $\lambda_1 \gg \lambda_2$.

Summing up, we have presented a novel model of a statistical driven system in 2+1 dimensions which turned out to be a direct generalization of the conventional one-dimensional TASEP model. In the hydrodynamic limit we derived the differential equation for the particle displacement in this model, and we have made a conjecture concerning the flow-on-density dependence of the model (the so-called fundamental diagram). Clearly, the model suggested here deserves further investigations in various directions. In particular, a detailed investigation of the limiting cases $\rho_x = \rho_y \to 0$ and $\rho_x = \rho_y \to 1$ can give valuable understanding of the flow-density dependence; the study of the displacement fluctuations in different regimes (compared to [3]) should be very fruitful, and the problem of proving conjecture (16) or at least (15) sounds quite challenging. Some progress in this direction will be reported in a forthcoming longer paper [34].

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