CV & Research Statements

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1 Curriculum-Vitae (brief CV)

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39, rue Ducouedic,
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Date of Birth: 01-01-1966

Education:

08/87 – 09/92 **Tata Institute of Fundamental Research** **Bombay, India**
Ph.D. in Physics, September 1992.
Thesis: Self-organized Criticality in Sandpiles and Driven Diffusive Lattice Gases.
Advisor: Prof. Deepak Dhar, Tata Institute, Bombay, India.

09/85 – 07/87 **University of Calcutta** **Calcutta, India**
M.Sc in Physics, 1987 (ranked 1st among approx. 200)

09/81 – 08/85 **Presidency College** **Calcutta, India**
B.Sc in Physics, University of Calcutta, 1985 (ranked 1st among approx. 2500)

Employment:

10/11 – present: Directeur de Recherche (DR1) at **Laboratoire de Physique Théorique et Modèles Statistiques, Université Paris-Sud, Orsay, France**.

09/03 – 09/11: Directeur de Recherche (DR2) at **Laboratoire de Physique Théorique et Modèles Statistiques, Université Paris-Sud, Orsay, France**.

01/00 – 09/03:	Chargé de Recherche (CR1) at **UMR 5626 du CNRS, IRSAMC, Université Paul Sabatier, Toulouse, France**.
11/96 – 12/99	Reader at **Tata Institute of Fundamental Research, Bombay, India**.
10/94 – 10/96	Post Doctoral Associate at **Yale University, USA**.
10/92 – 09/94	Post Doctoral fellow at **AT&T Bell Labs, USA**.
08/87 – 09/92	Thesis student and Research Assistant at **Tata Institute of Fundamental Research, Bombay, India.**

Honors and Awards:

1. **Gay-Lussac Humboldt** prize (awarded by Alexander von Humboldt foundation, 2019).

2. **EPS** prize for Statistical and Nonlinear Physics (2019) (jointly with S. Ciliberto).

3. **CNRS** silver medal (medaille d'argent) (2019).

4. **VAJRA** fellowship (Visiting Adjoint Research Associate) awarded by the ministry of Science and Technology (Govt. of India) in 2018.

5. **Plenary** speaker at STATPHYS-25 (Seoul, South Korea, 2013).

6. **Plenary** speaker at 'Extreme Value Analysis' (EVA 2011) (Lyon, France, 2011).

7. **'Prime d'excellence Scientifique'** (PES) (2009-2012, 2013-2016, 2017-) awarded by CNRS.

8. **'Excellence Award'** (2009) for outstanding contributions to statistical physics, awarded by the Tata Institute Alumni Association.

9. **Paul Langevin Medal** for theoretical physics (2005) awarded by the French physical society.

10. **Young Scientist Medal** awarded by the Indian National Science Academy, 1998.

11. **Geeta Udgaonkar Award**, 1992 for outstanding thesis in the school of Physics (Tata Institute, Bombay, India).

12. **Calcutta University Gold Medal** for securing first position in M.Sc (Calcutta University, India, 1987).

Editor of Journals:

• Divisional Associate Editor (DAE) of Phys. Rev. Lett. (Jan. 2011-Dec. 2013).

• Associate editor of the Journal of Statistical Physics (since 2011). Previously, member of the editorial board of Journal of Statistical Physics (2008-2010).

• Member of the executive board of J. Phys. A: Math. Theor. since 2020. Previously, the chief Section editor (Statistical Physics) of the Journal of Physics A: Math. Theor. since 2019. Member of the editorial board (2010-2016). Since 2016, member in the panel 'Fast track communications' in J. Phys. A: Math. Theor. (2016–)

• Member of the editorial board of Journal of Statistical Mechanics: Theory and Experiment (since 2003).

Honorary Positions:

• Adjunct Professor at Tata Institute, Bombay (India) since 2005.

• Adjunct Weston Professor at the Weizmann Institute, Rehovot (Israel) since 2011.

• Adjunct Professor at the International Center for Theoretical Sciences (ICTS), Bangalore, (India) since 2011.

• Associate at the Higgs Center, the University of Edinburgh (UK) since 2012.

• Adjunct Professor at the Raman Research Institute (RRI), Bangalore, (India) since 2015.

• Visiting Simon chair at the International Center for Theoretical Sciences (ICTS), Bangalore, (India) during September-October, 2017.

Publications and Invited Talks:

• **309** publications in reviewed journals (including **68** PRL's, 1 Science, 1 PNAS, 1 Adv. in Phys.), 4 conference proceedings and 9 invited reviews/book chapters. *Total number of citations: 16042/10924 (source: Google Scholar (GS)/ISI web of Science (ISI), dated 06/07/2021) with an h-index:* 66/54 *(GS/ISI).*

• **152** invited talks in international conferences/workshops/summer schools since (1996).

• **Named Lectures:**

(i) M. L. Mehta memorial lecture (February, 2011) at the Tata Institute of Fundamental Research, Bombay, India.

(ii) Subhramanyam Chandrasekhar lectures (January, 2012) at the International Center for Theoretical Sciences (ICTS), Bangalore, India.

(iii) Higgs colloquium at the Higgs Center, the University of Edinburgh (UK) (May, 2013).

(iv) K. Lakshmanan memorial distinguished lecture at CMI (Chennai mathematical Insitute, Chennai) (India) (January, 2017).

(v) J. Mahanty memorial lecture at the Indian Institute of Technology (IIT) (Kanpur) (India) (October, 2017).

(vi) P. S. Narayanan memorial lecture at the Indian Institute of Science (IISC) (Bangalore) (India) (December, 2019).

(vii) Infosys-Chandrasekhar colloquiuum at the Tata Insititute of Fundamental Research (Bombay) (India) (March, 2021).

Supervision of students and postdocs:

4 current Ph.D students, 14 (completed) Ph.D students (direction and co-direction), 8 M.Sc students and 8 postdocs.

Referee of Journals and Grants:

Since 1992, papers were regularly reviewed for several international journals including Physical Review Letters, Physical Reviews (A, B and E), Nature Physics, Europhysics Letters, Journal of Physics-A, Journal of Statistical Physics, Journal of Statistical Mechanics, Physica-A, IEEE transactions in Information Theory etc. The grant proposals for the Agence National de Research (ANR, France), National Science Foundation (NSF, USA) and the Israel Science Foundation (ISF, Israel) were also reviewed.

Recognized as a 'distinguished referee' by the EPL (Europhysics Letters) (2010).

Administrative Responsibilities:

I have been a member of various scientific commissions in and outside France. These include the following:

• Member of the commission de spécialistes (section 29) at the Université Paul Sabatier, Toulouse, France (2004-2009).

• Member of the 'Comité d'évaluation' of the Laboratoire de Physique Théorique (LPTENS) at Ecole Normale Superieure, January (2005).

• Member of the 'Conseil de laboratoire LPTMS' since November, 2005.

• Member of the 'Comité d'évaluation' (AERES) of the Laboratoire de Physique Théorique (LPTENS) at Ecole Normale Superieure, January (2009).

• Member of the 'Appointments Committee' and 'Management Board' of the International Center for Theoretical Studies (ICTS) at Bangalore, India, since 2011.

• Member of the comite de selection (section 29) (concours de recrutement MCF) at the Universite' Aix Marseille, April, 2012.

• Member of the comite de selection (section 29) (concours de recrutment Prof.) at the Universite' de Cergy-Pontoise, April, 2014.

• Member of the evaluation committee for IRMP (Institut de Recherche en Mathematique et Physique, Université catholique de Louvain, Belgium), March, 2018. I am also a member of the scientific advisory board of IRMP since 2018.

• Member of the advisory board of "Fundamental Problems in Statistical Physics" (FPSP) since 2015.

• Member of the International Advisory Board of Statphys 28 (to be held in Tokyo, 2022).

Research Topics :

My research interests cover various problems in equilibrium and nonequilibrium statistical physics with applications ranging from granular medium to computer science. Some of the present and past research projects are listed below.

• *Extreme Value Statistics of Strongly Correlated Variables*: Two particularly interesting strongly correlated systems where we made much progress in recent years: (i) (i) Brownian Motion and various related stochastic processes (ii) Eigenvalues in random matrices. Other questions related to extremes have also been studied, such as the statistics of the fluctuations in the positions of fermions confined in an external potential (trapped cold atoms), number of extrema in a random landscape, integer partition problem, level density of Bose gas and its relation to extreme statistics, zeroes of random polynomials and longest excursions in stochastic processes in nonequilibrium systems etc.

• *Stochastic processes with resetting*: We have shown recently that stochastic processes subject to random resetting to its initial condition leads to novel nonequilibrium steady states. Also stochastic search algorithmics become more efficient in the presence of resetting. The phenomena of **'Resetting'** has become a very popular topic in recent years with diverse applications and this field of research was initiated by myself and collaborators about 5 years back.

• *Order, Gap and Record Statistics for Stochastic Time Series:* Various other questions related to extreme statistics have also been studied, such as the order and the gap statistics, record statistics, density of near-extreme events etc. in random walks and other stochastic processes.

• *Diverse applications of Random Matrix Theory:* My current/ongoing research involves various applications of the random matrix theory such as the study of (i) fluctuations of the number of eigenvalues in a given spectral interval–the so called Index problem (ii) transport in mesoscopic cavities (ii) distribution of entanglement entropy in random pure states of bipartite systems (iv) nonintersecting Brownian motions and its connection to Yang-Mills gauge theory (v) matrix integrals and the associated fluid dynamics (vi) application of random matrix theory in cold atoms (free fermions in a harmonic trap at zero and finite temperature). Also, I have studied various integrable models related to random matrices, such as random growth models, biological sequence matching problems, random permutations, free fermions in a harmonic trap etc.

• *Applications of Statistical Physics in Computer Science: Sorting and Search Algorithms.*

• *Persistence and first-passage properties in Nonequilibrium Systems:* Spin Models, Diffusion and Random Walks, Random Search problems, Non-Markov Processes etc.

• *Stress Propagation and Compaction in Granular Medium.*

• *Real-space Condensation in Nonequilibrium Steady-States:* Aggregation and fragmentation processes, Zero-range processes, Random Average processes etc.

• *Quantum Phase Transitions in Disordered Spin Chains.*

• *Coarsening and Phase Ordering Dynamics in Spin Systems.*

• *Transport Properties of Vortices in High-T_c Superconductors.*

• *Interacting Particle Systems:* Symmetric and Asymmetric Exclusion Processes, Vicious random walkers, Trapping problems etc.

• *Polymers and Self-Avoiding Walks.*

• *Self-organized Criticality in Sandpile Models* (Ph.D Thesis, 1992).

2 Evolution of research

This section partially summarizes how my research evolved in time with highlights on some of my principal scientific contributions.

I did my Ph.D thesis (1992) titled 'Self-organized Criticality in Sandpiles and Driven Diffusive Lattice Gases' under the supervision of Prof. Deepak Dhar at Tata Institute of Fundamental Research, Bombay, India. When I started my thesis, self-organized criticality (SOC) was a relatively new field that started with the seminal paper by P. Bak, C. Tang and K. Wiesenfeld (BTW), published in 1987. The main idea behind the subject was that several interacting many-body systems evolve under their natural dynamics to a stationary state where measurable observables, such as two-point correlation functions, exhibit power-law decay. These power-law behaviors are considered as the signatures of 'criticality' and this critical state is reached without the fine tuning of any external parameter such as temperature (as is common in traditional liquid-gas or magnetic systems undergoing phase transitions). The latter fact justifies the name 'self-organized'. There are many examples of such SOC systems such as the granular systems including sandpiles and ricepiles and several reaction-diffusion systems amongst others. This area had been and is still an active area of research in statistical physics. Prior to my thesis, most of the existing work in this area were numerical in nature and a concrete analytical understanding was missing. In collaboration with my advisor D. Dhar, we discovered an abelian group structure in one of the basic models of SOC, now called the abelian sandpile model. This helped us deriving many properties of sandpile models analytically for the first time and I believe that this work formed one of the cornerstones in understanding the mathematical mechanism behind SOC. Besides, we also established a connection between the abelian sandpile model (which is inherently non-equilibrium in nature) and the random spanning tree model in equilibrium statistical physics. This connection was particularly useful in 2-d where we could then use conformal field theory to understand the statistical properties of random spanning trees and hence those of the sandpile model. To my knowledge, this was the first time that conformal field theory was used to derive exact results for a nonequilibrium system.

Another offshoot of my Ph.D thesis was to prove, again using conformal field theory in 2-dimensions, that the fractal dimension of the loop-erased random walks (LERW) was exactly 5/4. This result was first conjectured by Gutmann and Bursill in the late 80's on the basis of numerical results. Loop-erased random walks became rather fashionable later as a concrete example belonging to the class of models evolving under the stochastic Loewner equation (SLE). To my knowledge, the result 5/4 was first proved in my paper [Phys. Rev. Lett., 68, 2329 (1992)], much before the current activities in SLE started.

Following my thesis, I did my first postdoctoral research (1992-1994) at AT&T Bell Labs which at that time was arguably one of the best laboratories in the world in condensed matter physics. While my Ph.D thesis had lots of mathematical components including many exact results, at Bell Labs I took the opportunity to work on rather different areas of condensed matter physics in realistic systems. These include the phase ordering dynamics in systems such as magnets and liquid crystals, transport properties in high-T_c superconductors, and also on the stress propagation in granular systems. At Bell Labs, I interacted with many theorists (most notably D. Huse), as well as with many experimentalists. I collaborated with the experimental group of D. Bishop and P. Gammel and studied the transport properties in high-T_c superconductors in the vortex liquid regime. Based on a simple model that D. Huse and myself introduced (Phys. Rev. Lett., 71, 2473 (1993)), we predicted that the resistivity in the vortex liquid regime is essentially non-local in nature and one needs to generalize the simple Ohm's law to take into account this non-local behavior. The analytical results based on our simple model were in good agreement with later experimenal results (Phys. Rev. Lett., 72, 1272 (1994)). I also collaborated with the experimental group of B. Yurke on the persistence properties and the phase ordering dynamics in liquid crystals. At the same time, I collaborated extensively with C. Sire (Toulouse), who was also a post-doc at Bell Labs at that time, on the phase ordering dynamics in nonequilibrium systems and related topics.

Let me just mention here one work on phase ordering dynamics (further details can be found in the Research Report in section 7). When a magnetic system is quenched rapidly from the high temperature disordered phase to its critical point, the correlation length $\xi(t) \sim t^{1/z}$ grows with time t as a power law, where z is the dynamical critical exponent. Also the spin-spin auto correlation function (say at site i), $A(t) = \langle s_i(0)s_i(t)\rangle$, decays with time as $A(t) \sim \xi(t)^{-\lambda_c}$ with a nontrivial nonequilibrium critical exponent λ_c. It was known that the exponent depends on the conservation laws satisfied by the dynamics. However, prior to our work, there was a lot of controversies surrounding the value of the exponent λ_c for the case of the conserved order parameter dynamics. In collaboration with D. Huse and B. Lubachevsky (Phys. Rev. Lett., 73, 182 (1994)), we showed that for the conserved case, and provided some technical conditions are satisfied, $\lambda_c = d$ (where d is the spatial dimension) is a universal exponent. This settled the existing controversies and our result was verified experimentally later. In collaboration with C. Sire, we were the first to derive analytically the nonequilibrium exponents associated with the phase ordering dynamics in the Potts model (Phys. Rev. Lett., 74, 4321 (1995)).

At the end of my first post-doc at Bell Labs, I wanted to return to India and I had a permanent job offer from Tata Institute, Bombay (India). However, I took a leave of absence for 2 years to continue my post-doctoral research in the US, mainly because I wanted to broaden my scientific horizon a bit more. So, I did a second postdoc (1995-1996) at Yale university in the group of S. Sachdev. At Yale also, I worked on a variety of condensed matter subjects. I had the opportunity there to work with two highly talented Ph.D students of S. Sachdev, namely K. Damle (now a faculty at Tata Institute, India) and T. Senthil (now a faculty at MIT, USA). At that time, new experiments on trapped cold atoms were begining to be carried out which later led to a revival of the old subject of Bose-Einstein condensation in dilute gases. In collaboration with K. Damle and S. Sachdev, we addressed the following natural question: Consider a 3-dimensional dilute Bose gas initially in thermal equilibrium at a high temperature so that the condensate density in the gas is zero. Then the gas is rapidly quenched to a temperature below its critical point following which it evolves in time freely, starting from the initial high-temperature configuration. Eventually, the gas will reach thermal equilibrium corresponding to the final temperature which is below the critical temperature, i.e., eventually there will be a nonzero condensate density. How does the condensate density grow with time? A naive classical analysis based on the dissipative nonconserved dynamics of a 3-d X-Y model predicts that the condensate density would grow as $t^{3/2}$ at late times. However, contrary to this naive expectation, we found that the condensate density grows much faster as t^3 [Phys. Rev. A, 54, 5037 (1996)]. This is due to an additional sound wave mode in the dilute Bose gas. In $d = 2$, our corresponding result also provided a new and faster mechanism for the annihilation of topological defects such as vortices in this system. We also worked on the thermodynamics of the Bose gas in presence of a harmonic trap and demonstrated how its critical properties get modified in the presence of a trap [Europhys. Lett. 36, 7 (1996)]. At Yale, I also worked with T. Senthil on disordered quantum spin chains. We were able to obtain exact results on disordered quantum Potts chain using a real space renormalization group scheme (Phys. Rev. Lett., 76, 3001 (1996)).

During my stay at Yale, I also collaborated with S.N. Coppersmith (now at Wisconsin), T. Witten (Chicago) amongst other theorists and also the experimental group of S. Nagel (Chicago) on the stress propagation in granular medium. In the experiment a weight was put on top of a granular pile and then the stress at different points at the bottom layer of the pile was measured. The distribution of the stress at the bottom layer suggested that the stress from the top to the bottom layer propagates along rather irregular paths. We constructed a simple model of this stress propagation (now called the "q-model" of force fluctuations in the literature) which explained this anomalous stress propagation. This model also has beautiful exact solutions. Our work (published in **Science**, 269, 513 (1995) and Phys. Rev. E, 53, 4673 (1996)) in this area remains the two pioneering papers in this field (with a total of 1489/976 citations (GS/ISI) dated 16/07/2020). **Our work was reviewed in the popular press article: "Clues About How a Sand Pile Holds Itself Up: Scientists Get 3-D View of Force Chains in Granular Materials" by S. Koppes (University of Chicago)**, see http://www-news.uchicago.edu/releases/95/950820.granular.forces.shtml

At around the same time, I also started my work on the subject of persistence in nonequilibrium systems, in collaboration with C. Sire and A. Bray (Manchester,UK), following my visit to Toulouse (Sept.-Nov. 1995) for 3 months. The field of persistence essentially started in France and became rather popular shortly. Over the last 20 years, there have been a lot of work in this area, amongst which many major contributions came from France. I believe that my collective work on persistence spanning over more than 15 years and carried out in collaboration with many theorists and experimentalists, have been one of the major contributions of my scientific career (>40 papers including >12 PRL's and an extensive review in Advances in Physics, carrying more than 1500 citations (GS/ISI), dated 16/07/2020). More details on persistence can be found in section 7.10. Here I just highlight the main problem.

Persistence is simply the fraction of points in space where a nonequilibrium field, fluctuating in space and time, has not changed sign upto some time t. In many nonequilibrium situations such as in systems undergoing coarsening after a rapid quench in temperature and even in the simple diffusion equation, the persistence decays with time t slowly as a power law $\sim t^{-\theta}$ for large t. This exponent θ is the simplest quantitative measure of the history dependence of the nonequilibrium process and is a new nonequilibrium exponent, not simply related to other dynamical exponents. Theoretical and experimental determination of θ has been the focus of intense research by a broad community of physicists in recent times. My collective contributions to the field of persistence have been considerable (more than 35 papers). To my knowledge, we were the first to make a connection between the persistence exponent and the late time decay of no barrier crossing probabilities in many stochastic Gaussian stationary processes. Thanks to this connection, it was realized that the reason this exponent θ is often nontrivial is due to the fact that the underlying Gaussian process in non-Markovian. Besides, this connection also led to many important new results in the field in addition to providing new analytical methods to calculate the persistence exponent [such as the independent interval approximation (Phys. Rev. Lett. , 77, 2867 (1996)), and a new perturbation theory around a Markov process (Phys. Rev. Lett., 77, 1420 (1996))]. Moreover, we showed that the persistence exponent associated with the no sign change of the global magnetization at the critical point is a new nonequilibrium critical exponent in the sense that it is not related to other critical exponents by a scaling relation (Phys. Rev. Lett., 77, 3704 (1996)).

Many of our results on persistence have been verified in diverse experiments. **Our work on persistence drew a lot of attention internationally and was reviewed in the research news section of Science [A. Watson, Science, 274, 919, (1996)]**. With A.J. Bray and G. Schehr, I have recently written an extensive review on the subject of persistence ["Persistence and first-passage properties in nonequilibrium systems", Adv. in Phys., 62, 225-361 (2013)]. As a theoretical physicist, I find this subject beautiful as it makes connections between the probability theory, statistical physics and realistic experimental systems. The subject of persistence remains one of my most favourite areas of research in which I continue to work. Currently, the subject of persistence has again seen a revival among mathematicians and I was recently invited to give a talk at the international mathematical workshop "Persistence probabilities and related fields", held at the Technical University, Darmstadt, Germany, July, 2014.

Following my postdoc at Yale, I took up the permanent faculty position at Tata Institute (1997-1999) where I continued to work on persistence and related problems and also started my research in a different area, namely the aggregation and fragmentation phenomena and a class of mass transport models, in collaboration with M. Barma and my Ph.D students S. Krishnamurthy and R. Rajesh. To my knowledge, our paper (S.N. Majumdar, S. Krishnamurthy and M. Barma, Phys. Rev. Lett. 81, 3691 (1998)) was one of the first (along with a paper in the same year by O.J. O'loan, M.R. Evans and M.E. Cates, Phys. Rev. E 58, 1404 (1998)) to predict the existence of the phenomenon of **real-space** condensation. This phenomenon of real-space condensation is encountered in a variety of situations such as aggregation and fragmentation processes, granular clustering, phase separation, traffic and networks. Unlike traditional Bose-Einstein condensation in the **momentum space**, a condensate in these systems forms in **real space**, e.g., upon increasing the density beyond a critical value a macroscopically large mass/cluster may form at a single site on a lattice. This is a very popular subject currently and I have continued to work on this subject in collaboration with M.R. Evans (Edinburgh), R.K.P. Zia (Virginia Tech), M. Barma (Tata Institute) and others. We found the necessary and sufficient conditions for the occurrence of the real-space condensation in a class of generic mass transport models. Such models are generalizations of classical models such as Zero Range processes. In collaboration with E. Trizac (LPTMS), I. Pagonabarraga (University of Barcelona) and M.R. Evans, we have shown that this real-space condensation also happens in a fluid of polydisperse hard rods, a system that plays a central role in simulations of soft matters. In July 2008, I gave a set of lectures on this phenomenon of real-space condensation at Les Houches summer school "Exact Methods in Low-dimensional statistical physics and quantum computing". (see my review in the Les-Houches book, Ref. [4] in Invited Reviews in the list of publications).

In 2000, I joined CNRS at LPT(Toulouse). I chose to go to Toulouse since I had continued my collaborations with C. Sire there. In Toulouse, I played a role in forming the statistical physics group there which included, apart from myself, C. Sire, D.S. Dean, P-H. Chavanis and N. Destainville. We had extended visits by several people at Toulouse including A.J. Bray (Manchester) (with whom I had continued collaborations extending over more than 10 years) and P.L. Krapivsky (Boston).

While at Toulouse, in collaboration with Krapivsky and Dean, I started working in a new area namely the 'sorting and search' problems in computer science. This is a very active area of research in computer science, but physicists never worked in this area prior to us. To my knowledge, we were the first to realize that many problems in this area can be studied using the techniques of statistical physics. The basic problem in 'sorting and search' is simple and its solution has practical importance. When new data arrive in a computer, the computer sorts this data and arranges them using a suitable 'sorting' algorithm such that when one needs a particular data later, the computer spends minimum time in 'searching' it. The efficiency of a 'sorting and search' algorithm is quantified typically by its 'search time' t_{search}, i.e., the time needed to search a typical element from the already sorted data. Of course t_{search} depends on the data size N. If the data is organized in a linear fashion, i.e., they are put in an array in the same order as they arrive, then $t_{\text{search}} \sim O(N)$. This is clearly not very good. It turns out that one of the most efficient ways to organize the data is to arrange them on a binary tree. For these binary search trees, $t_{\text{search}} \sim c \log(N)$ for large N. It turns out that usually it is very difficult to find an algorithm whose search time will be faster than $\log(N)$. Hence, all one can hope for is to find algorithms which will minimize the coefficient c in front of the $\log(N)$. It is for this reason that the exact determination of the coefficient c for a given algorithm is rather important in computer science.

When the incoming data is completely random, the corresponding binary trees are generated randomly also, i.e., each tree has the same statistical weight. For such 'random binary search trees', the computer scientists have previously derived upper and lower bounds for the constant c. By mapping this problem onto a directed polymer problem, we have determined the constant c exactly as a root of a transcendental equation [Phys. Rev. Lett., 85, 5492 (2000)]. More importantly, we showed that the cumulative probability $P(t_{\text{search}} > x, N)$ for a given data size N has the form, $P(t_{\text{search}} > x, N) = f[x - c\log(N)]$ for large N. Thus if one interprets $\log(N)$ as a 'time' variable, this describes a 'traveling front' propagating with a velocity c. This observation has rather profound consequence. Physicists, over the years, had developed techniques for computing this velocity c of a traveling front in the context of various nonlinear reaction-diffusion systems (such as the Fisher-Kolmogorov-Petrovsky-Piskunov equation). We used some of these techniques from statistical physics in this computer science problem to determine the constant c exactly for random binary search trees. Moreover, this connection with 'traveling fronts' turns out to be more general and one can similarly study more generalized search trees and also for trees generated with unequal weights. This connection has thus enabled us to derive a host of exact analytical results for many computer science problems, some of which had remained unsolved for more than 20 years.

Since then, we have worked extensively in this area over the past few years (see section 7.8 for more details). Our work in this area have been recognized by the computer science community. In France, many computer scientists have worked and are working in this area including P. Flajolet (INRIA), B. Chauvin and D. Gardy (Université de Versailles) amongst others. I have active interactions with B. Chauvin and D. Gardy. Over the past years, I have been invited to many of their conferences (for example, in 'Arbres Aléatoires et Algorithmes', Versailles, 2003, 'Analysis of Algorithms 2006' Alden Biesen (Belgium), 'Random Shapes' at UCLA, 2007 etc.). I have also given a set of invited lectures at two summer schools on the interface between physics and computer science, one in Beijing (2006) and one in Bremen (2007). The computer science community have followed up our work and have proved some of our exact 'physics' results via more rigorous mathematical methods.

Thus, in my opinion, I (along with my collaborators) have played an original role in establishing this new area of exciting research, bringing in a section of the physics and the computer science community together.

In 2004 February, I moved to LPTMS (Orsay). While I was at Toulouse, I already had started my collaborations at LPTMS with A. Comtet on a number of problems in disordered systems. So, it was rather easy for me to integrate into this new laboratory.

After moving to LPTMS I started a new area of research in which I am currently most active, namely the 'extreme value statistics of correlated variables'. This is a very general problem that appears in a number of areas ranging from disordered systems and random matrices to mesoscopic transport and computer science problems. Since these subjects cover the research interests of many people in LPTMS, it naturally led to several collaborations inside LPTMS (see later).

Let me briefly mention here the main interests in this extreme value problems. The statistics of the minimum or the maximum of a set of random variables is interesting in many systems and have many applications ranging from weather records, finance, earthquakes and oceanography. When the random variables are uncorrelated, it was known for a long time that the distribution of the maximum (say) (properly centered and scaled) follows one of the three universal laws, namely (i) the Gumbel (ii) the Frèchet and (iii) the Weibull. The question is what happens when the variables are strongly correlated? This is indeed the generic situation in many systems. For example, the height fluctuations in a random interface are strongly correlated. Similarly, the eigenvalues of a random matrix are also strongly correlated. For such strongly correlated systems, one can no longer apply the theory of uncorrelated variables and one needs new methods. There are very few exact results available for strongly correlated systems. In collaboration with A. Comtet (Phys. Rev. Lett., 92, 225501 (2004)), we computed exactly the distribution of maximal height fluctuations in a general class of one dimensional interfaces. Curiously, we found that the distribution function of the maximum height fluctuations in this class is universal and is the same as that of the area under a Brownian excursion and it is called the Airy-distribution function. It turns out that exactly the same function also appears in many problems in graph theory and computer science. To my knowledge, this is one of the first exact results for the extreme statistics in a strongly correlated system. Besides, our result showed for the first time that the Airy-distribution function, which so far appeared only in graph theory and computer science as a purely mathematical object, could possibly be measured in a real experimental system such as the fluctuating step edges in Si-Al surfaces.

Another correlated extreme value problem that I studied with D.S. Dean deals with the distribution of the largest eigenvalue of a random matrix. The problem is very simple to state and it arises in many areas of physics. 'Random landscapes' appear in many systems such as in glasses, spin glasses and even in string theory. Understanding the geometrical properties of such random landscapes have direct consequences in the macroscopic collective behavior in such systems. One such natural question is: Given a random landscape, what fractions of its stationary points are local maxima, local minima or saddles? To analyse the nature of a stationary point of a random surface $V(\{x_i\})$ in N dimensions, one needs to know the eigenvalues of the $(N \times N)$ Hessian matrix: $H_{i,j} = \partial^2 V/\partial x_i \partial x_j$. For example, if all the eigenvalues are positive (negative), one has a local minimum (maximum). If some of are positive and some are negative, one has a saddle. In a simple 'random Hessian model' where one assumes that the elements of the Hessian matrix are uncorrelated Gaussian variables, one is then led to the question: Given a random ($N \times N$) Gaussian matrix, what is the probability P_N that all its eigenvalues are positive (or equivalently negative)? Thus P_N is the fraction of local minima (or maxima). This problem had been open for quite some time (more than 10 years) and most of the previous results were numerical. In a joint work with D.S. Dean (Phys. Rev. Lett., 97, 160201 (2006)) we proved that $P_N \approx \exp[-\theta \beta N^2]$ for large N where $\beta = 1$, 2 or 4 is the Dyson index characterizing the orthogonal (GOE), the unitary (GUE) or the symplectic (GSE) nature of the Gaussian matrix and the exponent $\theta = (\ln 3)/4 = 0.274653...$ is universal. Our exact result shows that P_N is very small indicating that in this simple random Hessian model, there are very few local minima or maxima, most of them are saddles.

This work essentially started my interest in random matrix theory, in particular the study of the statistics of the largest eigenvalue in random matrix theory. This subject has become very fashionable these days, following the celebrated work of C. Tracy and H. Widom, who computed the limiting distribution of the largest eigenvalue for Gaussian random matrices. Since then, the Tracy-Widom distribution has surfaced in numerous other areas of statistical physics. In the context of the largest eigenvalue of a random matrix, Tracy-Widom describes the distribution of 'typical' small fluctuations of $\lambda_{\max}$ around its mean value. However, it does not describe the probability of 'atypical' large fluctuations, which is often of interest as mentioned before. Using Coulomb gas method, myself and collaborators, were able to compute exactly the large deviation functions of $\lambda_{\max}$. Our results were thus complementary to Tracy-Widom distribution. Moreover, we showed that the large deviation tails of $\lambda_{\max}$ are 'asymmetric': left is not similar to the 'right'. Indeed, our subsequent work revealed that the there is actually a 3-*rd order phase transition* separating the 'left' and the 'right' and the Tracy-Widom is just crossover function connecting the left (pushed Coulomb gas) and the right (pulled Coulomb gas) phases. Moreover, we have demonstrated that this 3-rd order phase transition in the distribution of $\lambda_{\max}$ iis quite generic and occurs in many other systems, notably in Yang-Mills gauge theory, distribution of conductance in mesoscopic systems, vicious walker problem amongst others. *On this subject, I gave a* **Plenary** *talk at STATPHYS-25 (Seoul, South Korea, 2013) and with G. Schehr (LPTMS), we wrote a recent review on this very exciting and rapidly evolving field [Ref. 188 in the list of publications].*

Based on my recent talk on this subject at the international conference "Viewpoints on Emergent Phenomena in Non-equilibrium Systems", held at the Higgs Centre for Theoretical Physics, University of Edinburgh (UK, June, 2014), Mark Buchanan (a science writer with Nature Physics and Science) wrote an essay "Equivalence Principle" in Nature Physics (vol-10, 543, (2014)). This is available at: http://www.nature.com/nphys/journal/v10/n8/pdf/nphys3064.pdf

Also, a related popular science article by Natalie Wolchover appeared in the Quanta magazine (published by Simon's foundation) in the October 15 (2014) issue, with the title "At the Far Ends of a New Universal Law". This can be found online at: https://www.quantamagazine.org/20141015-at-the-far-ends-of-a-new-universal-law/

My work with Gregory Schehr on the large deviations of the top eigenvalue of a random matrix and the ubiquity of third order phase transitions (see Ref. [188] in the list of publications) was recently highlighted by the CNRS-Institut

National de Physique (INP) with the title "Luniversalité de la distribution de Tracy-Widom proviendrait dune transition de phase"

see online at: http://www.cnrs.fr/inp/spip.php?article3403

Even though random matrix theory is rather a classical area of research in physics, I believe that our work in random matrix theory brought new tools and methods to the existing traditional tools in this area and also posed new kinds of questions. In collaboration with P. Vivo (now at King's College, London), O. Bohigas (LPTMS) and M. Vergassola (Institut Pasteur, now at UCSD) (J. Phys-A, 40, 4317 (2007) and Phys. Rev. Lett., 102, 060601 (2009)), we applied the Coulomb gas method to understand the large deviation properties of the extreme eigenvalues in other classes of random matrices (beyond the Gaussian ensemble), such as Wishart matrices which are random covariance matrices used routinely in statistical data analysis. Our work brought out interesting applications of these large deviation probabilities in the so called Principal Component Analysis (PCA) that is used in practical methods such as in image processing. *Our large deviation results on Wishart matrices have subsequently been verified experimentally in coupled fiber laser systems by the group of N. Davidson at the Weizmann Institute (Israel) (see e.g., arXiv: 1012:1282).* With my former Ph. D student C. Nadal, we showed how Wishart matrices also appear in physics problems such as nonintersecting fluctuating interfaces that are used as models of fluctuating step edges on a crystalline surface (Phys. Rev. E., 79, 061117 (2009)).

Another system where our techniques were very useful is in the problem of estimating the entanglement entropy in a coupled random quantum bipartite system. In collaboration with O. Bohigas and A. Lakshminarayan (I.I.T Madras, India) we computed the exact probability distribution of the minimum eigenvalue of the reduced density matrix in this entanglement problem (J. Stat. Phys. 131, 33 (2008)). With my former Ph.D student C. Nadal and in collaboration with M. Vergassola, we have succeeded [Phys. Rev. Lett., 104, 110501 (2010)] in computing the exact probability distribution of the entanglement entropy in such bipartite systems (the previous results were known only for the mean and the variance of the entropy). In collaboration with O. Bohigas and P. Vivo at LPTMS, we applied similar techniques to compute the distribution of conductance and current fluctuations in an open mesoscopic system such as a quantum dot (Phys. Rev. Lett., 101, 216809 (2008)). In the latter problem, we have discovered new singularities in the conductance distribution which are the direct consequence of two new phase transitions in an associated Coulomb gas problem (Phys. Rev. B, 81, 104202 (2010)). Subsequently, we have used random matrix theory in other related problems in mesoscopic transport, such as in computing the distribution of Andreev conductance across the normal-superconductor junctions [Ref. 162] and also in computing the distribution of Wigner-time delay [Ref. 181].

Another interesting extreme value problem with connections to random matrix theory appears in the so called 'vicious walker' problem introduced by P.G. de. Gennes and later studied by M.E. Fisher. This problem has many applications most notably in characterizing the topological excitations (the so called 'watermelons') that appear in systems with a commensurate-incommensurate phase transitions. 'Watermelon' simply refers to the configuration of N non-crossing Brownian motions that start at the origin (in one dimension) at time 0 and regroup again at the origin at a fixed time t. One of the questions that concerned mathematicians and computer scientists alike and that remained open for many years: what is the distribution of the maximal height of the watermelon, i.e., the distribution of the maximal distance travelled in time t by the rightmost walker. With my former Ph.D student J. Randon-Furling, and in collaboration with A. Comtet (LPTMS) and G. Schehr (LPT), we computed this distribution exactly (Phys. Rev. Lett., 101, 150601 (2008)). Moreover, we also showed that at any fixed intermediate time $0 \leq \tau \leq t$, the joint distribution of the positions of the non-intersecting walkers in the watermelon geometry is given exactly by the joint distribution of the eigenvalues of the Wigner-Dyson Gaussain matrices. In addition, if there is a hard wall at the origin so that the walkers move only in the positive semi-axis (the half-watermelon geometry), the joint distribution of the positions is isomorphic to the joint distribution of the eigenvalues of the Wishart matrix mentioned before, thus providing yet another physical application of the Wishart matrix. **Our work was reviewed in the 'research highlights' section of Nature Physics (vol-4, page 829, November 2008 issue).**

In collaboration with P.J. Forrester (Melbourne), G. Schehr (LPTMS), and A. Comtet, we have established an exact mapping between the vicious walker problem and the Yang-Mills gauge theory in two dimensions [Refs. 154, 175]. This mapping helped us understand why Tracy-Widom distribution appears as the limiting distribution of the maximal height in the vicious walker problem.

On this subject, we obtained the ANR grant (2011) "Marcheurs browniens rpulsifs et matrices alatoires" acronymed 'WALKMAT' (projet blanc) [Principal investigators: S.N. Majumdar (LPTMS, Orsay) and G. Schehr (formerly at LPT (Orsay), now at LPTMS (Orsay)). We recruited A. Kundu in 2012 as a postodoc under this ANR grant. Kundu has since successfully worked on a number of problems related to this project and has now obtained (since 2015) a faculty position at ICTS (Bangalore, India).

Very recently, we have also applied the ideas on extremes from the random matrix theory to the physical system of trapped fermions in a confining potential. This has led to exciting new developments (see later for details). On this subject, we have just obtained a new ANR grant (2017) ""Random matrices and trapped fermions" (RAMATRAF) jointly with D. Dean (Bordeaux), P. Le Doussal (ENS, Paris) and G. Schehr (LPTMS, Orsay, coordinator). We will recruit a postdoc under this grant.

Another extreme value problem that I studied in collaboration with S. Nechaev (LPTMS) is worth mentioning. This problem appears in the context of biological sequence matching (for details see section 6.2), where one tries to optimize the number of matches between two random DNA sequences. For two random sequences, it is important to understand the statistics of the number of maximal matches. A particularly useful and well studied model is the so called Bernoulli matching model. While the average maximal match length in this model was well known from the early 90's, the variance was unknown prior to our work. We computed the variance exactly (Phys. Rev. E, 72, 020901(R), (2005)). Moreover, it was believed for a long time that the distribution of the number of maximal matches was Gaussian around its mean. We computed this distribution exactly and showed that the limiting distribution is precisely the same as the Tracy-Widom distribution that describes the probability distribution of the largest eigenvalue of a random matrix (as described earlier) and hence is non-Gaussian. This work made a link between two apriori unrelated fields of research namely the sequence matching problems and the random matrices.

Amongst other extreme value problems that we studied, let me briefly mention some of our exact results concerning Brownian motions with 'constraints' and their various applications. With my former Ph.D student J. Randon-Furling we have used path-integral methods to study analytically the distribution of the time at which a Brownian motion achieves its maximum within a fixed time interval. While for a simple unconstrained Brownian motion the result was known since P. Lévy (1941), it was not easy to extend the available mathematical methods in probability theory to study this distribution to more generalized Brownian motions such as Brownian bridge, Brownian excursion and Brownian meander. We succeeded in computing exactly this distribution for a variety of constrained Brownian motions. Use of the path integral method in this problem led to the discovery of new mathematical identities in number theory. This is a very successful project and led to our joint collaboration with M. Yor (Laboratoire de Probabilités et Modèles Aléatoires, Jussieu) (J. Phys. A. Math. Theor. 41, 365005 (2008)). These results also have important applications in finance where it gives a precise prediction about the time at which one should sell one's stock. The application of these path-integral methods in finance led me to start a new collaboration with J-P. Bouchuad (Capital Fund Management) (Quantitative Finance, 8, 753 (2008)). In collaboration with A. Comtet and J. Randon-Furling, we have applied some of these results to compute exactly the mean and the perimeter of the convex hull of N independent Brownian motions (Phys. Rev. Lett., 103, 140602 (2009)) with important applications in the estimation of the home range of foraging animals in the context of ecolgical conservation. In collaboration with A. Rosso (LPTMS) and A. Zoia and E. Dumonteil (CEA, Saclay), we have extended our method to compute the mean perimter and the mean area of a two-dimensional stochastic branching Brownian motion, which has interesting applications in estimating the range of epidemic spread in animal polpulations. Our results appeared in **PNAS**, 110, 4239 (2013).

We have since studied the extreme statistics and its different aspects in many other systems (for details see 7.1, 8.1, 8.2, 8.3 for details). This is a very active area of research worldwide where I believe that I have made very significant and important contributions (a total of ~ 70 papers including > 25 PRL's). On my research in this area, I gave a set of invited lectures at Les Houches summer school on 'Complex Systems' (2006), organized by M. Mèzard and J-P. Bouchaud, a set of 8 lectures on "extreme statistics of correlated random variables" at the Beg-Rohu summer school (June, 2008) organized by G. Biroli and A. Lefèvre, a set of 4 lectures on this subject at the summer school "Fundamental problems in statistical physics XII" at La Foresta, Belgium (2009). In May 2011, I have given a set of 5 lectures on this subject at the Les Houches summer school "Vicious Walkers and Random Matrices". In June 2011, I have given a plenary lecture at the international conference 'Extreme Value Analysis" held at the University of Lyon (This is one of the biggest conferences in Statistics in the area of extremes). I also gave a set of 9 lectures at the Beg-Rohu school in 2013 on this subject. As mentioned earlier, I gave a plenary talk at STATPHYS-25 (Korea, 2013) on this subject. Recently, I gave a set of lectures on this subject at the Galileo Galilei Institute (Florence, Italy, May 2014) and a set of 10 lectures at the Spring College at GGI (Florence, February 2018). Also, with C. Godreche (CEA, Saclay) and G. Schehr (LPT, Orsay), I organized the 16-th Claude Itzykson meeting "Extremes and Records" at Saclay in 2011 on this subject.

I have had many fruitful collaborations with several people at LPTMS on a variety of subjects listed below.

• O. Bohigas (on extreme value problems in random matrices, applications to mesoscopic transport and also to entanglement problems in quantum bipartite systems)

(Refs. [107], [118], [120], [133], [145] in the list of publications).

• A. Comtet (on a variety of extreme value problems in disordered systems and in fluctuating interfaces, on various statistical aspects of Brownian functionals and their applications, on reaction-diffusion problems, on integer partition problem and its applications etc.)

(Refs. [58], [62], [63], [74], [80], [87], [90], [96], [103], [105], [112], [115], [122], [130], [137], [141], [147], [175], [185], [197], [217])

• J. Desbois (on the occupation time problems on graphs)

(Ref. [63])

• P. Leboeuf (on integer partition problem and its connection to level density of Bose and Fermi gases)

(Ref. [105])

• O. C. Martin (on the statistics of maxima and minima in random landscapes in glassy systems)

(Ref. [102])

• S. Nechaev (on growth models, biological sequence-matching problems and the Tracy-Widom distribution in random matrices)

(Refs. [73], [82], [119])

• S. Ouvry (on integer partition and exclusion statistics problems)

(Refs. [112], [115])

• A. Rosso (on the translocation dynamics of a polymer chain (DNA) through a pore and the associated first-passage properties of non-Markovian processes, 2-d spread of epidemics in animal populations etc.)

(Refs. [136], [146], [148], [151], [160], [161], [177], [184], [225])

• G. Schehr (on a variety of extreme value problems from Brownian motion to random matrix theory)

(Refs. [95], [113], [126], [130], [138], [139], [150], [154], [165], [170], [171], [174], [175], [180], [182], [183], [185], [187], [188], [189], [190], [191], [192], [195], [196], [202], [205], [206], [207], [208], [211], [215], [216], [217], [218], [220], [221], [223], [224], [225], [227], [228], [229], [234], [236], [237], [238], [239], [241], [244], [245], [246], [248], [249], [250], [252])

• C. Texier (on the distribution of Wigner time-delay in chaotic cavities and constrained Coulomb gases)

Refs. ([181], [232], [235], [246])

• E. Trizac (on the real-space condensation in polydisperse hard rod systems)

(Ref. [143])

Apart from the permanent members at LPTMS, I also worked with several Ph.D students and postdocs at LPTMS. My graduate students include

- J. Randon-Furling (graduated in 2009) (who worked on random convex hulls and extreme value statistics and his Ph.D had the title "Statisques d'extremes du mouvement brownien et applications").
- C. Nadal (graduated in 2011) (who worked on "Matrices aléatoires et leurs applications à la physique statistiques et physique quantique").
- R. Marino (graduated in 2015) worked on "Number Statistics in random matrices and applications to qunatum systems").
- C. De Bacco (graduated in 2015) (jointly with S. Franz) (who did her Ph.D on "Decentralized network control, optimization and random walks on networks"). Caterina has been awarded the EPS-SNPD early carrer prize in 2021.
- J. Bun (graduated in 2016) (jointly with M. Potters from CFM) (who worked on "Application de la théorie des matrices aléatoires pour les statistiques en grande dimension")
- A. Grabsch (graduated in 2018) (jointly with C. Texier) (who worked on "Random matrices in statistical physics: quantum scattering and disordered systems").
- B. Lacrioix-A-Chez-Toine (gradauted in 2019) (officially the Ph.D student of my colleague G. Schehr), who has collaborated with me extensively during his Ph.D work on "Extreme value statistics of strongly correlated systems : fermions, random matrices and random walks".
- F. Mori is currently doing his Ph. D with me (2nd year) on "Extremal properties of Brownian motion".
- B. De Bruyn (1st year) (officially the Ph. D student of G. Schehr) is collaborating with me on a number of topics on Brownian motion and random walks.
- P. Mergny is currently doing his Ph. D (2nd year) with me and M. Potters on "Applications of random matrix theory".
- T. Gautie is a current 3rd year Ph. D student (jointly with P. Le Doussal and J.-P. Bouchaud) working on "Nonintersecting Brownian motions".
- A. Flak is a current 1st year Ph. D student (jointly with S. Nechaev) working on "Truncated linear statistics in one dimensional 1-component plasma" with myself and G. Schehr.
- V. Maria Schimmenti is a Ph.D student of A. Rosso, and is also working with me

In addition, several Ph.D students have worked with me during their Ph.D thesis. They include

- H. Kallabis (former Ph.D student of J. Krug from the University of Cologne, 1997)
- G.C.M.A Ehrhardt (former Ph.D student of A.J. Bray from the University of Manchester, 2001)
- M. Constantin (former Ph.D student of S. Das Sarma from the University of Maryland, 2003)
- P. Vivo (former Ph.D student of G. Akemann, Brunel University, UK, 2007)
- R. Allez (former mathematics Ph.D student of M. Gubinelli and V. Vargas from Universite' Paris-Dauphin, 2009)
- G. Borot (former Ph.D student of B. Eynard from IPhT, Saclay, 2010)
- J. Franke (former Ph.D student of J. Krug from the University of Cologne, 2012)
- A. Gabel (former Ph.D student of S. Redner from Boston University, 2012)
- G. Wergen (former Ph.D student of J. Krug from the University of Cologne, 2012)
- A. Perret (former Ph.D student of G. Schehr from LPTMS, Univerite' Paris-Sud, 2013)
- J. Whitehouse (former Ph.D student of M. R. Evans from the University of Edinburgh, 2013)
- Y. Edery (former Ph.D student of B. Berkowitz from the Weizmann Institute, 2013)
- M. Chupeau (former Ph.D student of O. Benichou from LPTMC, UPMC, 2015)
- G. Claussen (former Ph.D student of A.K. Hartmann from the University of Oldenburg, 2015)
- A. Bar (former Ph.D student of D. Mukamel from the Weizmann Institute, 2016)
- H. Schawe (ex Ph. D student of A.K. Hartmann from the University of Oldenburg, since 2017 and is currently a postdoc at Univ. of Cergy Pontoise, France).

My postdocs at LPTMS include

- S. Sabhapandit (2006-2008, currently a faculty member at Raman Research Institute, Bangalore, India) who worked on disordered systems, extreme value statistics, integer partition problems
- D. Villamaina (2011-2013, currently at CFM) who worked with me on extreme eigenvalues of Cauchy random matrices
- A. Kundu (2012-2014, currently a faculty member at ICTS, Bangalore, India) who has worked on vicious walker problem as well as on the number of distinct and common sites visited by random walkers
- S. Gupta (2012-2014, currently a faculty member at Belur University, Calcutta, India) who worked on nonequlibrium steady states induced by 'resetting' dynamics in fluctuating interfaces

- K. Ramola (2012-2014, currently postdoc at Brandeis University, USA), who has worked on the order and gap statistics in one dimensional branching Brownian motion.
- J. Grela (2016-2018), currently a postdoc at Queen Mary College (London), who worked with me on linear statistics of trapped fermions at finite temperature.
- U. Basu (2017-2018), currently a faculty member at Raman research Institute, Bangalore, India who worked with me on "active Brownian motion".
- T. Banerjee (2018-2019), who is going to University of Louvain for his next postdoc.
- N. Smith (2019-) who has just finished his postdoc with us at LPTMS under the ANR grant RAMATRAF and has just obtained a faculty position at Ben-Gurion University in Israel.

The research environment at LPTMS is exciting and I hope to have brought new ideas and initiated new directions of research in this dynamic laboratory. I have also maintained continuing collaborations with my colleagues M. Barma and D. Dhar at Tata Institute (Bombay, India). I visit Tata Institute on a regular basis as an adjunct professor to the department of theoretical physics. Several graduate students there are working under my co-direction. In addition, I am an adjunct professor at the International Center for Theoretical Sciences (ICTS, Bangalore, India) and the Raman Research Institute (Bangalore, India). I visit both Institutes once every year. I am also a visiting Weston professor at the Weizmann Institute, Israel where I visit annually for a month and collaborate with D. Mukamel and his graduate students. In addition, I'm a Higgs associate (since 2012) at the Higgs Center for Theoretical Physics at the University of Edinburgh (UK), where I have regular collaborations with M. R. Evans and his group.

3 Principal Scientific Contributions, Achievements and Impact

To summarize, I have worked on a broad range of subjects in statistical physics with applications ranging from granular systems all the way to computer science and biology. Despite the diversity of the subjects, there are two basic themes that are common to most of my work:

(i) the physical systems that I am interested in usually involve many degrees of freedom which are '**interacting**' or '**correlated**' and

(ii) there is usually an element of '**randomness**' in them. I mean '**randomness**' in a rather broad sense here: for example, it can be the 'frozen' disorder as in spin-glasses or it can be the 'noise' or 'stochasticity' in the temporal evolution of a system. In the context of data structures and algorithm problems in computer science, 'randomness' is in the underlying data structures.

In the general spirit of statistical physics, my research methods consist in constructing simple yet nontrivial models that essentially capture the physics of the more complicated real systems and then try to solve these models analytically to understand the basic physical mechanism involved behind the collective behavior of the underlying many body systems. The results I have obtained are predominantly analytical in nature. *Many of them are first exact results in the fields.* In my opinion, my principal and most original contributions so far have been in the following areas:

(1) *Self-organized criticality in sandpile models:* (Ph.D thesis, 1992) Exact solution of the Abelian sandpile model (ASM) on the Bethe lattice, exact height-height correlations in ASM, an exact mapping between the avalanche size distribution in ASM and the clusters in the q-state Potts model in the limit $q \to 0$. These results have made impact both in Physics and Mathematics.

(2) *Stress propagation in granular systems:* We contributed two pionnering papers in this field. Our model, now known as the q-model in the literature, describes how an applied stress on a granular heap propagates through the heap our analytical results are verified in experiments in Chicago. Refs: Science, 269, 513 (1995) and Phys. Rev. E, 53, 4673 (1996).

(3) *Persistence in nonequilibrium systems:* Exact solutions for the first-passage probability for various Markov and Non-Markovian stochastic processes: contributed $>$ 50 papers including 15 PRL's with more than 1500 citations and an extensive recent review on the subject jointly with A.J. Bray and G. Schehr: "Persistence and first-passage properties in nonequilibrium systems", Adv. in Phys. 62, 225-361 (2013).

(4) *Real-space condensation in nonequilibrium steady states:* In many systems where there is a transfer of mass or energy from one sppace point to another, a condensation phenomenon can occur in real space, i.e., at long times the system might develop a single point in space with a thermodynamically large fraction of mass condensing there. Our exact solutions of various mass transport models have shed important lights on this phenomena, in particular in characterizing the nature of the condensate.

(5) *'Sorting and Search' problems in computer science:* An exact mapping between binary search tree problem and the directed polymer problem in random medium helped identify a fundamental 'travelling front' structure in sorting and search algorithms. Many asymptotic results on the efficiency of search algorithms could be derived using this method.

(6) *Extreme value statistics of correlated variables:* with applications in Diffusion, Brownian motion and other stochastic processes such as fluctuating interfaces (60 papers including 19 PRL's). Very recently, we have written a comprehensive review on this subject: S. N. Majumdar, A. Pal, G. Schehr, "Extreme value statistics of correlated random variables: A pedagogical review", Phys. Rep. v-840, 1 (2020).

(7) *Random Matrix Theory:* with applications that involve (i) growth models (ii) bilological sequence matching (iii) large deviations of the extreme eigenvalues (iv) conductance distribution in quantum dots (v) bipartite entanglement in random pure quantum states (vi) non-intersecting Brownian motions and Yang-Mills gauge theory (vii) Trapped fermions and the physics of ultracold gases (40 papers including 15 PRL's).

(8) *Stochastic resetting and its applications:* It is a new field of research in statistical physics, in connection with random search via diffusive processes, introduced in collaboration with M. R. Evans (Edinburgh, UK) in: M. R. Evans and S. N. Majumdar, "Diffusion with Stochastic Resetting", Phys. Rev. Lett. v-106, 160601 (2011). **In 2021, Phys. Rev. E has declared it as the 'PRE spotlights of 'emergent areas' and J. Phys. A is bringing out a special issue honoring this work**. We have recently written an exhaustive review on the subject: M. R. Evans, S. N. Majumdar, and G. Schehr, "Stochastic resetting and applications", J. Phys. A. : Math. Theor. v-53, 193001 (2020).

(9) *Order, Gap and Record statistics for stochastic time series* We have introduced new analytical tools to study the statistics of records and orders for strongly corelated time series with a wide ranging applications from climates to finance. Currently, with G. Schehr, we are writing a book on this subject (Oxford University Press).

The strong points of my scientific research are:

(1) *Diversity of subjects* (Extreme Value Statistics, Random Matrix theory and applications, first-passage properties in out of equilibrium systems, Brownian motion and its diverse applications, stochastic resetting, statistics of records, gaps and orders in stochastic processes, sorting and search problems in computer science, transport in high temp. superconductors, stress propagation in granular systems, self-organized criticality in sandpile models, etc.)

(2) *Development of novel analytical techniques* for simple yet nontrivial and outstanding problems

(3) *Strong international collaborations* (more than 100 collaborators internationally) and (¿30 collaborators in France). In LPTMS I have collaborated with 12 permanent members

(4) Many of my results are *first exact results* in the field

(5) *Useful applications* of my results : many of my analytical predictions have been verified

experimentally.

Achievements and Impact:

My collective work includes **309** publications (till 06/07/2021) in reviewed journals (including **68** PRL's, 1 Science, 1 PNAS etc.), 4 conference proceedings and 9 invited reviews and book chapters. They have gathered 16042/10924 citations (source: Google Scholar/ISI web of Science, dated 06/07/2021) with an h-index: 66/54 (GS/ISI).

I also gave 152 invited talks over the last 15 years in international conferences/workshops/summer schools (including the **plenary** talks at STATPHYS-25 (Seoul, 2013) and 'Extreme Value Analysis' (Lyon, 2011). I have also taught in various prestigious summer schools in statistical physics including Les Houches, Beg-Rohu, Leuven, Bangalore etc. In addition, I have been invited to give several prestigious named lectures/colloquiuum across the world, including M. L. Mehta lectures (Tata Institute, India), S. Chandrasekhar lectures (ICTS, India), Higgs Colloquiuum (Edinburgh University, UK) etc.

I have received several international honors/awards during my career. Very recently, I gave a **plenary** lecture at STATPHYS-25 (Seoul, South Korea, 2013)–the biggest conference in statistical physics held every three years in different countries. Also, I was a **plenary** speaker at one of the important conferences in Statistics, namely, 'Extreme Value Analysis' (EVA) (Lyon, France, 2011), held every three years in different countries. I received the 'Geeta Udgaonkar medal' (for outstanding Ph.D thesis at Tata Institute in 1992) and later the 'Young Scientist medal' awarded by the Indian National Science Academy in 1998. In France I received the "Prix Paul Langevin" of the French Physical Society in 2005. Since 2009, I am a recipient of the 'Prime d'excellence Scientifique' (PES) awarded by CNRS in France. Also, in 2009, I recieved the 'Excellence Award' (2009) for outstanding contributions to statistical physics from the prestigious Tata Institute Alumni Association. In 2018, I was awarded the prestigious VAJRA fellowship of the department of Science and Technology of the government of India.

In 2019, I received three prestigious prizes. I was the recipient of the **EPS (European Physical Society) prize** for Statisical and Nonlinear Physics (jointly with S. Ciliberto from ENS-Lyon).

The citation for my EPS prize reads:

The 2019 EPS-SNPD prize is awarded to Satya Majumdar

" *for his seminal contributions to non-equilibrium statistical physics, stochastic processes, and random matrix theory, in particular for his groundbreaking research on Abelian sandpiles, persistence statistics, force uctuations in bead packs, large l* deviations of eigenvalues of random matrices, and applying the results to cold atoms and other physical systems."

http://www.epsnews.eu/2019/03/eps-snpd-prizes-2019/

In 2019, I was awarded the **CNRS silver medal (medaille d'argent)**. Also, in 2019, I received the **Gay-Lussac-Humboldt prize** from the Alexander von Humboldt foundation in Germany for my outstanding contributions to Statistical Physics.

I have been the divisional associate editor of PRL (2011-2013). Currently I am an associate editor of J. Stat. Phys. (since 2008) and also a member of the editorial board of J. Phys. A: Math. Theor. (since 2010), J. Stat. Mech. (since 2003) and Pramana: J. of Phys.(since 2014).

The impact and the international visibility of my work is reflected in a number of news articles/research highlights in science journals. Here are few examples:

• The citation for my EPS prize can be found in the official EPS website:

http://www.epsnews.eu/2019/03/eps-snpd-prizes-2019/

• Following my EPS prize and CNRS silver medal in 2019, the following press article appeared in the official Universite Paris-Saclay website:

https://www.universite-paris-saclay.fr/en/news/satya-majumdar-modeling-the-random

• My recent work with E. Trizac on 'When random walkers help solving intriguing integrals' (Phys. Rev. Lett., 123, 020201 (2019)) got highlighted in Physics Today (Search and Discovery):

https://physicstoday.scitation.org/do/10.1063/PT.6.1.20190808a/full/

It also got a press coverage in Phys.org:

https://phys.org/news/2019-07-illusive-patterns-math-ideas-physics.html

• A popular science article on our work on third order phase transition behind the universality of Tracy-Widom distribution, written by Natalie Wolchover appeared in the Quanta magazine (published by Simon's foundation) in the October 15 (2014) issue, with the title "At the Far Ends of a New Universal Law".

This can be found online at: https://www.quantamagazine.org/20141015-at-the-far-ends-of-a-new-universal-law/

• My work with Gregory Schehr on the large deviations of the top eigenvalue of a random matrix and the ubiquity of third order phase transitions (see Ref. [188] in the list of publications) was recently highlighted by the CNRS-Institut National de Physique (INP) with the title "Luniversalité de la distribution de Tracy-Widom proviendrait dune transition de phase"

see online at: http://www.cnrs.fr/inp/spip.php?article3403

• "Equivalence Principle", an essay by M. Buchanan, published in **Nature Physics**, **10**, 543 (August, 2014), based on my talk on "KPZ/Tracy-Widom story" given at the international conference "Viewpoints on Emergent Phenomena in Non-equilibrium Systems", held at the Higgs Centre for Theoretical Physics, University of Edinburgh, UK, June, 2014.

see http://www.nature.com/nphys/journal/v10/n8/pdf/nphys3064.pdf

• An article 'A walk in the park' that appeared in the research highlights section of **Nature Physics** [Nature Phys., **4**, 829 (2008)] that reviewed my work on the connection between vicious walkers problem and random matrix theory [with G. Schehr, A. Comtet and J. Randon-Furling, published in Phys. Rev. Lett. 101, 150601 (2008)].

see http://www.nature.com/nphys/journal/v4/n11/full/nphys1119.html

• "Une puce qui saute au hasard" (**La Recherche**, mathmatiques - 01/10/2005 par Benot Rittaud dans mensuel n390 la page 26), that reviewed my work with A. Comtet on us the maximum of a random walk (published in J. Stat. Mech. P06013 (2005)).

see http://www.larecherche.fr/actualite/mathematiques/ puce-qui-saute-au-hasard-01-10-2005-71971

• "Persistence Pays Off in Defining History of Diffusion" by A. Watson in the research news section of **Science** [vol-274, page 919-920, 1996], that reviewed my work on persistence in diffusion equation [with A.J. Bray, S.J. Cornell and C. Sire, published in Phys. Rev. Lett. **77**, 2867 (1996)].

see https://www.sciencemag.org/content/274/5289/919

• "Clues About How a Sand Pile Holds Itself Up:Scientists Get 3-D View of Force Chains in Granular Materials" by S. Koppes (University of Chicago press news) that reviewed our work on the force chains in granular materials [published in **Science**, 269, 513 (1995)].

see http://www-news.uchicago.edu/releases/95/950820.granular.forces.shtml

4 Scientific Animation

4.1 Research Initiatives:

Since my arrival in France in 2000, first at LPT (Toulouse) and then at LPTMS (Orsay), I have started many new collaborations and initiated several new directions of research. At Toulouse, I have collaborated with D.S. Dean and C. Sire on a variety of problems ranging from persistence in nonequilibrium systems to algorithms in computer science. At LPTMS (Orsay) (since 2004), I have collaborated with many people and started new research projects. My collaborators at LPTMS include O. Bohigas, A. Comtet, J. Desbois, P. Leboeuf, O.C. Martin, S. Nechaev, S. Ouvry, A. Rosso, G. Schehr, C. Texier, E. Trizac, P. Vivo (now at king's College, London) among the permanents. I also had a number of Ph.D students and postodoc. Ph.D students include J. Randon-Furling (former Ph.D thesis student, graduated in 2009), C. Nadal (former Ph.D thesis student, graduated in 2011), R. Marino (former Ph.D student, graduated in 2015), C. De Bacco (former Ph.D student jointly with S. Franz, gradutaed in 2015), J. Bun (former Ph.D student jointly with M. Potters at CFM, gradutaed in 2016) and A. Grabsch (former Ph.D student jointly with C. Texier, graduated in 2018). Currently, I am supervising and co-supervising the Ph. D these of F. Mori, B. De Bruyn, P. Mergny, T. Gautie and A. Flak. My postdocs at LPTMS include S. Sabhapandit (2006-2008), D. Villamaina (2011-2013), A. Kundu (2012-2014), S. Gupta (2012-2014), K. Ramola (2012-2014), J. Grela (2016-2018), U. Basu (2017-2018), T. Banerjee (2018-2019), N. Smith (2019-2020). In addition, R. Allez (Ph.D student in mathematics from Univ. de Paris-Dauphin), J. Franke (former M.Sc thesis student) and G. Wergen (Ph.D student of J. Krug from Cologne) also worked with me.

Outside LPTMS (in France), I have ongoing collaborations with O. Benichou and G. Oshanin (LPTMC, Jussieu), J.-P. Bouchaud and M. Potters (CFM), D. S. Dean (University of Bordeaux), E. Dumonteil and A. Zoia (CEA, Saclay), P. Mounaix and D. S. Grevenkov (Ecole Polytechnique), C. Godrèche (IPhT, Saclay), P. Le Doussal (ENS, Paris), K. Wiese (ENS, Paris), K. Mallick (IPhT, Saclay), H. Orland (IPhT, Saclay). Besides, I have also collaborated in the past with M. Yor (Laboratoire de Probabilités et Modèles Aléatoires, Univ. Paris VI et VII), G. Borot and B. Eynard (IPHT, Saclay), C. Sire (Université Paul-Sabatier, Toulouse) and M. Vergassola (Institut Pasteur, now at ENS, Paris).

Apart from carrying out research and writing articles, I believe I have played a major role in establishing these collaborations and also in bringing new directions of research.

4.2 Collective Responsibilities:

I have been members of various scientific commissions in and outside France. These include the following:

• Member of the commission de spécialistes (section 29) at the Université Paul Sabatier, Toulouse, France (2004-present).

• Member of the 'Comité d'évaluation' of the Laboratoire de Physique Théorique (LPTENS) at Ecole Normale Superieure, January (2005).

• Member of the 'Conseil de laboratoire LPTMS' since November, 2005.

• Member of the 'Comité d'évaluation' (AERES) of the Laboratoire de Physique Théorique (LPTENS) at Ecole Normale Superieure, January (2009).

• Member of the 'Appointments Committee" and the 'Management Board" of the International Center for Theoretical Studies (ICTS) at Bangalore, India, since 2011.

• Member of the comite de selection (section 29) (concours de recrutement MCF) at the Universite' Aix Marseille, April, 2012.

• Member of the comite de selection (section 29) (concours de recrutment Prof.) at the Universite' de Cergy-Pontoise, April, 2014.

• Member of the advisory board of "Fundamental Problems in Statistical Physics" (FPSP) since 2015.

• Member of the International advisory board of Statphys 28 (to be held in Tokyo, 2022).

4.3 Scientific Grants:

(1) We have obtained an ANR grant (2017) "Random matrices and trapped fermions" (RAMATRAF) jointly with D. Dean (Bordeaux), P. Le Doussal (ENS, Paris) and G. Schehr (LPTMS, Orsay, coordinator). We recruited N. Smith as a postdoc under this grant.

(2) We have obtained a 4-year research grant (2016-2019) from the Indo-French Center for the promotion of Advanced research (IFCPAR) (project number (5604-2). On the French side the researchers are: C. Bernadin (Nice), S.N. Majumdar (LPTMS), K. Mallick (IPHT, Saclay), A. Rosso (LPTMS) and G. Schehr (LPTMS, coordinator). On the Indian side, the researchers are A. Dhar (ICTS, Bangalore), A. Kundu (ICTS, Bangalore) and S. Sabhapandit (RRI, Bangalore). We recruited U. Basu as a postdoc at LPTMS under this grant (starting September, 2017). In addition, this grant gave us travel money for several travels on both sides.

(3) We obtained the ANR grant (2011) "Marcheurs browniens repulsifs et matrices aleatoires" acronymed 'WALKMAT' (projet blanc) [Principal investigators: G. Schehr (LPT (now at LPTMS), Orsay) and S.N. Majumdar (LPTMS, Orsay)]. We recruited A. Kundu as a postdoc under this ANR grant.

(4) We obtained a 3-year research grant (2012-2014) from the Indo-French Center for the promotion of Advanced research (IFCPAR) (project number (4604-3). On the French side the researchers are: A. Comtet, S.N. Majumdar, A. Rosso, S. Ouvry (all LPTMS), G. Schehr (LPT), K. Mallick (CEA Saclay) and on the Indian side the researchers are D. Dhar (Tata Institute), S. Sabhapandit (RRI, Bangalore), D. Das (IIT-Mumbai), P.K. Mohanty (Saha

Institute, Calcutta). We recruited S. Gupta as a postdoc at LPTMS under this grant. In addition, this grant gave us travel money for several travels on both sides. Our grant got an 'excellent' rating after the completion of the project.

(5) We obtained a 3-year research grant (2006-2009) from the Indo-French Center for the promotion of Advanced research (IFCPAR) (project no: 3404-2). On the French side the researchers are: A. Comtet, J. Jacobsen, and myself, all from LPTMS (ORSAY) and on the Indian side: M. Barma (Tata Institute, Bombay), D. Das (IIT, Bombay) and D. Dhar (Tata Institute, Bombay). The grant started in January, 2006. S. Sabhapandit worked as a postdoc with me at LPTMS (2006-2008) under this IFCPAR project. Sabhapandit has now obtained a permanent faculty position at Raman Research Institute (Bangalore, India).

(6) I was the 'responsible' (French side) for the Ecos-Sud (Franco-Argentinian) project A12E05 titled "Physique des extremes et des records: avalnaches, chocs, diffusion anormale et fonctions de grandes deviations" (2013-2014).

(7) I was a member of the EVERGROW project funded by the EC in the FP6 program.

(8) I was part of the European program 'NETADIS' (Statistical Physics Approaches to Networks Across Disciplines) which started from September, 2012. Under this program, we had two foreign Ph.D students this year at LPTMS. Caterina Di Bacco did her Ph.D thesis (from September, 2012 to September, 2015) jointly with myself and S. Franz at LPTMS.

(9) A postdoctoral research grant was obtained from the French "Ministère de la Recherche" for the year 2003-2004 and the candidate Dr. Swati Khandelwal worked in our group at Toulouse starting January, 2003.

4.4 Supervision of Students:

Ph.D students (direction and co-direction):

1. Francesco Mori has joined me for his Ph. D work on extremal properties of Brownian motion in 2019. Francesco has already wriiten 7 papers with me.

2. T. Gautie is currently in 3rd year of Ph. D (jointly with P. Le Doussal and J.-P. Bouchaud). Tristan has so far written one paper with me.

3. P. Mergny is currently in 2nd year of Ph. D (jointly with M. Potters).

4. A. Flak is currently in 1st year of Ph. D (jointly with S. Nechaev). She is also collaborating with G. Schehr.

5. Aurelien Grabsch did his Ph.D thesis at LPTMS (defended in 2018) under the joint supervision of myself and C. Texier. He worked on the applications of random matrix theory in mesoscopic transport, with title "Random matrices in statistical physics: quantum scattering and disordered systems". Aurelien is curently a posidoc at the University of

Leiden in the group of Prof. Carlo Beenakker. For his Ph. D thesis, Aurelien obtained the prestigious 'Prix de these PhoM' (2018) awarded by the Universite Paris-Saclay.

Grabsch has written 4 papers with me ([232], [235], [245], [246]).

6. Joel Bun finished his Ph.D thesis (jointly with myself and M. Potters from CFM) in 2016. His thesis is titled "Application de la theorie des matrices aleatoires pour les statistiques en grande dimension". Joel is curently employed in a financial company in New York.

Joel has so far written one paper with me ([198]).

7. Ricardo Marino did his Ph.D thesis at LPTMS (starting September, 2012) under the joint supervision of myself and P. Vivo. His thesis is titled "Number statistics in random matrices and applications to quantum systems". He defended his thesis in October, 2015 and after a year's postdoc at the Weizmann Institute (Israel), he is currently employed by Googles Research in Paris.

Ricardo has written three papers with me ([187], [192], [227]).

8. Caterina De Bacco finished her Ph.D at LPTMS (september, 2015) under the joint supervision of myself and S. Franz. Her thesis title is "Decentralized network control, optimization and random walks on networks". Caterina is currently at the Max planck Institite Tuebingen (Germany). In 2021, Caterina has been awarded the prestigious EPS-SNPD early career prize.

Caterina has written one paper with me ([209]).

9. Celine Nadal did her Ph.D thesis under my supervision at LPTMS. She defended her thesis titled "Matrices Aléatoires et leurs applications à la physique statistique et physique quantique" in June, 2011. Celine obtained the prestigious All-Souls fellowship for 5 years at Oxford University (UK). She then successffully pursued an alternative career in Archeology and is currently the head of the Archeology deprtament at Troy museum in France.

Celine has written 7 papers with me ([140], [142], [149], [153], [156], [157], [159]).

10. Julien Randon-Furling defended his Ph.D thesis (2006-2009) at LPTMS under the joint supervision of myself and A. Comtet. His thesis title was: " Statisques d'extrêmes du mouvement Brownien et applications" He did a postdoc at Saarbrucken (Germany) and has since obtained (June, 2010) a MCF position at the laboratoire SAMM (Statistique, Analyse, Modélisation Multidisciplinaire (EA 4543)) at the university Paris-1.

Julien has written 5 papers with me ([116], [125], [130], [141] and [147]).

11. Shamik Gupta at Tata Institute (Bombay) did his Ph.D thesis on 'Finite Size Effects in the Tagged Particle Correlations in Interacting Particle Systems" under the joint supervision of myself and Prof. M. Barma (Tata Institute). Shamik is currently a faculty member at the Belur University at Calcutta (India).

Shamik has written three papers with me ([110], [117], [190]).

12. Co-direction (along with M. Barma at Tata Institute) of the Ph.D thesis of Apoorva Nagar during 2003-2005 on "Passive Sliders on Fluctuating Surfaces and Strong-Clustering States". Apoorva now has a permanent faculty position at ICSAR, Bhopal in India.

Apoorva has written 3 papers with me (see [88],[92] and Conf. Proc. [2]).

13. Co-direction (along with M. Barma at Tata Institute) of the Ph.D thesis of Dibyendu Das during 1999-2002 on "Fluctuation dominated phase ordering dynamics". Dibyendu now has a permanent faculty position at IIT, Mumbai in India.

Dibyendu has written 4 papers with me (see [48], [76], [83], [111]).

14. Supervision of the thesis of R. Rajesh (official director of the thesis was Prof. D. Dhar) at Tata Institute during 1999-2000. Rajesh worked with me on a number of projects including the statistical properties in the random average process, exact corelation functions in granular systems and nonequilibrium phase transition in aggregation models. Rajesh is currently a permanent member of the theoretical physics group at IMSC, Chennai, India.

Rajesh has written 4 papers with me (see [38], [40], [44], [50]).

15. Partial supervision of the thesis of Supriya Krishnamurthy (official director of the thesis-Prof. M. Barma) at Tata Institute during 1997-1999. Supriya worked with me on various aggregation models with applications in cluster dynamics. Currently she has a permanent position at the university of Stockholm, Sweden.

Supriya has written 3 papers with me (see [33], [37], [39]).

16. Partial supervision of Abhishek Dhar, a graduate student of Prof. D. Dhar at Tata Institute. He worked with me on the problems of persistence and occupation time. Abhishek is now a professor at ICTS (Bangalore, India).

Abhishek has written 4 papers with me (see [34], [238], [247], [250]).

17. Kedar Damle, a graduate student of Prof. S. Sachdev at the Yale University worked with me during 1995-1996 on the nonequilibrium dynamics of a dilute Bose gas in a harmonic trap and a number of other problems on quantum spin chains. Kedar is currently a professor at Tata Institute, Mumbai.

Kedar has written 3 papers with me (see [24], [25], [162]) and a conference proceeding [1].

18. T. Senthil, a graduate student of Prof. S. Sachdev at the Yale University worked with me during 1995-1996 on the random quantum Potts and clock model and also on the critical behavior of trapped atomic gases. Senthil is currently a professor at MIT (USA).

Senthil has written 2 papers with me (see [23], [25]).

Master students:

1. M. Biroli (M1) did his master's thesis (2021) on the "Number of distinct sites visited by a resetting random walk in d-dimensions".

2. C. Di Bello (M2, Complex Systems) did his master's thesis (2021) on "Currebt distribution for noninteracting particles in a semi-infinite system".

3. Francesco Mori has done his Master's thesis (International master course in physics of complex systems, UPMC, Jan-June 2018) on "Distribution of the time difference between the maximum and the minimum of a Brownian motion of fixed duration". He is starting as my Ph. D thesis student in October 2019.

4. M. Magoni has done his Master's thesis (International master course in physics of complex systems, UPMC, Jan-June 2018) on "Glauber-Ising dynamics in the presence of stochastic resetting", under my supervision.

5. Lorenzo Palmieri has done his Master's thesis (International master course in physics of complex systems, UPMC, Jan-June 2016) on "Record statistics" under the joint supervision of myself and G. Schehr.

6. Daming Li has done his Master's thesis (M2, MSA internship, Universite' Paris-VII) on "One-dimensional random walk with stochastic resetting" (Jan-June, 2016) under the supervision of myself and G. Schehr.

7. Irene Marzouli (from the University of Padova, Italy) did her Master's thesis (2015) at LPTMS under my supervision on "Statistics of forward and backward records in random time series".

8. G. Wergen, a Ph.D student of J. Krug at Cologne, spent 6 months at LPTMS (2012) and carried out part of his Ph.D thesis under my supervision and also in collaboration with G. Schehr.

Gregor Wergen has written 2 papers with me ([170], [171]).

9. Jasper Franke did his Master's thesis (2007) at LPTMS under my supervision. The title of his thesis is: "Effects of discrete time in a class of reaction-diffusion systems".

Jasper has written a paper with me ([169]).

10. Emeric Thibaud has done his stage (de première année de Magistère de Mathématiques) jointly with me and S. Ouvry at LPTMS (May, 2007), title: " Partitions Entières'.

11. Direction of the M.Sc thesis project of Ajay Gopinathan on "Persistence in Nonequilibrium Systems" during the summer of 1997 at Tata Institute, India. Ajay has written a paper on his M.Sc thesis and then went to do his Ph.D thesis at the University of Chicago and now he is a faculty at the University of California at Merced (USA).

12. Direction of the M.Sc thesis project of Shilpa Jain on "Stochastic Processes in Physics and Mathematics" during the summer of 1992 at Tata Institute. The formal director of the thesis was Prof. S. Wadia. Shilpa went to do her Ph.D thesis at Harvard University.

4.5 Supervision of Postdocs:

1. Supervision of Dr. N. Smith (2019-2020), who has just obtained a tenure track faculty position at the Ben-Gurion University (Israel).

2. Supervision of Dr. Tirthankar Banerjee (2018-2019), who is currently at the Cambridge University (UK) as a postdoc.

3. Supervision of Dr. Urna Basu (2017-2018), curently a faculty member at the Raman Research Institute, Bangalore, India.

Urna has so far written one paper with me (see [259])

4. Supervision of Dr. Jacek Grela (2016-2018), who just started a second postdoc at Queen Mary College, London, UK.

Jacek has so far written one paper with me (see [239]).

5. Supervision of Dr. Anupam Kundu (2012-2014, currently a faculty at ICTS, jointly with G. Schehr under the ANR grant WALKMAT).

Anupam has written 10 papers with me (see [180], [196], [197], [219], [222], [238], [247], [250], [262], [276]).

6. Supervision of Dr. Shamik Gupta (2012-2014, currently a faculty member at the Belur University, Calcutta, India), jointly with A. Rosso and G. Schehr, under the Indo-French CEFIPRA grant).

Shamik has written 3 papers with me (see [110], [117], [190]).

7. Supervision of Dr. Kabir Ramola (2012-2014, currently a postdoc at Brandeis university (USA)), jointly with G. Schehr and C. Texier.

Kabir has written three papers with me (see [189], [205], [208]).

8. Supervision of Dr. Dario Villamaina (2011-2013, currently a postdoc at ENS).

Dario has written 1 paper with me (see [174]).

9. Supervision of Dr. Sanjib Sabhapandit (2006-2008), currently a faculty at Raman Research Institute, Bangalore, India.

Sanjib has so far written 15 papers with me (see [96], [108], [115], [121], [122], [128], [135], [202], [211], [216], [229], [238], [247], [250], [262]).

10. Supervision of Dr. Swati Khandelwal at LPT, Toulouse (2003-2004) under a postdoctoral grant from the French "Ministère de la Recherche".

5 Scientific Commitments

5.1 Organization of Conferences and Seminars:

1. Co-organization of the international conference, ”Modern Field Theory Colloquium” held at Tata Institute, Bombay, 1990.

2. Co-organization of the international conference, ”Recent Developements in Theoretical Physics” held at Tata Institute, Bombay, 1999.

3. Co-organization and running of a physics seminar series ‘Random Interaction’ at Tata Institute, Bombay, India during 1997-1999.

4. Co-organization of the international conference, ”India and Abroad: Perspectives in Condensed Matter Physics-II” held at the Indian Institute of Science, Bangalore, January 2-5, 2002.

5. Co-organization of the international conference, ”India and Abroad: Perspectives in Condensed Matter Physics-III” held at the S.N. Bose Center for Basic Sciences, Calcutta, India, January 2-5, 2003.

6. Co-organizer (along with C. Godrèche and S. Redner) of the international conference, “First-Passage and Extreme Value Problems in Random Processes” at the Isaac Newton Institute, Cambridge (UK), June 26-30, 2006.

7. Co-organizer (along with C. Godrèche and G. Schehr) of the 16-th Itzykson meeting “Extremes and Records”, held at IPhT, Saclay, June 14-17, 2011.

8. Co-organizer of the summer school followed by an international conference on “Random matrix theory and applications” at the International Center for Theoretical Sciences (ICTS), Bangalore, India, January 16-30, 2012.

9. Co-organizer (along with D. Mukamel, H. Park, J.D Noh) of the international conference “Nonequilibrium Statistical Physics of Complex Systems” (NSPCS) (the 5-th KIAS conference on Statistical Physics) held at Korea Institute of Advanced Studies (KIAS), Seoul, South Korea, July, 2012.

10. Co-organizer (along with D. Mukamel, S. Ruffo, R. Livi)) of the international workshop “Advances in Nonequilibrium Statistical Mechanics” at the Galileo Galilei Institute at Florence, Italy, May-June, 2014.

11. Co-organizer (along with D. Mukamel, H. Park, J.D Noh) of the international conference NSPCS (the 6-th KIAS conference on Statistical Physics) held at Korea Institute of Advanced Studies (KIAS), Seoul, South Korea, July, 2014.

12. Co-organizer (along with D. Mukamel, H. Park, J.D Noh) of the international conference NSPCS (the 7-th KIAS conference on Statistical Physics) held at KIAS, Seoul, South Korea,

July, 2016.

13. Co-organizer (with D. Mukamel and Y. Kafri) of the international workshop "Correlations, Fluctuations and anomalous transport in systems far from equilibrium" held at SRITP, Weizmann Institute of Science, Israel, December 2017- January 2018.

14. Co-organizer (along with D. Mukamel, H. Park, J.D Noh) of the international conference NSPCS (the 8-th KIAS conference on Statistical Physics) held at KIAS, Seoul, South Korea, July, 2018.

5.2 Courses and Lectures:

1. In 1991-92, a set of lectures on "The Bethe Ansatz and Exactly Solvable Models in Statistical Mechanics" was given as part of a graduate course at Tata Institute, India.

2. In 1996, a set of lectures on "Nonequilibrium Dynamics in the Ising Model was given at Yale University, USA as part of the statistical mechanics graduate course conducted by Prof. R. Shankar.

3. In 1997, a full graduate course on "Modern Classical Mechanics and Dynamical Systems" was given at the Tata Institute, India.

4. In 1998, a set of lectures on Abelian Sandpile Model was given at the summer school "Temporal-Spatial Patterns" at the University of Twente, the Netherlands.

5. In 2003, a DEA (graduate level) course on 'Systems out of equilibrium' was given at the Universite Paul Sabatier, Toulouse (France).

6. In Feb. 2004, a course on "Stochastic Processes and Nonequilibrium Dynamics" was given at the SERC school held at Tata Institute of Fundamental Research, Bombay, India.

7. In March 2006, a set of 3 lectures were given in a summer school on "The Principles of the Dynamics of Non-Equilibrium Systems" held at the Isaac Newton Institute, Cambridge, UK.

8. In July 2006, a course of 3 lectures on "Tracy-Widom distribution of Random Matrices and its applications" was given at the summer school on "Complex Systems" at Les Houches, France.

9. In October-November 2006, a set of 6 lectures was given for the Ecole Doctorale Course at SPhT, Saclay (France) on "Brownian functionals and their applications".

10. In June 2007, a set of lectures was given at the international summer school "Physics and Computer Science" at Jacobs University, Bremen (Germany) on "Understanding search trees via statistical physics".

11. In June 2008, a set of 8 lectures was given at the Beg-Rohu summer school "Manifolds

in random media, random matrices and extreme value statistics" at Beg Rohu (France) on "Extreme statistics of correlated random variables".

12. In July 2008, a set of 3 lectures was given at Les Houches summer school "Exact Methods in Low-dimensional statistical physics and quantum computing" on "Real-space Condensation in Mass Transport Models", Les Houches (France).

13. In September 2009, a set of 4 lectures was given at the international summer school "Fundamental Problems in Statistical Physics XII" (Leuven, Belgium) on "Brownian Functionals and their Applications: From Records to Comets".

14. In May 2011, a set of 5 lectures was given at the Les Houches summer school "Vicious Walkers and Random Matrices" held at Les Houches (France).

15. In March 2013, a set of 4 guest lectures ($4 \times 2 = 8$ hours) on 'random walks' was given for the Master course on 'Complex Systems' at King's College (London, UK).

16. In April 2013, a set of 6 lectures + 2 tutorials (a total of 12 hours) on 'models of nonequilibrium physics' were given at the 4-th RRI school on Statistical Physics at the Raman Research Institute (RRI) (Bangalore, India).

17. In June 2013, a set of 9 lectures (a total of 13.5 hours) on 'Random matrix Theory and its applications' were given at the Beg-Rohu summer school on 'Disordered Systems' (Beg-Rohu, France, 2013).

18. In August 2013, a set of 4 lectures (total of 4 hours) on '3rd order phase transitions in random matrix madels' were given at the Bielefeld summer school on ' Randomness in Physics and Mathematics: From Quantum Chaos to Free Probability', held at the University of Beilefeld (Bielefeld, Germany, 2013).

19. In November 2013, a set of 2 lectures on "Extreme statistics" were given at the University of Calcutta, India (2013).

20. In April 2014, a set of 6 lectures + 2 tutorials (a total of 12 hours) on 'Random Matrix Theory and its Applications' were given at the 5-th RRI school on Statistical Physics at the Raman Research Institute (RRI) (Bangalore, India).

21. In May 2014, a set of 2 lectures on "Extreme value statistics of correlated variables" were given at the Gallileo Gallili Institute (Florence, Italy).

22. In May 2014, a set of 5 lectures (a total of 7.5 hours) on 'Random Matrix Theory and its Applications' were given at the Universitu of Los Andes (Bogota, Colombia).

23. In June 2015, a set of 9 lectures (a total of 18 hours) on 'Random Matrix Theory and its Applications' were given at the "Spring College on Complex Systems" held at ICTP (Trieste, Italy).

24. In November-December 2015, a course (approved by the Ecole Doctoral "Physique en Île-de-France- ED PIF) on "Introduction to Random Matrix Theory and its Varios Applications"

was given at IPhT, Saclay (a course of 10 lectures, lasting in total 12.5 hours).

25. In December 2016, a set of 4 lectures (a total of 6 hours) on "Random matrices and cold atoms" was given at Raman research Institute (RRI, Bangalore, India), intended for Ph.D and Master students.

26. In March 2017, a set of 5 lectures (a total of 7.5 hours) on "Statistics of Extremes in Random Sequences" was given at the Mathematics summer school at the Technical University of Darmstadt (Damstardt, Germany), for Ph.D and Master students.

27. In February 2018, a set of 10 lectures (a total of 10 hours) on "Introduction to Random Matrix Theory" was given at the summer school "Statistical Field Theories: 2018", held at the Galileo Galilei Institute (Florence, Italy) for Ph.D and Master students.

28. In September-October 2018, the Masters M2 course on 'Advanced Statistical Mechanics" was given at ENS-Lyon (a total of 24 hours), followed by an examination taken by 43 students.

29. In June 2019, a set of 6 lectures + 2 tutorials (a total of 12 hours) on 'Random Matrix Theory and its Applications' were given at the 10-th ICTS-RRI school on Statistical Physics at the Raman Research Institute (RRI) (Bangalore, India).

30. In September-October 2019, the Masters M2 course on 'Advanced Statistical Mechanics" was given at ENS-Lyon (a total of 24 hours), followed by an examination taken by 50 students.

5.3 Editor of Journals:

(1) Divisional Associate Editor (DAE) of Phys. Rev. Lett. (January, 2011 to December, 2013).

(2) Associate editor of Journal of Statistical Physics (since 2011). Previously, member of the editorial board of Journal of Statistical Physics (2008-2010).

(3) Since 2010, member of the editorial board of the Journal of Phys. A: Math. Theor (IOP). During (2019-2020), the Section editor (Statistical Physics) for J. Phys. A. Currently a member of its executive board.

(4) Member of the editorial board of Journal of Statistical Mechanices: Theory and Experiment (since 2003).

(5) Member of the editorial board of Pramana–Journal of Physics (since June 2014).

5.4 Referee of Journals:

Since 1992, papers were regularly reviewed for several international journals including Physical Review Letters, Physical Reviews (A, B, and E), Nature Physics, Europhys. Lett., Journal of Physics-A, Journal of Statistical Physics, Journal of Statistical Mechanics, Physica-A, IEEE transactions in information theory etc. The grant proposals for the Agence National de Research (ANR, France), National Science Foundation (NSF, USA) and the Israel Science Foundation (ISF, Israel)) were also reviewed.

Recognized as a 'distinguished referee' by the EPL (Europhysics Letters) (2010).

5.5 Other Scientific Activities: Thesis Examinations

• Member of the thesis committee of a number of Ph.D students at Tata Institute, Bombay during the period 1997-1999. I was one of the reporters (and a member of the jury) for the following thesis in France:

1. "Problèmes Stochastiques Associés à l'équation de Langevin: Persistence et Processus de Réaction-Diffusion" (Université Paris 6) by O. Deloubriére at the LPT, Orsay in September, 2001.

2. "Systèmes hors d'équilibre: persistence et métastabilité" (Docteur de l'Ecole Polytechnique) by G. De Smedt at SPhT at Saclay in September, 2002.

3. "Nonequlibrium Dynamics in Fiber Networks, aggregation, and Sand Ripples" by E. Hellen at the Helsinki University of Technology in November, 2002.

4. "Thermodynamique et Dynamique hors d'équilibre de systèmes élastiques désordonnés" (Docteur de l'université Paris 6) by G. Schehr at LPTENS, September 2003.

5. "Modèles dilués en physique statistique: Dynamiques hors d'équilibre et algorithmes d'optimisation" (Docteur de l'université Paris 6) by G. Semerjian at LPTENS, June 2004.

6. "Vicious walkers in one-body potentials" (M.Sc thesis at the University of Manchester, UK) by K. Winkler, September 2004.

7. "Méthodes Probabilistes pour la Vérification des Systèmes Distribués" (ENS de Cachan) by S. Messika, December 2004.

8. " Milieux granulaires denses, gaz granulaires: des systèmes modèles hors de l'equilibre" (Habilitation 'a diriger des recherches, Université de Paris XI, Orsay) by A. Barrat, April 2005.

9. " États Stationnaires hors de l'équilibre: quelques propriétés génériques" (Habilitation à diriger des recherches, Université de Paris XI, Orsay) by F. Van Wijland, September 2006.

10. "Thermodynamique des histoires et fluctuations hors d'équilibre" (Docteur de l'université Paris VII) by V. Lecomte, March 2007.

11. "Approximations simples d'intégrals de chemins 'a température finie" (Docteur de Laboratoire de Physique de l'Ecole Normale Supérieure de Lyon) by S. Paulin, December 2007.

12. “A Study of Some First Passage Problems Using Fokker-Planck Methods” (University of Manchester, UK) by R. Smith, March 2008.

13. “Dynamique hors équilibre à travers une transition de phase” (Thèse de Doctorat de L’université Pierre et Marie Curie) by A. Sicilia, September 2009.

14. “ Reaction-diffusion Kinetics” (Habilitation ‘a diriger des recherches, Université de Paris VI, Jussieu) by G. Oshanin, March 2010.

15. “Vieillissement dans les processus réaction-diffusion sans bilan détaillé ” (Thèse de Doctorat de L’université Henri Poincaré, Nancy I en Physique) by X. Durang, September 2011.

16. “Sommes et extremes en physique statistique et traitment du signal: Ruptures de convergences, effets de taille finie et representations mattricielles” (Thèse de Doctorat de l’ENS de Lyon) by F. Angeletti, December 2012.

17. “Dynamic cavity method and problems on graphs” (Thése de Doctorat d’Université Paris-Sud) by Andrey Y. Lekhov, November 2014 (president de jury).

18. Member of the jury and the co-directeur de these: “Decentralized network control, optimization and random walks on networks” (Thèse de Doctorat de L’université Paris-Sud XI) by C. De Bacco, October 2015.

19. Member of the jury and the directeur de these: “Number statistics in random matrices and applications to quantum systems” (Thèse de Doctorat de L’université Paris-Sud XI) by R. Marino, October 2015.

20. Member of the jury: “Comportement microscopique de particules en interaction: gaz de Coulomb, Reisz et log-gases” (Thèse de Doctorat de UPMC) by T. Leblé, February, 2016.

21. Member of the jury and the co-directeur de these: “Application de la théorie des matrices aléatoires pour les statistiques en grande dimension” by J. Bun, 2016.

22. Member of the jury: “ À l’interface entre systèmes physiques et modèles mathématques: Propriétés de premier passage d’interfaces fractionnaires et grandes déviations de moddeles cinétiquement constraints” (Thèse de Doctorat de Université Paris Diderot-Paris 7), by Arturo Leos Zamorategui, November, 2017.

23. Member of the jury and the co-directeur de these: “Random matrices in statistical physics: quantum scattering and disordered systems” by A. Grabsch, July, 2018.

24. Member of the jury de these: “Extreme value statistics of strongly correlated systems: fermions, random matrices and random walks” by B. Lacroix-a-chez-toine, LPTMS, Université Paris-Sud/Paris-Saclay, June, 2019.

25. Member of the jury for the HDR: “De la méchanique des interfaces liquides á la microstrucure des marchés financiers, quelques problémes de modélisation” by M. Benzaquen, Sorbonne Université, Paris, June, 2019.

6 Scientific Collaborations

I have been collaborating with many theorists and experimentalists (more than 100 collaborators internationally and (>40 in France including >12 amongst permanents in LPTMS), since 1996), including my Ph.D students and postdocs. The names and the current affiliations of some of my collaborators are given below.

In France:
My collaborations in France and in LPTMS have already been mentioned in section 3.1. Below I highlight my collaborations outside France.

Outside France: I have collaborated with several theorists and experimentalists as listed below.

• Theory:

• D. Dhar, M. Barma, K. Damle, V. Tripathi, T. Sadhu (Tata Institute, India), C. Dasgupta (Indian Institute of Science, Bangalore, India), A. Dhar, M. Kulkarni, and A. Kundu (ICTS, Bangalore, India), R. Rajesh (IMSC, Chennai, India), D. Das (Indian Institute of Technology, Bombay, India), A. Lakshminarayan (IIT Madras, India), S. Sabhapandit and U. Basu (Raman Research Institute, Bangalore, India), K. Sengupta and B. Mukherjee (IACS, Calcutta, India).

• A.J. Bray, S.J. Cornell and G. Ehrhardt (Manchester University, UK), M.J. Kearney and R. Martin (University of Surrey, UK), M.R. Evans, T. Hanney, R. Allen, J. Whitehouse, J. Szavits-Nossan (University of Edinburgh, UK), P. Sollich and P. Vivo (Kings College, London, UK), R. Horgan and I. Smith (Cambridge University, UK).

• D.A. Huse (Princeton University, USA), S. Sachdev (Harvard University, USA), T. Senthil (MIT, USA), S. Redner (Santa Fe), P.L. Krapivsky (Boston University, USA), R.M. Ziff (University of Michigan, USA), A. Kostinski (Michigan State, USA), S.N. Coppersmith (University of Wisconsin, USA), S. Nagel and T. Witten (University of Chicago, USA), O. Narayan (UCSC, USA), A.M. Sengupta (Rutgers University, USA), B. Lubachevsky (Bell Labs, USA), B. Chakraborty (Brandeis University, USA), M. Constantin and S. Das Sarma (University of Maryland, USA), V. Privman and D. Ben-Avraham (Clarkson University, USA), E. Ben-Naim (Los Alamos, USA), R.K.P. Zia (Virginia Tech, USA).

• J. Krug, J. Franke, G. Wergen, P. von Bomhardt (Cologne, Germany), P. Grassberger (Julich, Germany), A.K. Hartmann, G. Claussen, T. Dewenter and H. Schwae (Oldenburg, Germany).

• A. Scardichio (ICTP, Trieste, Italy), P. Calabrese (Sissa, Trieste, Italy), G. Gradenigo (Univ. of Aquilla, Italy), R. Livi and S. Iubini (University of Florence, Italy).

• I. Bena (University of Geneva, Switzerland).

• F. D. Cunden and N. O'Connell (Dublin, Ireland).

• J. M. Meylahn and H. Touchette (University of Stellenbosch, South Africa).

• F. den Hollander (Leiden University, The Netherlands)

• A. Bar, J. Cividini, D. Mukamel, Y. Edery, B. Berkowitz (Weizmann Institute, Rehovot, Israel), B. Meerson (Hebrew University of Jersusalem, Israel), A. Pal (Tel-Aviv University, Israel).

• S. Krishnamurthy (University of Stockholm, Sweden).

• P. J. Forrester (University of Melbourne, Australia).

• D. Boyer, A. Falcon-Cortes, I. P. Castillo (UNAM, Univ. of Mexico, Mexico).

• J. Grela (University of Krakow, Poland).

• **Experiment:** I have also collaborated with several experimentalists during the last two decades:

(i) D. Bishop and his group (at Bell Labs, USA): On transport properties in high-T_c superconductors in the vortex liquid regime.

(ii) S. Nagel and his group (Univ. of Chicago, USA): On stress propagation in granular media.

(iii) B. Yurke and his group (at Bell Labs, USA): On persistence properties and phase ordering dynamics in liquid crystals.

(iv) B. Berkowitz and Y. Edery (Dept. of Environmental Sciences, Weizmann Institute, Rehovot, Israel): on measuring the statistics of records in particle tracking experiments in porous media.

(v) I have an ongoing collaboration with the experimental group of S. Ciliberto (ENS-Lyon) since 2019. Recently, we verified our theoretical predictions for the optimal resetting paradigm for diffusing particles in an experiment in laser traps performed at ENS-Lyon. These results are published in a series of recent papers (see my list of publication).

7 Research Report

My research interests cover various problems in equilibrium and nonequilibrium statistical physics with applications ranging from granular medium to computer science. Some of the past and present projects, along with the main results and the collaborators, are highlighted below.

New themes developed after 2004: The following new directions of research were developed over the last few years.

• *Extreme Value Statistics of Strongly Correlated Variables*: two particularly interesting strongly correlated systems where we made much progress in recent years: (i) Brownian Motion and various related stochastic processes (ii) Eigenvalues in random matrices. Other questions related to extremes have also been studied, such as the statistics of the number of extrema in a random landscape, integer partition problem, level density of Bose gas and its relation to extreme statistics, zeroes of random polynomials and longest excursions in stochastic processes in nonequilibrium systems.

• *Order, Gap and Record Statistics for stochastic time series:* Various other questions related to extreme statistics have also been studied, such as the order and the gap statistics, record statistics, density of near-extreme events etc. in random walks and related stochastic processes. Since this is part of the ongoing projects, it will be discussed in detail in Section 8.

• *Diverse applications of random matrix theory*: My current/ongoing research involves various applications of the random matrix theory such as the study of (i) fluctuations of the number of eigenvalues in a given interval–the so called *Index problem* (ii) transport in mesoscopic cavities (ii) distribution of entanglement entropy in random pure states of bipartite systems (iv) nonintersecting Brownian motions and its connection to Yang-Mills gauge theory (v) matrix integrals and associated fluid dynamics (vi) application of the random matrix theory to the physics of cold atoms, in particular to free fermions in a confining trap. This is also part of ongoing and future projects and will be discussed in detail in Section 8.

• *Random Growth Models, Biological Sequence Matching Problems and relation to Random Matrices.*

• *Bose-Einstein Condensation in Real Space.*

• *Passive Sliders on Fluctuating Surfaces: the phenomenon of Strong Clustering.*

• *Discrete-time Random Walk and Flux to a Trap.*

• *A class of Combinatorial Optimization problems.*

• *Applications of Brownian Functionals in Pure and Disordered Systems.*

Themes developed before 2004 : I have also continued to work on previously developed themes listed below:

- *Applications of Statistical Physics in Computer Science.*
- *Extreme Value Statistics and Traveling Fronts.*
- *Persistence in Nonequilibrium Systems.*
- *Nonequilibrium Phase Transitions in Models of Aggregation.*
- *Stress Propagation and Compaction in Granular Medium.*
- *Quantum Phase Transitions in Disordered Spin Chains.*
- *Coarsening and Phase Ordering Dynamics.*
- *Transport Properties of Vortices in High-T_c Superconductors.*
- *Interacting Particle Systems.*
- *Polymers and Self-Avoiding Walks.*
- *Self-organized Criticality in Sandpile Models (Ph.D. Thesis, 1992).*

7.1 Extreme Value Statistics of Strongly Correlated Variables

Extreme events are ubiquitous in nature. They may be rare events but when they occur, they may have devastating consequences and hence are rather important from practical points of view. To name a few, different forms of natural calamities such as earthquakes, tsunamis, extreme floods, large wildfire, the hottest and the coldest days, stock market risks or large insurance losses in finance, new records in major sports events like Olympics are the typical examples of extreme events. There has been a major interest to study these events systematically using statistical methods and the field is known as Extreme Value Statistics (EVS). This is a branch of statistics dealing with the extreme deviations from the mean/median of probability distributions. The general theory sets out to assess the type of probability distributions generated by processes which are responsible for these kind of highly unusual events. In recent years, it has been realized that extreme statistics and rare events play an equally important role in various physical/biological/financial contexts as well. A typical example can be found in disordered systems where the ground state energy, being the minimum energy, plays the role of an extreme variable. In addition, the dynamics at low temperatures in disordered are governed by the statistics of extremely low energy states. Hence the study of extremes and related quantities is highly important in the field of disordered systems. An important physical system where extreme fluctuations play an important role correspond to fluctuating interfaces of the Edwards-Wilkinson/Kardar-Parisi-Zhang varities. Another exciting recent area concerns the distribution of the largest eigenvalue in random matrices: the limiting Tracy-Widom distribution and the associated large deviation probabilities of the largest eigenvalue and its various applications. Extreme value statistics also appears in computer science problems such as in binary search trees and the associated search algorithms. We have recently written an exhaustive review on this subject:

S. N. Majumdar, A. Pal, G. Schehr, "Extreme value statistics of correlated random variables: A pedagogical review", Phys. Rep. v-840, 1 (2020).

In the classical extreme value theory, one is concerned with the statistics of the maximum (or minimum) of a set of *uncorrelated* random variables $\{x_1, x_2, \ldots, x_N\}$, where each x_i is drawn from the identical parent pdf $p(x)$ (i.i.d). For the *uncorrelated* case, the statistics of the maximum (or the minimum) can be studied exactly: the distribution of the maximum $M = \max(x_1, x_2, \ldots, x_N)$, appropriately centered and scaled, approaches in the large N limit to one of the three classical limiting distributions: Fisher-Tippett-Gumbel, Fréchet or Weibull depending on the tail of the parent pdf $p(x)$.

In contrast, in most of the physical systems mentioned above, the underlying random variables are typically *strongly correlated*. In such correlated systems, the probability distribution law of the maximum or minimum is, in general, not known. So, the general goal is to understand how the statistics of extremes behave when there are *correlations* between the random variables. The correlations between the variables can be either *weak* or *strong*. Suppose that the connected part of the correlation function decays fast (say exponentially) over a certain finite correlation length ξ

$$C_{i,j} = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle \sim \; e^{-|i-j|/\xi} \tag{1}$$

Clearly, when two variables are separated over a length scale larger than ξ, i.e., when $|i - j| >> \xi$, then they essentially get uncorrelated. Now *weak* correlation implies that $\xi << N$, where N is the total size of the sample. In contrast if $\xi \sim N$, the system is *strongly* correlated.

For the *weakly* correlated case, one can construct a heuristic argument to study the extreme statistics [80]. Consider $N' = \xi << N$ and break the system into identical blocks each of size ξ (see Fig. 1). There are thus N/ξ number of blocks. While the random variables inside each box are still strongly correlated, the variables that belong to different boxes are approximately uncorrelated. So, each of these boxes are non-interacting. Now, for each box i, let y_i denote the 'local maximum', i.e., the maximum of all the x-variables belonging to the i-th block, where $i = 1, 2, \ldots, N' = N/\xi$. By our approximation, y_i's are thus essentially *uncorrelated*. So, we have

$$M = \max[x_1, x_2, ..., x_N] = \max[y_1, y_2, ..., y_{N'}] \tag{2}$$

So, in principle if one knows the PDF of y, then this problem is essentially reduced to calculating the maximum of N' uncorrelated random variables $\{y_1, y_2, \ldots, y_{N'}\}$, which has already been discussed before. So, we know that depending on the tail of $p(y)$, the limiting distribution of M for N weakly correlated variables will, for sure, belong to one of three (Fisher-Tippett-Gumbel, Fréchet, or Weibull class) limiting extreme distributions of i.i.d random variables. To decide the tail of $p(y)$, of course one needs to solve a *strongly* correlated problem since inside each block the variables are strongly correlated. However, one can often guess the tail of $p(y)$ without really solving for the full pdf of $p(y)$ and then one knows, for sure, to which class the distribution of the maximum belongs to. However, this argument breaks down when $\xi \sim N$, i.e., when the system is *strongly* correlated. The real challenge

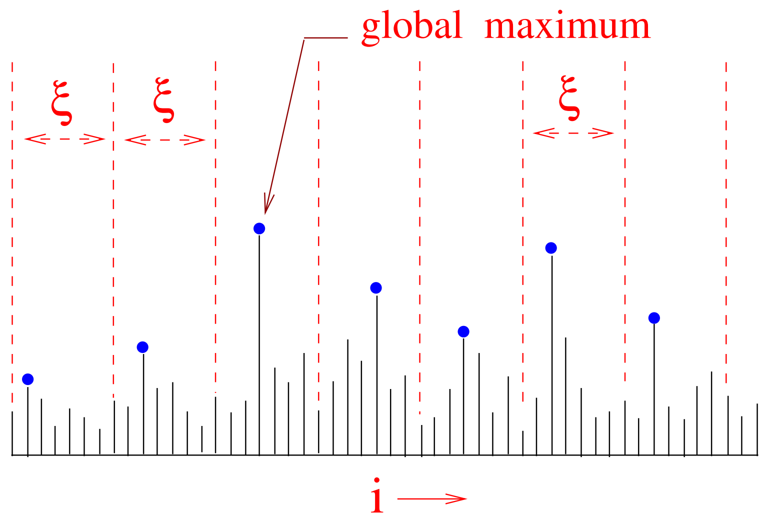


Figure 1: The sequence of N weakly correlated random variables are broken into $N' = N/\xi$ blocks, where ξ denotes the correlation length. Within each block, the variables are strongly correlated, while across blocks they are essentially uncorrelated.

then is to compute the distribution of the maximum when the underlying random variables are *strongly* correlated.

In recent years, there have been some advances in the understanding of EVS of *strongly* correlated variables and our group at Orsay played a pioneering role in this field which are briefly highlighted below [*a recent review is available in arXiv:1406.6768, based on my lectures at Galileo Galilei Institute, Florence (May, 2014) during the workshop on 'Advances in Nonequilibrium Systems'*. With G. Schehr, I am now writing a book on this subject to be published by the Oxford University Press.]

Two particularly simple physical systems where the underlying random variables are *strongly correlated* and yet, one can make some analytical progress in studying their extremal statistics, are (i) Constrained Brownian motions (arising for example in the stationary state of one dimensional fluctuating interfaces) and (ii) Statistics of the largest eigenvalue in random matrix theory.

(i) **Constrained Brownian motion in Fluctuating Interfaces:**

Fluctuating $(1+1)$-dimensional interfaces (or lines) are abundant in nature. For example, it can be the domain wall in a magnet, the dipolar chain in a ferrofluid or a fluctuating step-edge in a crystal. There are many experimental systems of $(1+1)$-d lines and their static and dynamic properties have been studied extensively for the past two decades. A particularly simple model of such a fluctuating line, that describes well the experimental system of step edges on crystals, is called the Edwards-Wilkinson model where the height field $H(x,t)$, defined over a linear substrate of size L, evolves via a stochastic equation which has a diffusion term and an additive thermal noise. Another extremely well studied model is the so called Kardar-Parisi-Zhang (KPZ) interface, where the height field $H(x,t)$ evolves via the stochastic nonlinear partial differential equation

$$\frac{\partial H}{\partial t} = \frac{\partial^2 H}{\partial x^2} + \lambda \left(\frac{\partial H}{\partial x}\right)^2 + \eta(x,t) \tag{3}$$

where $\eta(x,t)$ is a Gaussian white noise with zero mean and a correlator $\langle \eta(x,t)\eta(x',t')\rangle = 2\delta(x-x')\delta(t-t')$. For $\lambda = 0$, the equation becomes linear and reduces to the Edwards-Wilkinson (EW) equation.

The height is usually measured relative to the spatially averaged height i.e.

$$h(x,t) = H(x,t) - \frac{1}{L}\int_0^L H(y,t)dy \tag{4}$$

$$\text{with } \int_0^L h(x,t)dx = 0 \tag{5}$$

It can be shown that the joint PDF of the relative height field $P(\{h\},t)$ reaches a steady state as $t \to \infty$ in a finite system of size L. Also the height variables are strongly correlated in the stationary state. Again in the context of the EVS, a quantity that has created some interests recently is the PDF of the maximum relative height in the stationary state, i.e. $P(h_m, L)$ where

$$h_m = \lim_{t\to\infty} \max_x [h(x,t), 0 \leq x \leq L]. \tag{6}$$

This is an important physical quantity that measures the extreme fluctuations of the interface heights. We assume that initially the height profile is flat. As time evolves, the heights of the interfaces at different spatial points grow more and more correlated. The correlation length typically grows as $\xi \sim t^{1/z}$ where z is the dynamical exponent ($z = 3/2$ for KPZ and $z = 2$ for EW interfaces). For $t << L^z$, the interface is in the 'growing' regime where the height variables are weakly correlated since $\xi \sim t^{1/z} << L$. In contrast, for $t >> L^z$, the system approaches a 'stationary' regime where the correlation length ξ approcahes the system size and hence the heights become strongly correlated variables.

In the weakly correlated 'growing' regime ($\xi \sim t^{1/z} << L$), one can divide the system into $M = L/\xi(t) >> 1$ blocks which are essentially uncorrelated. Hence, one would expect that global maximal relative height, appropriately centered and scaled, should have the Fisher-Tippett-Gumbel distribution. In contrast, in the stationary regime, the height variables are strongly correlated and the maximal relative height h_m should have a different distribution. This distribution was computed numerically by the Rochester group [PRL, 87, 136101 (2001)]. In collaboration with A. Comtet (LPTMS), we were able to compute the distribution $P(h_m, L)$ exactly (Refs. [74] and [80]) using path integral techniques. This then presents one of the rare solvable cases for the EVS of strongly correlated random variables.

The joint PDF of the relative heights in the stationary state can be written putting all the constraints together [74, 80],

$$P_{st}[\{h\}] = C(L) e^{-\frac{1}{2}\int_0^L (\partial_x h)^2\ dx} \times \delta\Big[h(0) - h(L)\Big] \times \delta\Big[\int_0^L h(x,t)dx\Big] \tag{7}$$

where $C(L) = \sqrt{2\pi L^3}$ is the normalization constant and can be obtained integrating over all the heights. Note that this stationary measure of the relative heights is independent of the coefficient λ of the nonlinear term in the KPZ equation, implying that the stationary measure of the KPZ and the EW interface is the same in $(1+1)$-dimension. This is a special property only in $(1+1)$-dimension. The stationary measure indicates that the interface locally behaves as a Brownian motion in space. For an interface with periodic boundary condition, one would then have a Brownian bridge in space. However, the presence of the constraint $\int_0^L h(x,t)\,dx = 0$ (the zero mode being identically zero), as shown explicitly by the delta function in Eq. (7), shows that the stationary measure of the relative heights corresponds to a Brownian bridge, but with a global constraint that the area under the bridge is strictly zero–a fact that plays a crucial role for the extreme statistics of relative heights.

Using path integral techniques, we showed [74, 80] that the PDF of h_m has the scaling form for all L

$$P(h_m, L) = \frac{1}{\sqrt{L}} f\left(\frac{h_m}{\sqrt{L}}\right) \tag{8}$$

where the scaling function can be computed explicitly as [74, 80]

$$f(x) = \frac{2\sqrt{6}}{x^{10/3}} \sum_{k=1}^{\infty} e^{-\frac{b_k}{x^2}}\, b_k^{2/3}\, U\left(-\frac{5}{6}, \frac{4}{3}, \frac{b_k}{x^2}\right) \tag{9}$$

where $U(a,b,y)$ is the confluent hypergeometric function and $b_k = \frac{2}{27}\alpha_k^3$, where α_k's are the absolute values of the zeros of Airy function: $\mathrm{Ai}(-\alpha_k) = 0$. A plot of this scaling function $f(x)$ is shown in Fig. 2. It is easy to obtain the small x behavior of x since only the $k = 1$ term dominates as $x \to 0$. Using $U(a,b,y) \sim y^{-a}$ for large y, we get as $x \to 0$,

$$f(x) \to \frac{8}{81}\alpha_1^{9/2}\, x^{-5}\, \exp\left[-\frac{2\alpha_1^3}{27x^2}\right] \tag{10}$$

The asymptotic behavior of $f(x)$ at large x is more tricky to derive [80]. However, one can show that [74, 80]

$$f(x) \xrightarrow{x\to\infty} e^{-6x^2} \tag{11}$$

Surprisingly, this Airy-distribution function also appears in a number of seemingly unrelated problems, for instance, in computer science and graph theory [for a review see Ref. 80]. Thus our results show that this nontrivial function also appears in a physical system and for the first time provides a possibility of measuring this function in real experiments on fluctuating step-edges. Furthermore, in collaboration with G. Schehr (LPTMS, Orsay) (Ref. [95]), we have shown that this Airy-distribution function also appears in a wide class of one dimensional solid-on-solid models, and not just restricted to Edwards-Wilkinson or Kardar-Parisi-Zhang interfaces. For all these lattice models, in the large L limit, a central limit argument shows that, for periodic boundary conditions, $P(h_m, L)$ takes a universal scaling form $P(h_m, L) \sim (\sqrt{12}w_L)^{-1} f(h_m/(\sqrt{12}w_L))$, with w_L the width of the fluctuating interface and $f(x)$ the Airy distribution function. For one instance of these models, corresponding to the extremely anisotropic Ising model in two dimensions, this result is obtained by an exact computation using transfer matrix technique, valid for any $L > 0$. These arguments and exact analytical calculations are supported by numerical simulations, which show in addition that the subleading scaling function is also universal, up to a non universal amplitude, and

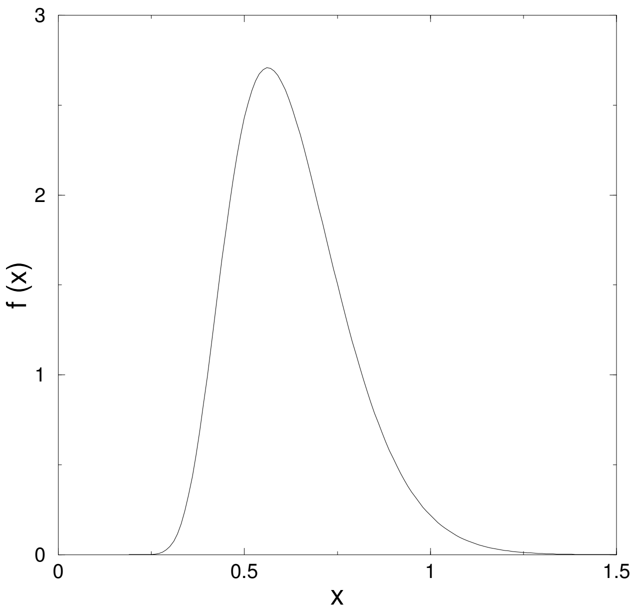


Figure 2: A Mathematica plot of the Airy distribution function $f(x)$ vs. x.

simply given by the derivative of the Airy distribution function $f'(x)$. Many beautiful results regarding the nonlinear recursion relation satisfied by the moments of this function have also been derived in Ref [114].

Collaborators: A. Comtet (LPTMS), G. Schehr (LPTMS), M.J. Kearney (Surrey), R.J. Martin (Surrey).

(ii) **Largest eigenvalue in the random matrix theory:**

Another beautiful solvable example of the extremal statistics of strongly correlated variables can be found in the distribution of the top eigenvalue in a random matrix [*for a recent review of the results and its various applications, see Ref. [188], based on my* **plenary** *lecture at STATPHYS-25 (Seoul, 2013) on this subject*].

Let us consider a $N \times N$ Gaussian random matrices with real symmetric, complex Hermitian, or quaternionic self-dual entries $X_{i,j}$ distributed via the joint Gaussian law:

$$\Pr[\{X_{i,j}\}] \propto \exp\left[-\frac{1}{2\sigma^2}\,\mathrm{Tr}(X^2)\right], \tag{12}$$

where σ sets the scale of the fluctuations of the matrix entries. The distribution is invariant respectively under orthogonal, unitary and symplectic rotations giving rise to the three classical ensembles: Gaussian orthogonal ensemble (GOE), Gaussian unitary ensemble (GUE) and Gaussian symplectic ensemble (GSE). The eigenvalues and eigenvectors are random and their joint distributions decouple. One can integrate out the eigenvectors, and focus only on the statistics of N eigenvalues $\lambda_1,\ \lambda_2, ...,\ \lambda_N$. The joint PDF of these eigenvalues is given by the classical result of Wigner-Dyson

$$P_{\rm joint}(\lambda_1,\ \lambda_2, ...,\ \lambda_N) = B_N(\beta) \exp\Big[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\lambda_i^2\Big]\prod_{i<j}|\lambda_i-\lambda_j|^\beta, \tag{13}$$

where $B_N(\beta)$ is a normalization constant and β is called the Dyson index, that measures the strength of the repulsion between pairs of eigenvalues. For the three rotationally invariant Gaussian ensembles, the index β takes quantized values: $\beta = 1$ (GOE), $\beta = 2$ (GUE) and $\beta = 4$ (GSE). For convencience, we will choose, without any loss of generalities, the scale $\sigma^2 = 1/\beta N$ and then we can rewrite the statistical weight as

$$P_{\text{joint}}(\lambda_1,\ \lambda_2, ...,\ \lambda_N) = B_N(\beta) \exp\left[-\beta\Big(\frac{N}{2}\sum_{i=1}^{N}\lambda_i^2 - \frac{1}{2}\sum_{i\neq j}\ln|\lambda_i - \lambda_j|\Big)\right], \tag{14}$$

The choice $\sigma^2 \sim O(1/N)$ ensures that the eigenvalues are typically of $O(1)$ in the large N limit.

Hence, this joint law can be interpreted as a Gibbs-Boltzmann measure (Dyson, 1962), $P_{\text{joint}}(\{\lambda_i\}) \propto \exp\left[-\beta\, E\left(\{\lambda_i\}\right)\right]$, of an interacting gas of charged particles on a line where λ_i denotes the position of the i-th charge and β plays the role of the inverse temperature. The energy $E\left(\{\lambda_i\}\right)$ has two parts: each pair of charges repel each other via a 2-d Coulomb (logarithmic) repulsion (even though the charges are confined on the 1-d real line) and each charge is subject to an external confining parabolic potential. Note that while $\beta = 1$, 2 and 4 correspond to the three classical rotationally invariant Gaussian ensembles, it is possible to associate a matrix model to (14) for any value of $\beta > 0$ (namely tridiagonal random matrices introduced by Dimitriu and Edelman in 2002). Here we focus on the largest eigenvalue $\lambda_{\max} = \max_{1\leq i\leq N}\lambda_i$: what can be said about its fluctuations, in particular when N is large ? This is a nontrivial question as the interaction term, $\propto |\lambda_i - \lambda_j|^\beta$, renders inapplicable the classical results of extreme value statistics for i. i. d. random variables.

The two terms in the energy of the Coulomb gas in (14), the pairwise Coulomb repulsion and the external harmonic potential, compete with each other. While the former tends to spread the charges apart, the later tends to confine the charges near the origin. As a result of this competition, the system of charges settle down into an equilibrium configuration on an average and the average density of the charges is given by

$$\rho_N(\lambda) = \frac{1}{N}\Big\langle \sum_{i=1}^{N}\delta(\lambda - \lambda_i)\Big\rangle \tag{15}$$

where the angular brakets denote an average over with respect to the joint PDF in Eq. (14). For such Gaussian matrices (14), it is well known (Wigner, Dyson) that as $N \to \infty$, the average density approaches an N-independent limiting form which has a semi-circular shape on the compact support $[-\sqrt{2}, +\sqrt{2}]$

$$\lim_{N\to\infty}\rho_N(\lambda) = \rho_{sc}(\lambda) = \frac{1}{\pi}\sqrt{2-\lambda^2} \tag{16}$$

where $\rho_{sc}(\lambda)$ is called the Wigner semi-circular law. Hence our first observation is that the maximum eigenvalue resides near the upper edge of the Wigner semi-circle:

$$\lim_{N\to\infty}\langle\lambda_{\max}\rangle = \sqrt{2} \tag{17}$$

However, for large but finite N, $\lambda_{\max}$ will fluctuate from sample to sample and the interseting thing would be to compute the cumulative distribution which is

$$Q_N(w) = \text{Prob}[\lambda_{\max} < w] \tag{18}$$

which can be written as a ratio of two partition functions

$$Q_N(w) = \frac{Z_N(w)}{Z_N(z \to \infty)}, \tag{19}$$

$$Z_N(w) = \int_{-\infty}^{w} d\lambda_1 ... \int_{-\infty}^{w} d\lambda_N \exp\left[-\beta\Big(\frac{N}{2}\sum_{i=1}^{N} \lambda_i^2 - \frac{1}{2}\sum_{i\neq j} \ln|\lambda_i - \lambda_j|\Big)\right] \tag{20}$$

where the partition function $Z_N(w)$ describes a 2-d Coulomb gas, confined on a 1-d line and subject to a harmonic potential, *in the presence of a hard wall at w*. The study of this ratio of two partition functions reveals the existence of two distinct scales correspond to (i) *typical* fluctuations of the top eigenvalue, where $\lambda_{\max} = \mathcal{O}(N^{-2/3})$ and (ii) *atypical* large fluctuations, where $\lambda_{\max} = \mathcal{O}(1)$. It can be shown that the typical fluctuations are governed by

$$\lambda_{\max} = \sqrt{2} + \frac{1}{\sqrt{2}} N^{-2/3} \chi_\beta \tag{21}$$

where χ_β is an N-independent random variable. Its cumulative distribution function (CDF), $\mathcal{F}_\beta(x) = \text{Prob}[\chi_\beta \leq x]$, is known as the β-Tracy-Widom (TW) distribution which is known only for $\beta = 1,\ 2,\ 4$ (Tracy & Widom, 1994). For arbitrary $\beta > 0$, it can be shown that this PDF has asymmetric non-Gaussian tails

$$\mathcal{F}'_\beta(x) \approx \begin{cases} \exp\left[-\frac{\beta}{24}|\mathrm{x}|^3\right], & x \to -\infty \\ \\ \exp\left[-\frac{2\beta}{3}\mathrm{x}^{3/2}\right] & x \to +\infty \end{cases} \tag{22}$$

These TW distributions also describe the top eigenvalue statistics of large real and complex Gaussian Wishart matrices. Amazingly, the same TW distributions have emerged in a number of a priori unrelated problems [*a partial review can be found in my Les Houches lecture notes, arXiv:cond-mat/0701193, Ref. 3 in Invited Reviews*] such as the longest increasing subsequence of random permutations, directed polymers, and growth models in the Kardar-Parisi-Zhang (KPZ) universality class in $(1+1)$ dimensions as well as for the continuum $(1+1)$-dimensional KPZ equation, sequence alignment problems, mesoscopic fluctuations in quantum dots, height fluctuations of non-intersecting Brownian motions over a fixed time interval, height fluctuations of non-intersecting interfaces on a substrate, and also in finance. Remarkably, the TW distributions have been recently observed in experiments on nematic liquid crystals [Takeuchi & Sano, 2010] (for $\beta = 1, 2$) and in experiments involving coupled fiber lasers [Fridman et. al. 2012] (for $\beta = 1$).

Large deviation tails:

A natural question then arises: Why is the Tracy-Widom distribution so ubiquitous and occurs in such diverse systems? In statistical physics, we are taught that near a critical point systems behave universally, and details become irrelevant. This raises the question if the Tracy-Widom distribution in all these systems can be viewed as an observable associated to a 'hidden' critical point? For the past several years, we have studied this question extensively, and have found that indeed in all these systems wherever the Tracy-Widom distribution appears, there is a hidden 3-rd order critical point, separating a 'strong coupling' phase from a 'weak coupling phase'. To detect these phases and the associated phase transition, one needs to study *atypical* large fluctuations of the basic observable, in this case $\lambda_{\max}$, say over a wider region of width $\sim O(1)$ around the mean? These 'atypical' large fluctuations, far from the mean, are not described by the Tracy-Widom distribution and one has to go beyond the standard Tracy-Widom to describe the law of such atypical fluctuations (see Fig. 3). Questions related to large deviations of extreme eigenvalues have emerged in a variety of contexts including cosmology, disordered systems such as spin glasses and in the assessment of the efficiency of data compression algorithms such as in Principal Component Analysis (PCA).

In my work over the past several years, we have shown that the probability of atypically large fluctuations, to leading order for large N, is described by two large deviations (or rate) functions $\Phi_-(x)$ (for fluctuations to the *left* of the mean) and $\Phi_+(x)$ (for fluctuations to the *right* of the mean). More precisely, the behavior of the cumulative distribution function $Q(w, N) = \text{Prob.}[\lambda_{\max} \leq w, N]$ of $\lambda_{\max}$ for large but finite N is described as follows

$$Q(w,N) \approx \begin{cases} \exp\left[-\beta N^2 \Phi_-(w)\right], & w < \sqrt{2} \text{ and } |w-\sqrt{2}| \sim \mathcal{O}(1) \\ \mathcal{F}_\beta\left(\sqrt{2} N^{\frac{2}{3}}(w-\sqrt{2})\right), & |w-\sqrt{2}| \sim \mathcal{O}(N^{-\frac{2}{3}}) \\ 1-\exp\left[-\beta N \Phi_+(w)\right], & w > \sqrt{2} \text{ and } |w-\sqrt{2}| \sim \mathcal{O}(1)\,. \end{cases} \tag{23}$$

Equivalently, the PDF of $\lambda_{\max}$, obtained from the derivative $P(w, N) = dQ(w, N)/dw$ reads (keeping only leading order terms for large N)

$$P(\lambda_{\max}=w,N) \approx \begin{cases} \exp\left[-\beta N^2 \Phi_-(w)\right], & w < \sqrt{2} \text{ and } |w-\sqrt{2}| \sim \mathcal{O}(1) \\ \sqrt{2} N^{\frac{2}{3}} \mathcal{F}'_\beta\left(\sqrt{2} N^{\frac{2}{3}}(w-\sqrt{2})\right), & |w-\sqrt{2}| \sim \mathcal{O}(N^{-\frac{2}{3}}) \\ \exp\left[-\beta N \Phi_+(w)\right], & w > \sqrt{2} \text{ and } |w-\sqrt{2}| \sim \mathcal{O}(1)\,. \end{cases} \tag{24}$$

where the parameter $\beta = 1$, 2 and 4 correspond respectively to the GOE, GUE and GSE. A schematic picture of this probability density is shown in Fig. (3).

The left rate function $\Phi_-(w)$ was first explicitly computed by D.S. Dean and myself using Coulomb gas method (Refs. [100] and [123]) and it reads

$$\begin{aligned} \Phi_-(w) = \quad & \frac{1}{108}\Big[36w^2 - w^4 - (15w + w^3)\sqrt{w^2+6} \\ & +27\left(\ln 18 - 2\ln\left(-w+\sqrt{w^2+6}\right)\right)\Big]\,,\ w < \sqrt{2}\,. \end{aligned} \tag{25}$$

Note in particular the behavior when w approaches $\sqrt{2}$ from below:

$$\Phi_-(w) \sim \frac{1}{6\sqrt{2}}(\sqrt{2}-w)^3\,,\ w \to \sqrt{2}\,. \tag{26}$$

for $w < \sqrt{2}$, the Coulomb gas is *pushed* by the hard wall at w to the left of the semi-circular edge $\sqrt{2}$. This leads to a re-organization of all N charges (and a new equilibrium

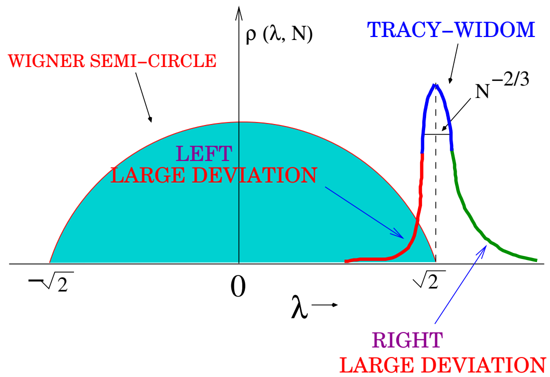


Figure 3: Sketch of the pdf of $\lambda_{\max}$ with a peak around the right edge of the Wigner semicircle $\langle\lambda_{\max}\rangle = \sqrt{2}$. The typical fluctuations of order $\mathcal{O}(N^{-2/3})$ around the mean are described by the Tracy-Widom density (blue), while the large deviations of order $\mathcal{O}(1)$ to the left and right of the mean $\langle\lambda_{\max}\rangle = \sqrt{2}$ are described by the left (red) and right (green) large deviation tails.

configuration), causing an energy cost of $O(N^2)$ (see the first line of Eq. 23). The left large deviation function $\Phi_-(w)$ has the physical interpretation of being this cost of energy in pushing the Coulomb gas.

On the other hand, the right large deviation function $\Phi_+(w)$ was computed explicitly by myself and M. Vergassola [Ref. 134]. It reads

$$\Phi_+(w) = \frac{1}{2}w\sqrt{w^2-2} + \ln\left[\frac{w-\sqrt{w^2-2}}{\sqrt{2}}\right] , \tag{27}$$

with the asymptotic behavior

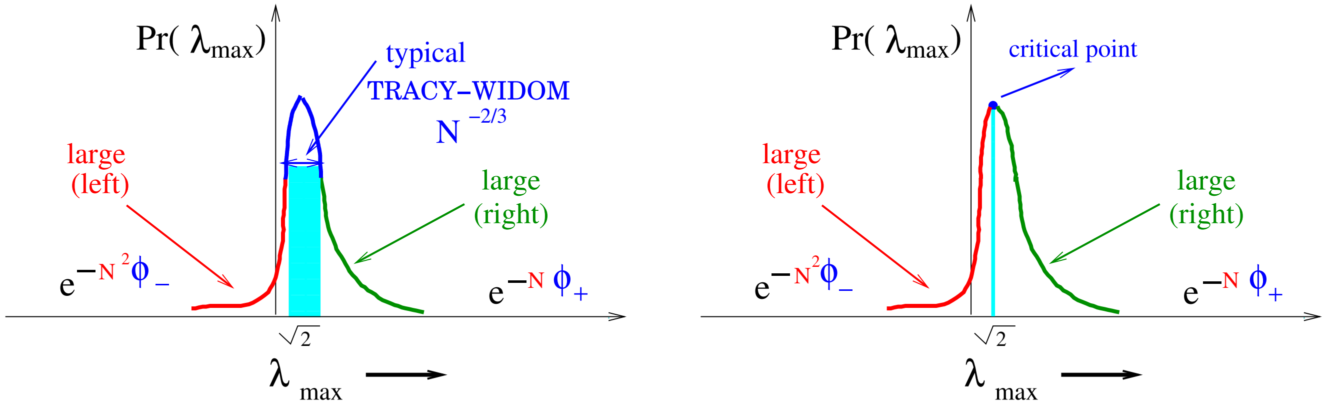
$$\Phi_+(w) \sim \frac{2^{7/4}}{3}(w-\sqrt{2})^{3/2} \ , \ w\to\sqrt{2} \, . \tag{28}$$

For $w > \sqrt{2}$, the dominant configuration is the one where $(N-1)$ charges stay in the semi-circle, while one (the maximum one) gets free of the semi-circle and becomes larger than $\sqrt{2}$. Thus, the energy cost of $O(N)$ (see the third line of Eq. 24) corresponds to just *pulling* a single charge out of the semi-circle and the right large deviation function $\Phi_+(w)$ is precisely this cost of energy [134].

Furthermore, the sub-leading corrections to the leading behavior have been explicitly computed using more sophisticated loop equation methods for the left tail (in collaboration with G. Borot, B. Eynard and C. Nadal in [159]) and by a completely new orthogonal polynomial method for the right tail (in collaboration with C. Nadal in [157]). Interestingly, this latter method also provided a simple derivation of the Tracy-Widom function for $\beta = 2$ [157]. These results have subsequently been used in a number of applications from disordered systems, all the way, to cosmology.

3-rd order phase transition:

The physics of the two large deviation tails are thus quite different. On the left side, the configurations that dominate in the large N limit corrspond to a 'collective' movement of all



(1a) (1b)

Figure 4: (a) Schematic plot of $P(\lambda_{\max}, N)$ vs. $\lambda_{\max}$ for finite but large N. The central (blue) part is described the Tracy-Widom distrivution. The left (red) and right (green) large deviation tails are indicated (b) as $N \to \infty$, the central region shrinks to zero around the critical point $\lambda_{\max} = \sqrt{2}$

charges to the left of w–the cost of energy is of $O(N^2)$. We call this a 'strong coupling' phase as it involves collective movement. In contrast, on the right side, the dominant configuration is the one where one single eigenvalue (the top one) splits off the Wigner sea and the energy cost in pulling this charge is of $O(N)$. We call this 'weak coupling' phase as it involves a single degree of freedom. The picture that emerges can then be summarized as follows. As N increases, and if one observes the histogram on a scale of $w \sim O(1)$, this middle region shrinks to a point at $w = \sqrt{2}$ —this is the critical point (see Fig. 1b). The large deviation functions on both sides are the analogues of two 'phases', separated by the 'critical' point $w = w_c\sqrt{2}$. If one zooms out this point (by making N large but finite), one observes the Tracy-Widom distribution as a *finite size crossover function* connecting the free energies of these two phases. Thus the pdf $P(w, N) = \text{Prob.}(\lambda_{\max} = w, N)$, for large N, has a non-analytic behavior at $w = \sqrt{2}$. One thus has to think of w as the control parameter of this phase transition. As it passes through the critical point $w = \sqrt{2}$, the distribution $P(w, N)$ becomes non-analytic in the large N limit.

To understand why the transition is of 3-rd order, it is useful to consider the cumulative probability distribution of the top eigenvalue $Q(w, N) = \text{Prob.}[\lambda_{\max} \geq w, N] = \int_w^\infty P(w', N)\, dw'$. This quantity has a physical meaning: for $w < w_c = \sqrt{2}$, the quantity $\ln Q(w, N)$ can be interpreted as the free energy cost to push the Coulomb gas from its equilibrium configuration given by the Wigner semi-circular law. The cost of energy is of $O(N^2)$ as discussed above. One has indeed, from (23):

$$\lim_{N\to\infty} -\frac{1}{N^2} \ln Q(w, N) = \begin{cases} \Phi_-(w)\,, & w < \sqrt{2}\,, \\ 0 & w > \sqrt{2}\,, \end{cases} \tag{29}$$

where $\Phi_-(w)$ is given in (25). Since $\Phi_-(w) \sim (\sqrt{2} - w)^3$ when w approaches $\sqrt{2}$ from below (26), the third derivative of the free energy of the Coulomb gas at the critical point $w_c = \sqrt{2}$ is discontinuous: this can thus be interpreted as a *third order phase transition.* This third order phase transition is similar, in spirit, to the so called Gross-Witten-Wadia phase transition which was found in the 80's in the context of two-dimensional $U(N)$ lattice quantum chromodynamics (QCD) [Gross & Witten, 1980; Wadia, 1980]. Indeed, our *pushed* Coulomb gas (for $w < \sqrt{2}$) corresponds to the *strong coupling phase* of QCD, while our pulled Coulomb gas (for $w > \sqrt{2}$) corresponds to the *weak coupling phase* of QCD.

We have pointed out that this 3-rd order phase transition as $N \to \infty$ is rather ubiquitous and occurs in a broad class of systems including dynamical systems of ecospecies, conductance distribution in cavities, non-intersecting Brownian motions, all the way to two dimensional Yang-Mills gauge theory. The basic mechanism behind this 3-rd order transition is also identified: it happens when the gap between the soft edge (characterizing the equilibrium charge density of an underlying Coulomb gas with a single support and with a square root singularity at the support edge, such as in the Wigner semi-circular case) and a hard wall vanishes. In all these transitions, the critical crossover function (interpolating between the 'strong coupling' and the 'weak coupling' phases for finite but large N) is given by the Tracy-Widom distribution. Based on our work over the past years and gathered evidences from the study of several models, we can make a clear hypothesis that the **Tracy-Widom distribution is always accompanied by a 3-rd order critical point.** In all systems that exhibit Tracy-Widom distribution, we have found a hidden 3-rd order critical point separating a strong-coupling and a weak-coupling phase and the Tracy-Widom distribution acts like a a universal crossover function at this 3-rd order critical point. With Gregory Schehr, I have written a recent review on this subject [Ref. 188].

Thus our main conclusion is that **the Tracy-Widom distribution is more like a universal crossover function at a 3-rd order critical point**. Historically, it was first found in the distribution of the largest eigenvalue of Gaussian matrices, but extreme statistics is not really responsible for its universality. The fact that the pdf of $\lambda_{\max}$ has a singular 3-rd order critical point at $\lambda_{\max} = \sqrt{2}$, which was first realized by our series of works, places this problem in the general class of system showing a 3-rd order critical transition, and hence naturally they all share the Tracy-Widom distribution.

To summarize, the large deviation functions associated with the probability distribution of an observable in RMT (such as $\lambda_{\max}$) carry crucial informations concerning phase transitions in the system in the form of singularities in the associated large deviation functions and hence are useful and important objects to study. Our Coulomb gas method turns out to be very general and can be extended beyond the Gaussian ensembles of random matrices. For instance, the large deviation functions associated with $\lambda_{\max}$ have been computed for Wishart matrices in [107, 134] and for Cauchy ensembles of random matrices in [174]. *Our results for $\lambda_{\max}$ in Wishart matrices have been verified experimentally in coupled fiber laser systems [Fridman et. al., 2012].*

3-rd order phase transition in Kardar-Parsi-Zhang growth models:

To test our proposed hypothesis discussed above, namely that generically the Tracy-Widom distribution is accompanied by a 'hidden' 3-rd order phase transition, we studied very recently [Ref. 223] another system where the Tracy-Widom distribution appears. This corresponds to growth models in (1+1)-dimensions that belong to the so called Kardar-Parisi-Zhang (KPZ) universality class. In these models, growth of an interface occurs on a one dimensional substrate via random deposition and evaporation of atoms, with certain local constraints on the evolution of the height field $h(x, t)$. While it was well known from the

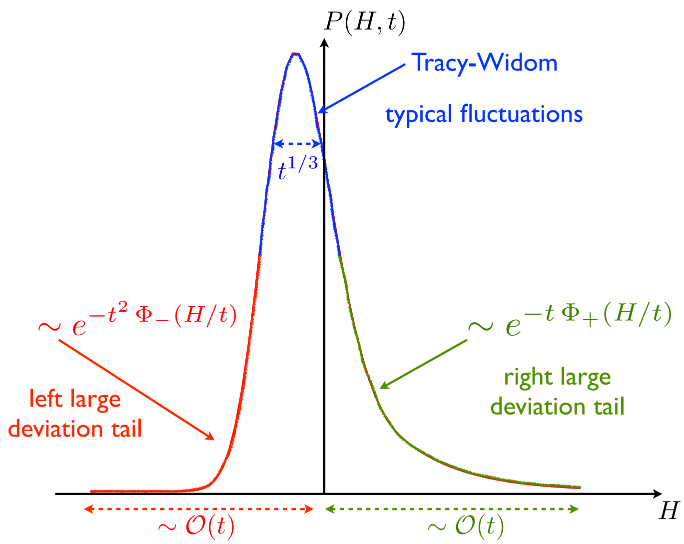


Figure 5: Schematic behavior of the height distribution in KPZ growth models

90's that the typical height fluctuation (width) of the interface grows univerally at late times algebraically as $t^{1/3}$ in these models, only in late 90's it was realized that not just the second moment of the height, but even the full probability distribution of the typical height fluctuation (suitably scaled) is universal at late times and is described by the Tracy-Widom distribution. It turns out that initial conditions play a crucial role. For models growing from a curved initial condition, the height distribution is given by Tracy-Widom (GUE), while for the flat initial condition, it corresponds to the Tracy-Widom GOE. These results have been proved rigorously by several groups for difdrent discrete and continuous growth models belonging to the KPZ class and have also been verified experimentally (in the liquid crystal experiment by Takeuchi and Sano mentioned previously).

In any case, the presence of the Tracy-Widom distribution naturally raises the question: *is there an underlying* 3*-rd order phase transition in these growth models?* If so, what are these phases and how do we detect them? Recently, in collaboration with P. Le Doussal and G. Schehr, we have investigated [Ref. 38] this interesting question in a class of growth models belonging to the KPZ universality class (including the continuum KPZ equation itself). Our main colclusion was that indeed there is a 3-rd order phase transition that shows up when one investigates the large deviation tails of the height distribution (i.e., when the height fluctuation, say at the origin, $H(t) = h(0,t) - \langle h(0,t)\rangle$ is of $\sim O(t)$ that is much larger than the typical fluctuation $\sim O(t^{1/3})$). This 3-rd order phase transition occurs in much the same way as it manifests itself in the distribution of $\lambda_{\max}$ in RMT (discussed in the previous subsection).

We found that for generic models belonging to (1+1)-dimensional KPZ class, the distribution of the centered height at a fixed point in space (say at the origin) $H(t) = h(0,t) - \langle h(0,t)\rangle$

has the following generic behavior at late times t (see Fig. 5)

$$P(H,t) \sim \begin{cases} e^{-\mathbf{t^2}\,\Phi_-(H/t)} & , \quad H \sim \mathcal{O}(t) < 0 \quad \mathrm{I} \\ \\ \frac{1}{t^{1/3}} f\left[\frac{H}{t^{1/3}}\right] & , \quad H \sim \mathcal{O}(t^{1/3}) \quad \mathrm{II} \\ \\ e^{-\mathbf{t}\,\Phi_+(H/t)} & , \quad H \sim \mathcal{O}(t) > 0 \quad \mathrm{III} \end{cases}$$

where $\Phi_-(z)$ and $\Phi_+(z)$ are respectively the left and the right large deviation functions that we computed explicitly for several models. As a consequence,

$$\lim_{t\to\infty} -\frac{1}{t^2} \ln P(H \le \mathbf{z}\,t, t) = \begin{cases} \Phi_-(\mathbf{z}) & z \le 0 \\ 0 & z \ge 0 \end{cases}$$

where we have shown that $\Phi_-(z) \propto |z|^3$ as $z \to 0^-$ universally, indicating a 3-rd order phase transition at the critical point $z = 0$. In addition, we have shown that for $z < 0$, the system behaves *collectively* (to produce a height much less than the typical height, there should not be growth in a space-time area of $O(t^2)$ and the probability of this event is $\sim O(e^{-t^2})$)—this is the analogue of *strong coupling* phase. In contrast, for $z > 0$, only the single degree of freedom in space (the height at the origin) dominates (to generate a configuration where the height at a given point is much larger than typical height, it is enough to keep depositing at the single site irrespective of neighbours and the probability for the even is typically $O(e^{-t})$). Hence $z > 0$ corresponds to the *weak coupling* phase. **These exact results thus provide a strong support to our hypothesis concerning the Tracy-Widom distribution and the 3-rd order phase transition.**

Collaborators: O. Bohigas (LPTMS), A. Comtet (LPTMS), D.S. Dean (Toulouse-Bordeaux), P. Le Doussal (LPTENS), R. Marino (LPTMS, Weizmann), C. Nadal (former Ph.D student at LPTMS), J. Randon-Furling (former Ph.D student at LPTMS), G. Schehr (LPTMS), D. Villamaina (ENS), P. Vivo (LPTMS) and M. Vergassola (Institut Pasteur, now at ENS, Paris).

This area of the top eigenvalue of a random matrix, third order phase transitions and the ubiquity of the Tracy-Widom distribution has become very popular in recent days—a subject in which we have made very significant contributions. I gave a plenary talk at STATPHYS-25 (Seoul, South Korea, 2013) on this subject. Also, based on my recent talk on the subject at the international conference "Viewpoints on Emergent Phenomena in Non-equilibrium Systems", held at the Higgs Centre for Theoretical Physics, University of Edinburgh (UK, June, 2014), Mark Buchanan (a science writer with Nature Physics and Science) wrote an essay "Equivalence Principle" in Nature Physics (vol-10, 543, (2014)). This is available at: http://www.nature.com/nphys/journal/v10/n8/pdf/nphys3064.pdf

A popular science article on our work on third order phase transition behind the universality of Tracy-Widom distribution, written by Natalie Wolchover appeared in the Quanta magazine (published by Simon's foundation) in the October 15 (2014) issue, with the title "At the Far Ends of a New Universal Law".

This can be found online at: https://www.quantamagazine.org/20141015-at-the-far-ends-of-a-new-universal-law/

Also, my work with Gregory Schehr on the large deviations of the top eigenvalue of a random matrix and the ubiquity of third order phase transitions (see Ref. [188] in the list of publications) was recently highlighted by the CNRS-Institut National de Physique (INP) with the title "Luniversalité de la distribution de Tracy-Widom proviendrait dune transition de phase"

see online at: http://www.cnrs.fr/inp/spip.php?article3403

(iii) **Other problems in extreme value statistics:** I have also worked on a number of other problems related to extreme value statistics over the past few years. I highlight some of them below.

The Statistics of the Number of Minima in a Random Energy Landscape:

In collaboration with O.C. Martin (LPTMS), we considered random energy landscapes constructed from d-dimensional lattices or trees. The distribution of the number of local minima in such landscapes follows a large deviation principle and we derived the associated law exactly for dimension 1. Also of interest is the probability of the maximum possible number of minima; this probability scales exponentially with the number of sites. We calculated analytically the corresponding exponent for the Cayley tree and the two-leg ladder; for 2 to 5 dimensional hypercubic lattices, we computed the exponent numerically and compare to the Cayley tree case [Ref: 102].

In collaboration with A.J. Bray (Manchester, UK) and P. Sollich (Kings College, UK) we found [131] an interesting phase transition in the number of stationary points in a class of lattice models of random landscapes. The implication of this finding in the context of glassy dynamics is one of the areas that I would like to work in future.

Integer Partitions, Level Density of a Bose Gas and Extreme Value Statistics:

In collaboration with A. Comtet (LPTMS) and P. Leboeuf (LPTMS), we have established [105] a connection between the level density of a gas of non-interacting bosons and the theory of extreme value statistics [Phys. Rev. Lett., 98, 070404 (2007)] . This problem has interesting connection to the celebrated integer partition problem in number theory on one hand and also possible experimental realization on the other hand. Depending on the exponent that characterizes the growth of the underlying single-particle spectrum, we show that at a given excitation energy the limiting distribution function for the number of excited particles follows the three universal distribution laws of extreme value statistics, namely Gumbel, Weibull and Fréchet [105]. Implications of this result, as well as general properties of the level density at different energies and also the effects of interactions are the subjects of ongoing research.

In collaboration with A. Comtet (LPTMS), S. Ouvry (LPTMS) and S. Sabhapandit (former postdoc at LPTMS) , we have also found an interesting combinatorial interpretation of exclusion statistics (satisfied by a gas of anyons for example) in terms of integer partition problem with the restriction that successive parts differ at least by a positive number $p \geq 0$. We have computed the probability distribution of the number of parts in a random minimal

p partition. It was shown that the bosonic point $p = 0$ is a repulsive fixed point for which the limiting distribution has a Gumbel form. For all positive p the distribution was shown to be Gaussian [Ref: 112]. Also, the limit shapes and the largest parts of the associated Young diagrams in the partition problem for general p was computed exactly [115, 122].

The integer partition problem and its many different versions and their applications in physics problems is one of my favourite current and future research programmes.

Random Polynomials and Their Applications:

In collaboration with G. Schehr (LPTMS), we have studied analytically various properties of the real roots of random polynomials [Phys. Rev. Lett. 99, 060603 (2007)]. We considered a class of real random polynomials, indexed by an integer d, of large degree n and focus on the number of real roots of such random polynomials. For n even, the probability that such polynomials have no real root decays as a power law $n^{-2(\theta(d)+\theta_2)}$ where $\theta(d) > 0$ is the exponent associated to the decay of the persistence probability for the diffusion equation with random initial conditions in space dimension d. Considering the particular case $d = 1$, this connection allows for a physical realization of real random polynomials. We further show that the probability that such polynomials have exactly k real roots (n and k having the same parity) in the interval $[0, 1]$ has an unusual scaling form given by $n^{-\tilde{\phi}(k/\log n)}$ where $\tilde{\phi}(x)$ a universal large deviation function [Refs: 113, 126]. We also found a very interesting phenonmenon of the condensation of the roots of a polynomial onto the real axis for a class of random polynomials [Ref: 139]. Many of our results have recently been proved 'rigorously' by mathemticians, such as A. Dembo and his group at Standford university, USA.

Longest Excursion of Stochastic Processes in Nonequilibrium Systems:

In collaboration with C. Godreche (IPHT, Saclay) and G. Schehr (LPTMS), we have studied the extreme statistics of excursions, i.e., the interval;s between consecutive zeros, of stochastic processes that arise in a variety of nonequilibrium systems [Phys. Rev. Lett. 102, 240602 (2009)]. In particular, we studied the temporal growth of the length of the longest excursion $l_{\max}(t)$ up to time t. Our results were very general and we found broadly two categories of temporal growth. For smooth processes (i.e., the ones with a finite density of zeros), we found a universal linear growth $\langle l_{\max}(t)\rangle \simeq Q_\infty t$ with a model dependent prefactor Q_∞. In contrast, for nonsmooth processes with a persistence exponent θ, we found that there is a critical value θ_c such that $\langle l_{\max}(t)\rangle$ has a linear growth if $\theta < \theta_c$ while $\langle l_{\max}(t)\rangle \sim t^{1-\psi}$ if $\theta > \theta_c$. The amplitude Q_∞ and the exponent ψ are novel quantities associated with nonequilibrium dynamics. Our theoretical predictions were verified numerically in a number of systems such as Ising model, diffusion equation, nonequilibrium critical dynamics etc.

7.2 Random Growth Models, Biological Sequence Matching Problems and Random Matrices

In collaboration with S. Nechaev (LPTMS, Orsay), we have computed the asymptotic distribution of scaled height in a (1+1)–dimensional anisotropic ballistic deposition model by mapping it to the Ulam problem of finding the longest nondecreasing subsequence in a random sequence of integers. Using the known results for the Ulam problem, we show that the scaled height in our model has the Tracy-Widom distribution appearing in the theory of random matrices near the edges of the spectrum. Our result supports the hypothesis that various growth models in $(1+1)$ dimensions that belong to the Kardar-Parisi-Zhang universality class perhaps all share the same universal Tracy-Widom distribution for the suitably scaled height variables (Ref. [73]).

Amazingly, the Tracy-Widom distribution appears in many other interesting problems. In collaboration with S. Nechaev (LPTMS, Orsay), we have shown [Ref. 82] that the Tracy-Widom distribution also appears in a sequence matching problem in evolutionary biology. Sequence alignment is one of the most useful quantitative methods used in evolutionary molecular biology. The goal of an alignment algorithm is to search for similarities in patterns in different sequences. A classic and much studied alignment problem is the so called 'longest common subsequence' (LCS) problem. The input to this problem is a pair of sequences $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_i\}$ (of length i) and $\beta = \{\beta_1, \beta_2, \ldots, \beta_j\}$ (of length j). For example, α and β can be two random sequences of the 4 base pairs A, C, G, T of a DNA molecule, e.g., $\alpha = \{A, C, G, C, T, A, C\}$ and $\beta = \{C, T, G, A, C\}$. A subsequence of α is an ordered sublist of α (entries of which need not be consecutive in α), e.g, $\{C, G, T, C\}$, but not $\{T, G, C\}$. A common subsequence of two sequences α and β is a subsequence of both of them. For example, the subsequence $\{C, G, A, C\}$ is a common subsequence of both α and β. There can be many possible common subsequences of a pair of sequences. The aim of the LCS problem is to find the longest of such common subsequences. This problem and its variants have been widely studied in biology, computer science, probability theory and also in statistical physics. A particularly important application of the LCS problem is to quantify the closeness between two DNA sequences. In evolutionary biology, the genes responsible for building specific proteins evolve with time and by finding the LCS of the same gene in different species, one can learn what has been conserved in time. Also, when a new DNA molecule is sequenced *in vitro*, it is important to know whether it is really new or it already exists. This is achieved quantitatively by measuring the LCS of the new molecule with another existing already in the database.

Finding analytically the statistics of the longest common subsequence (LCS) of a pair of random sequences drawn from c alphabets is a challenging problem in computational evolutionary biology. We proved exact asymptotic results for the distribution of the LCS in a simpler, yet nontrivial, variant of the original model called the Bernoulli matching (BM) model. We have shown [Ref. 82] that in the BM model, for all c, the distribution of the asymptotic length of the LCS, suitably scaled, is identical to the Tracy-Widom distribution of the largest eigenvalue of a random matrix whose entries are drawn from a Gaussian unitary ensemble. Our result disproved the previous conjecture, based on simulations, that the distribution of the LCS in the Bernoulli matching model is Gaussian. Moreover, in collaboration with S. Nechaev and K. Mallick (SPHT, Saclay), we have found an amazing connection between the BM model and the 5-vertex model of statistical physics that helped us understand many of the open issues (Ref. [119]).

Collaborators: S. Nechaev (LPTMS, Orsay), K. Mallick (SphT, Saclay).

7.3 Bose-Einstein Condensation in Real Space

The traditional Bose-Einstein condensation in an ideal quantum Bose gas occurs in momentum space, when a macroscopically large number of bosons condense onto the ground state. It is becoming increasingly clear over the last decade that condensation can also happen in real space (and even in one dimension) in the steady state of a broad class of physical systems. Condensation in real space means that a macroscopically large number of particles may condense onto a microscopically small region of the real space, such as a single lattice site. The systems where this phenomenon occurs are classical, generally lack a Hamiltonian and are defined by their microscopic kinetic processes. Examples include traffic jams on a highway, island formation on growing crystals, formation of gels etc. Various simple lattice models such as the Zero-range process and Chipping model have exhibited this kind of real-space condensation. The common microscopic kinetic process that all these different systems share is the stochastic transport of mass from one region of space to another.

In collaboration with M.R. Evans (University of Edinburgh, UK) and R.K.P. Zia (Virginia Tech, USA), we have introduced a very general mass transport model which includes as special cases all previously studied models. In a series of papers, we have studied analytically various aspects of the phenomenon of real-space condensation in this generalized model (Refs. [77], [78], [84], [93], [97], [98], [127]) In this model, at each time step, a continuous mass μ gets transported from a lattice site i with mass m to another site j (which can be the neighbour) with a certain probability $\phi(\mu|m)$ which we call the chipping kernel. For a general kernel $\phi(\mu|m)$, determining the steady-state mass distribution is nontrivial. We have derived the necessary and the sufficient condition on $\phi(\mu|m)$ such that the joint distribution of masses in the steady state is factorisable. Given this factorisable steady state, we have then addressed the following questions: (1) WHEN does a condensation in real space occur (the criterion) (2) HOW does the condensation happen (the mechanism) and (3) WHAT does the condensate look like (the nature of fluctuations and lifetime of the condensate etc.)? We have shown that these questions can be studied analytically for our generalized mass transport models.

Furthermore, we have gone beyond the simple product measure steady states in Ref. [98]. We have considered a more general class of mass transport models and have shown that the steady state of such models has a pair-factorised form which generalizes the standard factorized steady states. The condensation in this class of models is driven by interactions which give rise to a spatially extended condensate that differs fundamentally from the previously studied examples. We have presented numerical results as well as a theoretical analysis of the condensation transition and have shown that the criterion for condensation is related to the binding-unbinding transition of solid-on-solid interfaces.

In collaboration with E. Trizac (LPTMS) and I. Pagonabarraga (University of Barcelona, Spain) we have shown that such condensation also occurs in polydisperse hard rod mixtures (Ref. [143])

Collaborators: M.R. Evans (Edinbourg, UK), R.K.P. Zia (Virginia Tech, USA), S. Gupta and M. Barma (Tata Institute, Bombay), C. Godreche (SPht, Saclay), E. Trizac (LPTMS) and I. Pagonabarraga (University of Barcelona, Spain).

7.4 Passive Sliders on Fluctuating Surfaces: the phenomenon of Strong Clustering

The coupling of two or more driven diffusive systems can give rise to intricate and interesting behavior, and this class of problems has attracted much recent attention. Models of diverse phenomena, such as growth of binary films, motion of stuck and flowing grains in a sandpile, sedimentation of colloidal crystals and the flow of passive scalars like ink or dye in fluids involve two interacting fields. In collaboration with M. Barma and A. Nagar (Tata Institute, India), we have studied semiautonomously coupled systems — these are systems in which one field evolves independently and drives the second field (Refs. [88], [92]). Apart from being driven by the independent field, the passive field is also subject to noise, and the combination of driving and diffusion gives rise to interesting behavior, such as steady states with strong clustering property. Our aim was to understand and characterize such nontrivial steady states of a passive field of this kind.

In particular, as a model system representative of this generic phenomenon, we studied the clustering of passive, non-interacting particles moving under the influence of a fluctuating field and random noise, in one dimension. The fluctuating field in our case is provided by a surface governed by the Kardar-Parisi-Zhang (KPZ) equation and the sliding particles follow the local surface slope. As the KPZ equation can be mapped to the noisy Burgers equation, the problem translates to that of passive scalars in a Burgers fluid. We study the case of particles moving in the same direction as the surface, equivalent to advection in fluid language. Monte-Carlo simulations on a discrete lattice model reveal extreme clustering of the passive particles. The resulting Strong Clustering State is defined using the scaling properties of the two point density-density correlation function. Our simulations show that the state is robust against changing the ratio of update speeds of the surface and particles. In the equilibrium limit of a stationary surface and finite noise, one obtains the Sinai model for random walkers on a random landscape. In this limit, we obtain analytic results which allow closed form expressions to be found for the quantities of interest. Surprisingly, these results for the equilibrium problem show good agreement with the results in the non-equilibrium regime (Refs. [88], [92]).

Collaborators: A. Nagar (KIAS, Korea) and M. Barma (Tata Institute, Bombay).

7.5 Discrete-time Random Walks and Flux to a Trap

In collaboration with A. Comtet (LPTMS, Orsay) and R.M. Ziff (University of Michigan, USA), we have studied analytically two apriori unrelated random walk problems and found an amazing connection between them. These two random walk problems, one in one dimension and the other in three dimensions, seem to share a nontrivial constant whose numerical value is c=0.29795219028.. In the first problem, this constant shows up in the finite size correction to the expected maximum of a discrete-time random walk on a continuous line with unform jump density. This problem appeared first in the context of a packing algorithm problem in computer science and Coffmann et. al. computed this constant numerically. In the second problem, this constant appeared as the 'Milne extrapolation length' in the expression for flux of discrete-time random walkers to a spherical trap in three dimensions and its value was known for 15 years, though numerically only. We have derived an exact analytical formula for this constant valid for arbitrary jump distributions and also showed why the same constant appears in the two problems, a fact that was not evident at all (Refs. [87], [90], [103], [137]). In solving this problem, we needed to use and develop new techniques to solve special kinds of Wiener-Hopf problems that require solving integral equations over half-space and we believe that our methods will be very useful in other problems since such integral equations are quite common in many physics problems.

Our work [87] was reviewed in the article "Une puce qui saute au hasard" (La Recherche, mathmatiques - 01/10/2005 par Benot Rittaud dans mensuel n390 la page).

Collaborators: A. Comtet (LPTMS, Orsay) and R.M. Ziff (Michigan, USA).

7.6 A class of Combinatorial Optimization problems

In collaboration with D.S. Dean (Toulouse, France) and D.J. Lancaster (University of Westminster, UK) we have studied the stochastic versions of a class of combinatorial optimization problems (Refs. [79], [86]). We studied the statistical mechanics of a class of problems whose phase space is the set of permutations of an ensemble of quenched random positions. Specific examples analyzed are the finite temperature traveling salesman and various problems in one dimension such as the so called 'descent' model. We first motivated our method by analyzing these problems using the annealed approximation. In the limit of a large number of points we developed a formalism to carry out the quenched calculation. This formalism does not require the replica method and its predictions are found to agree with Monte Carlo simulations. In addition our method reproduces an exact mathematical result for the maximal travelling salesman problem in two dimensions and suggests its generalization to higher dimensions. The general approach may provide an alternative method to study certain systems with quenched disorder.

Collaborators: D.S. Dean (Toulouse-Bordeaux, France) and D.J. Lancaster (Westminster, UK).

7.7 Applications of Brownian Functionals in Pure and Disordered Systems

Stochastic processes appear in various contexts ranging from physics and biology to finance. In physics, stochastic processes have been studied in various contexts, in particular in nonequilibrium systems. These are usually classical many body intereacting systems undergoing relaxational dynamics. The time evolution of these systems can be most effectively studied by observing the temporal history of a single degree of freedom as a function of time. For example, in the Ising model undergoing zero temperature relaxation dynamics, one can observe the temporal evolution of a single tagged spin. The time evolution of this tagged spin is a complex stochastic process due to the interaction of the spin with its neighbours. Our goal is to characterize such a stochastic process arising out of a nonequilibrium system. Persistence, as discussed previously, is one such measure. Other quantities of interest are local time (the fraction of time spent by the process at a given value) and the occupation time (the fraction of time during which the process is positive). These quantities are of general interests and have important applications in other fields outside nonequilibrium systems such as in weather fluctuations and finance. The calculation of these quantities are difficult even for pure systems. The local and the occupation time have important applications in disordered systems as well. In collaboration with A. Comtet (LPTMS, Orsay) and S. Sabhapandit (LPTMS, Orsay) we have computed analytically the distribution of these objects for a particle moving in a random potential (Ref. [96]). We came up with a formalism for obtaining the statistical properties of functionals and inverse functionals of the paths of a particle diffusing in a one-dimensional quenched random potential. We demonstrated the implementation of the formalism in two specific examples: (1) where the functional corresponds to the local time spent by the particle around the origin and (2) where the functional corresponds to the occupation time spent by the particle on the positive side of the origin, within an observation time window of size t. We computed the disorder average distributions of the local time, the inverse local time, the occupation time and the inverse occupation time, and showed that in many cases disorder modified the behavior drastically. *I wrote a review in Current Science ["Brownian Functionals in Physics and Computer Science", Current Science, v-89, 2076 (2005)] on the methods and applications of Brownian functionals in physics and computer science.*

Collaborators: A. Comtet (LPTMS, Orsay), S. Sabhapandit (LPTMS, Orsay), J. Randon-Furling (LPTMS, Orsay) and M.J. Kearney (Surrey, UK).

7.8 Applications of Statistical Physics in Computer Science

The basic problem in the area of 'sorting and search' has already been described in section 2.
Our analytical results in various search tree problems using the 'traveling front' method have been published in Refs. [41], [42], [47], [51], [54], [56], [57], [65], [94].

We have also used the techniques of statistical physics to study the statistics of the number of nodes in an m-ary search tree problem in computer science. In the m-ary search trees, a tree has m branches. The case $m = 2$ corresponds to binary trees. The number of nodes needed to store a given incoming data is an important observable in computer science. For randomly incoming data, this number of nodes $X_m(N)$ is a random variable, where N is the data size. It turns out that while the average $< X_m(N) >\sim N$ for all m, the fluctuations characterized by the variance of $X_m(N)$ undergoes a phase transition as a function of m. For $m < 26$, the variance $\sigma^2(N) =< X_m^2(N) > - < X_m(N) >^2\sim N$ for large N. However for $m > 26$, $\sigma^2(N) \sim N^{2\theta}$ where $\theta(m)$ is a nontrivial exponent. We have explained this rather 'unusual' phase transition using the techniques of statistical physics. We determined the critical point $m_c = 26.0461..$ (m has been analytically continued) as a solution of a transcendental equation and also determined the exponent $\theta(m)$ analytically. Besides we have shown that such a phase transition is quite generic and happens also when the incoming data is a d-dimensional vector. In this case, the variance of the number of nodes undergoes a phase transition at a critical value $d_c = \pi/\sin^{-1}(1/2\sqrt(2) = 8.69..$ and the exponent $\theta = 2\cos(2\pi/d) - 1$. This is a new prediction in the area of computer science. These results are summarized in [56].

We have also shown that the 'Linear Probing with Hashing' (LPH) algorithm widely used in computer science to store data in hashed addresses can be viewed as a correlated percolation problem. We call this new percolation model as a 'drop-push' model where one starts with an initially empty lattice and drops a particle at a randomly chosen site. If the chosen site is empty, the particle goes there. If not, the dropped particle performs a random walk till it finds an empty site and goes there. As one increases the number of particles, the particle clusters start percolating at a critical density. This 'drop-push' percolation turns out to be in a completely new universality class and its critical exponents are nontrivial even in one dimension. We have solved this problem exactly in one dimension [57].

Another area of computer science where we have used statistical physics techniques successfully is in the area of 'hardware'. More precisely, what is the best strategy to organize the data in the temporary 'cache' area of computers. We have mapped this problem onto some urn models which can then be solved exactly. Our method gives quantitative comparisons between different 'cache' management strategies.

Collaborators: P.L. Krapivsky (Boston, USA), E. Ben-Naim (Los Alamos, USA), D.S. Dean (Toulouse, France), J. Radhakrishnan (Tata Institute, India)

7.9 Extreme Value Statistics and Traveling Fronts

The statistics of the minimum or the maximum of a set of random variables is interesting in many systems. For example, in a disordered many body system, one is typically interested in the ground state energy. This problem is also important in many optimization problems. This extreme value statistics is well understood when the random variables are uncorrelated. But in most physics examples, the random variables are strongly correlated. While studying the 'sorting and search' algorithms in computer science, we found a rather interesting connection between the extreme value statistics of correlated variables and the problem of traveling fronts in nonlinear systems. This connection is rather general and goes beyond the computer science problems. Exploiting this connection, we have computed exactly the statistics of extreme values in a random fragmentation problem [41]. Using the same techniques, we have have studied analytically the depinning phase transition in the ground state of directed polymers on a Cayley tree with bimodal bond energy distribution [42]. We have also introduced and studied analytically a simple model that describes the dynamics of efficiencies between competing agents [47]. Since the individual agents attempt to maximize their efficiencies, this also reduces to an extreme value problem.

Collaborators: P.L. Krapivsky (Boston, USA), D.S. Dean (Toulouse-Bordeaux, France)

7.10 Persistence in Nonequilibrium Systems

Over the past few years we have been working on the subject of persistence which has attracted attention from a broad scientific community and with applications in diverse fields ranging from ecology to seismology (see Science, **v. 274**, p-919, 1996). Persistence is simply the fraction of points in space where a nonequilibrium field, fluctuating in space and time, has not changed sign upto some time t. In many nonequilibrium situations such as in systems undergoing coarsening after a rapid quench in temperature and even in the simple diffusion equation, the persistence decays with time t slowly as a power law $\sim t^{-\theta}$ for large t. This exponent θ is the simplest quantitative measure of the history dependence of the nonequilibrium process and is a new nonequilibrium exponent, not simply related to other dynamical exponents. Theoretical and experimental determination of θ has been the focus of the research of a broad community over the last two decades. We had earlier computed analytically the exponent θ for the Ising model undergoing coarsening dynamics [Majumdar and Sire, PRL, **77**, 1420 (1996)] as well as for the diffusion equation [Majumdar et. al., PRL, **77**, 2867 (1996)]. Our work on the persistence in the diffusion equation got reviewed in the research news section of Science [A. Watson, Science, **274**, p-919, (1996)]. Our prediction of θ for the 2-dimensional Ising model was later verified in the liquid crystal experiment conducted in Bell Labs [Yurke et. al., PRE, **56**, R40 (1997)]. Our prediction for θ in the diffusion equation has been verified in the experiment jointly conducted by a Harvard and a MIT group [Wong et. al., PRL, **86**, 4156 (2001)] in the laser polarized Xe gas using NMR spectroscopy. Our predictions for persistence in interfaces have been verified in the experiments on step-edge fluctuations by the Maryland group [Dougherty et. al., PRL, **89**, 136102 (2002)].

We also introduced the concept of persistence with partial survival weight $p < 1$ and shown that the persistence exponent $\theta(p)$ depends continuously on p which one can exploit to compute the usual persistence exponent via a systematic series expansion in powers of p [32]. We have also shown that fluctuating interfaces in their steady states have algebraic persistence in space [45]. We introduced the concept of discrete-time persistence that one measures in experiments and have shown that it is usually different from the continuous-time persistence exponent [46, 60]. Persistence of a discrete sequence as opposed to that of a continuous process is begining to initiate a different direction of research. We have studied the persistence of a specific sequence which appears in the weather records and also in the one dimensional Ising spin glass. We have calculated the persistence exponent of this sequence exactly [52]. Moreover the exact analytical expression of $\theta(p)$ along with the statistics of multiple crossings [55] were also obtained.

We then extended our studies of persistence in various new directions:

• In collaboration with A. J. Bray (University of Manchester, UK), we studied a non-Gaussian stochastic process where a particle diffuses in the y-direction, $dy/dt = \eta(t)$, subject to a transverse shear flow in the x-direction, $dx/dt = f(y)$. Absorption with probability p occurs at each crossing of the line $x = 0$. We treat the class of models defined by $f(y) = \pm v_{\pm}(\pm y)^{\alpha}$ where the upper (lower) sign refers to $y > 0$ ($y < 0$). We show that the particle survives up to time t with probability $Q(t) \sim t^{-\theta(p)}$ and we derive an explicit expression for $\theta(p)$ in terms of α and the ratio v_+/v_-. From $\theta(p)$ we computed exactly the mean and variance of the density of crossings of the line $x = 0$ for this class of non-Gaussian processes [Ref. 101]. These results for the mean and variance are, to our knowledge, the first such results for non-Gaussian processes.

• In collaboration with Maryland group, we studied various persistence and first-passage problems for fluctuating interfaces that are of direct experimental relevance [Refs. 72, 75, 91] in systems of fluctuating step-edges. In particular, we were able to explain theoretically several anomalous persistence behaviors in the experiments on equilibrium step fluctuations in Si (111) surface [72,75].

• In collaboration with D. Das (Indian Institute of Technology, Bombay, India), we studied the persistence properties in a simple model of two coupled interfaces characterized by heights h_1 and h_2 respectively, each growing over a d-dimensional substrate [Ref. 83]. The first interface evolves independently of the second and can correspond to any generic growing interface, e.g., of the Edwards-Wilkinson or of the Kardar-Parisi-Zhang variety. The evolution of h_2, however, is coupled to h_1 via a quenched random velocity field. In the limit $d \to 0$, our model reduces to the Matheron-de Marsily model in two dimensions. For $d = 1$, our model describes a Rouse polymer chain in two dimensions advected by a transverse velocity field. We show analytically that after a long waiting time $t_0 \to \infty$, the stochastic process h_2, at a fixed point in space but as a function of time, becomes a fractional Brownian motion with a Hurst exponent, $H_2 = 1 - \beta_1/2$, where β_1 is the growth exponent characterizing the first interface. The associated persistence exponent is shown to be $\theta_s^2 = 1 - H_2 = \beta_1/2$. These analytical results are verified by numerical simulations.

Apart from the persistence, we have also studied various other first passage properties of stochastic processes. An example includes the calculation of the residence time distribution for a class of Gaussian markov processes [34, 59] and non-Markov processes [61]. We have also studied first-passage properties in the problem of translocation of a polymer through a pore (Ref. [136]).

Collaborators: A.J. Bray (Manchester, UK), S.J. Cornell (Cambridge, UK), D. Das (IIT, Bombay, India), C. Dasgupta (IISC, Bangalore, India), A. Dhar (RRI, India), D. Dhar (Tata Institute, India), D.S. Dean (Toulouse-Bordeaux, France), H. Kallabis (Essen, Germany), J. Krug (Essen, Germany), G. Schehr (LPT, Orsay), C. Sire (Toulouse, France), B. Yurke and his group (Bell Labs, USA), A. Rosso (LPTMS), A. Zoia (Saclay).

Our contributions in the subject of 'persistence' have been reviewed in the research news section of the Science magazine: See **"Persistence Pays off in Defining History of Diffusion" by A. Watson, Science, 274, p-919, 1996**.

Recently, with A.J. Bray (Manchester) and G. Schehr (LPTMS), I have written an extensive survey on persistence, see " Persistence and first-passage properties in nonequilibrium systems", Adv. in Phys. v-62, 225-361 (2013).

7.11 Nonequilibrium Phase Transitions in Models of Aggregation

The phenomenon of aggregation of particles is rather common in nature. The classic work in aggregation dates back to Smoluchowsky. We have studied the dynamics of various aggregation models. We solved exactly the dynamics of the Takayasu model where particles diffuse on a lattice, aggregate upon contact and also there is a constant injection of unit mass particles everywhere [13]. We computed exactly the two-point mass-mass correlation function in this model [40]. We also studied analytically a model of mass transport and aggregation in one dimension and have shown its connection to interacting particle system [37, 50].

We studied another interesting model where masses diffuse and aggregate upon contact and also a given particle can chip off a single unit of mass to its neighbours. This model describes the dynamics of polymerization process in solutions. We have shown that as the density of particle changes, the model undergoes a nonequilibrium phase transition, even in one dimension [33, 37]. **This transition is similar to the Bose-Einstein condensation, but happens in real space rather than in the momentum space as mentioned earlier**. The exact phase diagram of this model was found in arbitrary dimensions [44]. We also found a new type of nonequilibrium phase transition in the Takayasu model in presence of desorption of particles [39].

I gave a set of lectures in Les Houches (2008) and a review based on my lectures was published in the Les Houches book [Ref. 4 in Invited Reviews].

Collaborators: M. Barma (Tata Institute, India), S. Krishnamurthy (Stockholm, Sweden), R. Rajesh (IMSC, Chennai, India)

7.12 Stress Propagation and Compaction in Granular Medium

Collaboration was carried out with the experimental group of S. Nagel at Chicago on the measurement of the distribution of the stresses at the bottom of a cylinder filled with glass beads [19]. The idea was to understand how forces or stresses propagate in granular medium. We also proposed and analytically solved [21] a simple model (so called q-model in the literature) which explained the force distribution observed in the experiment. We have also computed exactly the stress-stress correlation function in the q-model in any dimension and provided precise predictions for future experiments [40].

Even though we had originally proposed the q-model as a simple model of stress propagation in granular medium, this model has subsequently found applications in various other problems including traffic problems, random average process studied in the dynamics of wealth distribution, river models, models describing the surface of quantum Hall multilayers and passive scalar turbulence.

We also studied another aspect of granular medium namely the phenomenon of compaction. When a granular medium such as a box of powder is shaken, it gets more and more compact. Experiments show that the density of the granular particles compactifies extremely slowly with time as $1/\log t$ for large time t. We proposed a simple one dimension model of granular compaction and studied it both analytically and numerically by exploiting a mapping to a one dimensional spin model coarsening in presence of kinetic disorders. This model exhibits the inverse logarithmic decay with time and also establishes that such a decay in granular systems is rather generic [43].

Collaborators: S.N. Coppersmith (Wisconsin, USA), D.S. Dean (Toulouse-Bordeaux, France), P. Grassberger (Julich, Germany), S.R. Nagel and his group (Chicago, USA), O. Narayan (UCSC, USA), R. Rajesh (IMSC, Chennai India), T.W. Witten (Chicago, USA).

7.13 Quantum Phase Transitions in Disordered Spin Chains

We have studied analytically the quantum phase transition in quantum Potts and clock chains in presence of quenched ferromagnetic disorder. We employed a real space renormalization group scheme to calculate the critical properties and the exponents and argued that our results become asymptotically exact. A rather striking result is that the system flows into a strong disorder fixed point and both the Potts and clock chains share the same critical properties as the disordered Ising chain [23].

Collaborators: T. Senthil (MIT, USA).

7.14 Coarsening and Phase Ordering Dynamics

When a system such as a ferromagnet, with more than one ordered equilibrium phases at low temperature, is quenched rapidly from the high temperature disordered phase to the low temperature ordered phase, domains of competing equilibrium phases form and grow with time. The system dissipates energy by annealing the various topological defects such as domain walls or vortices and phase ordering dynamics describes precisely how this happens. We solved the phase ordering dynamics of the Ising model with conserved order parameter dynamics on the Bethe lattice [10]. We also developed a general theory of coarsening in 1-d systems for both conserved and non-conserved models with deterministic and stochastic dynamics [15,20]. Correlations and coarsening in the q-state Potts model in arbitrary dimensions were studied analytically and for the first time, q-dependent autocorrelation exponent $\lambda(q)$ was proposed and calculated [17,18]. We also studied analytically the coarsening dynamics near the boundary of a semi-infinite system [22] which illustrated important surface effects.

In view of the experiments on the Bose-Einstein condensation in dilute atomic gases, we addressed a natural question: If a gas consisting of interacting Bose particles is suddenly quenched from the high temperature (where there is no condensate) to very low temperature (where the equilibrium state of the system has a condensate), how does the condensate density grow with time? We studied this question for a dilute interacting Bose gas and showed analytically within a simple physical scenario (supported by numerical simulations of the nonlinear Schrodinger equation) that the density of the condensate, following a rapid quench in temperature, grows as t^d at late times in d-dimensions [24]. We also studied the equilibrium critical properties of a Bose gas in a harmonic trap [25] and established how the true thermodynamic phase transition occurs in the limit when the trap frequency goes to zero.

We found a new type of rather unusual phase ordering which is fluctuation dominated and driven by stochastically evolving surfaces [48]. This work has potentially important experimental significance in systems such as growing thin films.

Collborators: D.A. Huse (Princeton, USA), C. Sire (UPS, Toulouse, France), B. Lubachevsky (Bell Labs, USA), A.M. Sengupta (Bell Labs, USA), S. Sachdev (Harvard, USA), K. Damle (Tata Institute, India), V. Privman (Pottsdam, USA). M. Barma (Tata Institute, India), D. Das (IIT, Bombay, India)

7.15 Transport Properties of Vortices in High-T_c Superconductors

In collaboration with D. Huse, we studied the transport properties of vortices in the high-T_c superconductors within a simple phenomenological model and found that the resistivity in the vortex liquid regime in high-T_c superconductors is non-local [12]. To test this theoretical prediction, we proposed a simple experiment. Our proposed experiment was then actually carried out successfully by the group of D. Bishop in Bell Labs using four-proble tranport techniques and our theoretical predictions were **verified** [14, 16].

Collaborators: D.A. Huse (Princeton, USA), D. Bishop and his group (Bell Labs, USA).

7.16 Interacting Particle Systems

Simple exclusion process where hardcore particles perform biased random walks on a one dimensional lattice is one of the simplest many body systems with a nonequilibrium steady state, not described by the usual Gibbs measure. We studied the tagged particle correlations in both symmetric and asymmetric simple exclusion processes in one dimension. It was known by Mathematicians that the root mean square displacement of a single tagged particle grows subdiffusively as $t^{1/4}$ for large time t for the unbiased walk but becomes diffusive and grows as $t^{1/2}$ in presence of an infinitesimal bias. We explained this puzzling result in an explicit physical manner by mapping this problem to a model of fluctuating interface [4]. Furthermore, this exact mapping to the interface problem suggested that a new tag-tag correlation function in the particle problem should grow as $t^{1/3}$. This presence of this new correlation was verified in numerical simulations [4,5].

We have extended the study of the tagged particle correlations to another interacting particle system namely the random average process [50]. Various tagged particle correlations have been derived analytically [50]. In collaboartion with S. Gupta and M. Barma (Tata) and C. Godrèche (Saclay), we have studied the finite size effects in the tagged particle correlations in the asymmetric simple exclusion process [110].

During my yearly visit to Weizmann Institute (Israel), I have started a longstanding/ongoing collaboration with D. Mukamel, and his group. Our main interest was to understand how a local perturbation (induced by a *localized* drive that breaks detailed balance), in an otherwise diffusive system, can induce *long-range correlations* between particles in the resulting nonequilibrium steady state. For this purpose, we studied simple symmetric exclusion process (SEP) on a d-dimensional lattice. In SEP, each particle attempts a hop to a neighbouring site at rate 1 and actually moves to the target site provided it is empty. Each site can contain at most one particle. This system reaches a steady state with a uniform constant density everywhere. Now, we just introduce a *local* drive, whereby, the rate of hopping from the origin to its eastern nearest neighbour becomes $(1-\epsilon)$, while the reverse rate across the same bond becomes $(1+\epsilon)$. The rate across all other bonds remains 1. What happens to the density profile and the correlations between particles when the localised drive ϵ is switched on?

We solved this problem analytically by mapping it to an electrostatic problem [163]. We showed that in dimensions $d \geq 2$, the average density profile is no longer uniform, but approaches the constant bulk value in an algebraic manner, $\rho(r) \sim \rho[1 - C\,r^{-(d-1)}]$, for large distance r away from the driven bond. Indeed, this density profile corresponds precisely to the potential induced by a localised dipole in electrostatics. The driven bond effectively behaves like a dipole. In a recent work [194], we have computed the density-density correlation function and have shown that the connected part of the correlation function $C(\vec{r}, \vec{s})$ between two points $\vec{r}$ and $\vec{s}$ behaves as, $C(\vec{r}, \vec{s}) \sim (r^2 + s^2)^{-d}$ in $d > 1$, at large distances r and s, away from the drive with $|\vec{r} - \vec{s}| >> 1$. This is again derived using an electrostatic analogy whereby $C(\vec{r}, \vec{s})$ is expressed as the potential due to a conguration of electrostatic charges distributed in $2d$-dimension. At bulk density $\rho = 1/2$, we showed that the potential is that of a localized quadrupolar charge. At other densities the same is correct in leading order in the strength (ϵ) of the drive and is argued numerically to be valid at higher orders [194].

Collaborators: M. Barma (Tata Institute, India), C. Godrèche (Saclay), S. Gupta (currently postdoc at LPTMS), D. Mukamel (Weizmann Institute, Israel), R. Rajesh (IMSC, Chennai, India), T. Sadhu (currently postdoc at LPTENS, Paris)

7.17 Polymers and Self-Avoiding Walks

We studied a simple model of a Gaussian polymer chain on a d-dimensional lattice with an attractive interaction at the origin of the lattice. The polymer undergoes a phase transition from the low temperature globular state to a high temperature extended state. The model was solved exactly in arbitrary dimensions and the critical exponents were found [1]. We also studied a special class of self-avoiding random walks known as loop-erased self-avoiding walks (LESAW). An LESAW is obtained by erasing the loops formed by an ordinary random walker. An exact mapping was found between the ensembles of spanning trees on a graph and that of the LESAW's on the same graph [6]. This enabled the exact calculation of the fractal dimension of LESAW's in 2-d to be 5/4 [6] proving a conjecture by Guttmann and Bursill. The loop erased walks now have become fashionable due to the emergence of research in SLE (stochastic Loewner Equation). To the best of my knowledge, the result that the fractal dimension of loop erased walks in 2-dimensions is 5/4 was first derived by myself in [6] and later by B. Duplantier.

7.18 Self-Organized Criticality in Sandpile Models

In my Ph.D thesis, I studied the Abelian sandpile model (ASM) of Self-organized Criticality (SOC). In this simple height model of SOC, the threshold dynamics of the heights takes the system automatically to a self-organized *critical* steady state where various physical quantities exhibit *power-law* decays. The dynamics of this model has an abelian group structure which we exploited to derive various exact results. We solved the model exactly on the Bethe lattice [2] and all the critical exponents were obtained exactly. For a regular d-dimensional hypercubic lattice, the fraction of sites with the minimum height in the steady state was also calculated exactly and a general formula for an arbitrary graph was derived [3]. Also, the height-height correlation between two sites at a distance r away was shown to decay as r^{-2d} for large r in the steady state in d dimensions [3]. An exact one to one mapping was found between the configurations of ASM in the steady state and and those of spanning trees. A spanning tree on a given lattice is a tree (i.e., without any loop) that goes through all the lattice sites. Using this mapping and the fact that the fractal dimension of the chemical path on spanning trees in 2-d was known from conformal field theory, a new nontrivial scaling relation was derived between two critical exponents [7]. Numerical simulations confirmed this new scaling relation. Also it was shown that the 2-d ASM belongs to the central charge, $c = -2$ family [7]. Reference [7], in fact, contains the major results of my thesis. Several other geometrical properties of spanning trees in 2-d were studied both analytically and numerically [8].

Collaborators: D. Dhar (Thesis Advisor, Tata Institute, India), S.S. Manna (S.N. Bose Center, Calcutta, India).

These contributions are recognized as rather significant exact results in the field of self-organized criticality and are now part of a text book:

Self-Organized Criticality : Emergent Complex Behavior in Physical and Biological Systems (Cambridge Lecture Notes in Physics, 10) by H.J. Jensen.

8 Current and Future Research Projects

Currently I am working on a broad range of problems in stochastic processes and random matrix theory and its applications. I expect to continue working in these problems over the next few years. Some of the current/future themes are as follows.

(1) **Order, Gap and Record Statistics:** in random walks and other stochastic time series

(2) **Random matrix theory and its diverse applications:** My current/future projects in this general area includes the following specific problems:

(i) *free fermions in a harmonic trap, random matrix theory and determinantal point processes: application to cold atom physics*

(ii) *fluctuations of the number of eigenvalues in a given spectral interval–the so called* Index *problem*

(iii) *entanglement properties of bipartite quantum systems*

(iv) *nonintersecting brownian motions and its connections to Yang-Mills gauge theory*

(v) *transport in mesoscopic cavities such as quantum dots*

(vi) *matrix integrals and the associated fluid dynamics*

(3) **Extreme value statistics and persistence/first-passage properties for various stochastic processes (Markov and non-Markov):** In this area, my current/future projects concern the following problems:

(i) *Random convex hulls and extreme value statistics: application to ecology and epidemic spread in animals*

(ii) *First-passage properties in polymer translocation process*

(iii) *Stochastic search problems: diffusion with stochastic resetting*

(iv) *Branching Brownian motion: extremal and gap statistics*

(4) **Statistical physics problems with biological applications:** This involves the following two areas:

(i) *Switching and growth for microbial populations*

(ii) *Dynamics of DNA bubbles during thermal denaturation*

(iii) *Statics and dynamics of active particles*

(5) **Nonequilibrium steady state problems:** This is divided into two principal categories:

(i) *Long range correlations induced by a localised drive in interacting particle systems*

(ii) *Condensation phenomena in nonequilibrium steady states*

Below I highlight some of the current/future results/questions and a brief description of the ongoing work in these areas.

8.1 Order, Gap and Record Statistics for a stochastic time series

Consider any stochastic discrete-time series $\{x_0, x_1, x_2, \ldots, x_N\}$, of $(N+1)$ entries that may represent, e.g., the daily temperatures in a city, magnetization data at regular time intervals or the stock prices of a company etc. Extreme value statistics (EVS), discussed before, concerns the statistics of the global maximum (or minimum) of these N entries–a single value characterizing a rare event. A natural question is: are these rare events isolated, or are there many of them close to the global maximum? These questions led to the study of *near-extreme events*. For example, in an earthquake, there is the big event, but there are several aftershocks/preshocks around the big quake. This is also a crucial question in disordered systems, where the low temperature properties are governed by excited states close to the ground state.

A natural way to characterize this phenomenon of crowding of near-extreme events is via the order statistics, i.e., arranging the random variables x_m's in decreasing order of magnitude $M_{1,N} > \ldots > M_{k,N} > \ldots > M_{N+1,N}$ where $M_{k,N}$ denotes the k^{th} maximum of the set $\{x_0, x_1, \ldots, x_N\}$. Evidently, $x_{\max} = M_{1,N}$, while $x_{\min} = M_{N+1,N}$. A set of useful observables that are naturally sensitive to the crowding of extremum are the gaps between the consecutive ordered maxima: $d_{k,N} = M_{k,N} - M_{k+1,n}$ denoting the k-th gap (see Fig. 6).

Another interesting question concerns the statistics of *records*. The study of record statistics is also an integral part of diverse fields including meteorology, hydrology, economics, sports and entertainment industries among others. In popular media such as television or newspapers, one always hears and reads about record breaking events. It is no wonder that *Guinness Book of Records* has been a world's best-seller since 1955. A *record* happens at step i if the i-th entry x_i is bigger than all previous entries x_0, x_1, $\ldots$, x_{i-1} (see Fig. 7 for an illustration). Statisical questions that naturally arise are: (a) how many records occur in time N? (b) How long does a record survive? (c) what is the age of the longest or shortest surviving record? etc. Understanding these aspects of record statistics is particularly important in the context of current issues of climatology such as global warming. Record statistics also have natural applications in finance (models of stock prices), in evolutionary biology as well as in several physical contexts such as domain wall dynamics, spin-glasses and random walks.

The order, gap and record statistics have been well studied by statisticians when the en-

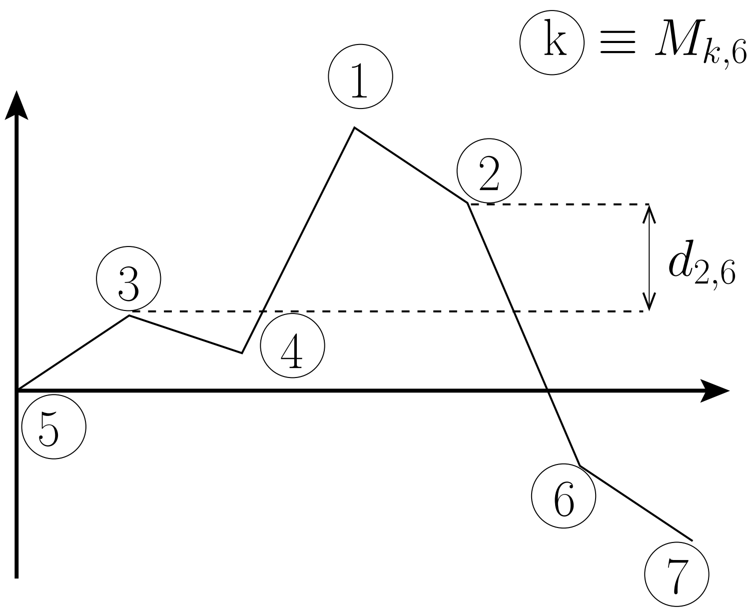


Figure 6: A realization of a time series of $N+1$ entries with $N=6$ and $x_0=0$. We denote by $M_{k,6}$ the k^{th} maximum and focus in particular on the gaps $d_{k,n} = M_{k,n} - M_{k+1,n}$.

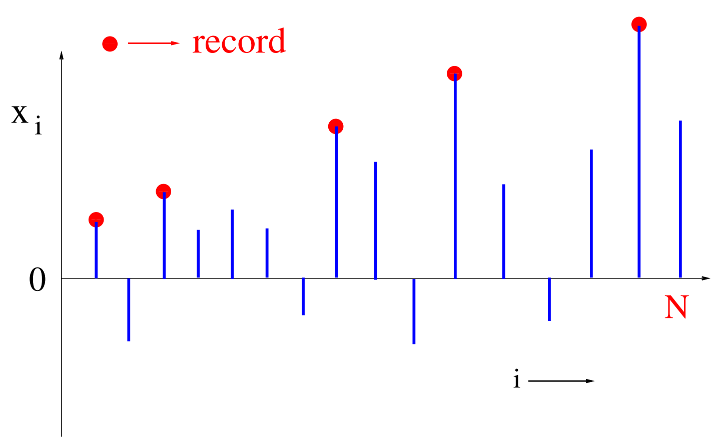


Figure 7: Records in a time series x_i shown by red dots. A record happens at step i if $x_i > \{x_0, x_1, \ldots, x_{i-1}\}$.

tries x_i's of the time series are i.i.d variables. The case of the i.i.d variables is thus fully understood [for a review, see G. Schehr and S.N. Majumdar, arXiv:1305.0639, Ref. [8] in Invited Reviews]. However, much less is known for the difficult case where x_i's are strongly correlated, which turns out to be the case of interest in many problems of statistical physics. One of our main current/future projects is to understand the role of *strong correlations* on the order/gap/record statistics.

Over the last few years, we have made considerable progress for a particular strongly correlated sequence, namely, when the entries x_i's correspond to the positions of a discrete-time random walk in one dimension. The walker starts at $x_0 = 0$ at time 0 and at each discrete step evolves via,

$$x_m = x_{m-1} + \eta_m \tag{30}$$

where the noise η_m's are i.i.d jump lengths each drawn from a symmetric distribution $\phi(\eta)$ with zero mean. If the second moment $\sigma^2 = \int d\eta\, \eta^2\, \phi(\eta)$ exists, the walk converges to a Brownian motion at late times. If σ^2 is divergent, e.g., if $\phi(\eta) \sim |\eta|^{-\mu-1}$ for large η with $0 < \mu < 2$, the jump distribution has *fat* tails and the walk belongs to the Lévy jump processes with Lévy index μ. Even though the jump lengths are uncorrelated, the entries x_m's are clearly correlated and represents perhaps the simplest, yet most ubiquitous correlated time series (discrete-time randoom walks) with a large variety of applications, including for instance in queuing theory – where x_m represents the length of a single server queue at time m – or in finance where x_m represents the logarithm of the price of a stock at time m. Even for this relatively simple correlated time series, we found that there is a *surprising* universality (i.e., independence on the jump distribution $\phi(\eta)$) in the order, gap and record statistics. Some of our main results and the aspect of universality is highlighted below.

8.1.1 Order and Gap statistics

In collaboration with G. Schehr (LPTMS), we have studied the order and the gap statistics for the simple random walk model stated above. We first focus on the case when the second moment σ^2 of the jump density $\phi(\eta)$ is finite. In Ref. [165], we showed that in this case, when $N \to \infty$,

$$\frac{\langle M_{k,N} \rangle}{\sigma} = \sqrt{\frac{2N}{\pi}} + \mathcal{O}(1) \;, \tag{31}$$

independently of k. Thus the property of the crowding of extremum (k-dependence) is not captured by the statistics of the maxima $M_{k,N}$ themselves, at least to leading order for large N. The simplest observable that is sensitive to the crowding phenomenon is thus the gap, $d_{k,N} = M_{k,N} - M_{k+1,N}$. The main result of Ref. [165] was to show that the statistics of the scaled gap $d_{k,N}/\sigma$ becomes stationary, i.e., independent of N for large N, but retains a rich, nontrivial k dependence which becomes *universal* for large k, i.e. independent of the details of the jump distribution $\phi(\eta)$.

In particular, using the so called Pollaczek-Wendel identity, the stationary mean gap $\bar{d}_k = \langle d_{k,\infty} \rangle$ was computed exactly [165] for all k and for arbitrary $\phi(\eta)$ [whose Fourier transform is denoted by $\hat{\phi}(q)$]

$$\bar{d}_k = \langle d_{k,\infty} \rangle = \frac{\sigma}{\sqrt{2\pi}} \frac{\Gamma(k+\frac{1}{2})}{\Gamma(k+1)} - \frac{1}{\pi k} \int_0^\infty \frac{dq}{q^2} \left[[\hat{\phi}(q)]^k - \frac{1}{(1+\frac{\sigma^2}{2}q^2)^k} \right] . \tag{32}$$

In the limit of large k, one finds from (32) that

$$\frac{\bar{d}_k}{\sigma} \sim \frac{1}{\sqrt{2\pi k}} , \tag{33}$$

independent of the details of $\phi(\eta)$, except for the overall scale set by σ. Furthermore, it was shown [165] that the full distribution of the stationary gap exhibits a scaling form for large k, $p_k(d_{k,\infty} = \delta) \simeq (\sqrt{k}/\sigma) P(\delta\sqrt{k}/\sigma)$, with a nontrivial *universal* scaling function

$$P(x) = 4 \big[\sqrt{\frac{2}{\pi}} (1+2x^2) - e^{2x^2} x(4x^2+3) \mathrm{erfc}(\sqrt{2}x) \big] , \tag{34}$$

where $\mathrm{erfc}(z) = (2/\sqrt{\pi}) \int_z^\infty e^{-t^2} \, dt$ is the complementary error function. Somewhat unexpectedly, we found that this universal scaling function has an algebraic tail $P(x) \sim x^{-4}$ for large x. For Lévy flights, where σ^2 is divergent, we have so far not been able to compute the gap distribution. This is a challenging open problem and is one of our ongoing projects.

In collaboration with P. Mounaix (CPT, Ecole Polytechnique) and G. Schehr (LPTMS), we have extended these calculations to finding analytically the joint distribution $P_N(g,l)$ of the first gap $d_{1,N} = G_N = M_{1,N} - M_{2,N}$ and the time $L_N = n_1 - n_2$ between the occurrence of these first two maxima [Refs. 183, 195]. This analysis was carried out for any value of the Lévy index $0 < \mu \leq 2$. In particular, it was shown that $P_N(g,l)$ converges to a stationary distribution, i.e. independent of N for large N, which displays a very rich behavior as a function of g and l as μ is varied [183, 195]. These results have interesting applications in the statistics of seismic events (earthquakes) [see**PRL**, 111, 070601 (2013), J. Stat. Mech. P09013 (2014)]. More recently, we have generalised these results to the case of continuous-time random walks [Ref. 220].

Density of near-extreme events: Another related observable that characterizes the crowding of rare events near the maximum is the so called density of near-extreme events. This quantity was introduced a few years back by myself and my former postdoc S. Sabhapandit [Ref. 108]. It is defined as

$$\rho(r,N) = \frac{1}{N} \sum_{x_i \neq x_{\max}} \delta(x_{\max} - x_i - r) \tag{35}$$

and measures, how many entries are at a value $[r, r+dr]$ per unit length below the global maximum. For i.i.d entries, we computed exactly [108] the average $\langle \rho(r,N) \rangle$ and found that it converges to three different limiting forms depending on whether the tail of the distribution of the random variables decays slower than, faster than, or as a pure exponential function. We argued that some of these results would remain valid even for certain *correlated* cases and verify it for power-law correlated stationary Gaussian sequences. Satisfactory agreement was found between the near-maximum crowding in the summer temperature reconstruction data of western Siberia and the theoretical prediction [108]. More recently, in collaboration with

A. Perret (Ph.D student of G. Schehr at LPTMS), A. Comtet and G. Schehr, we managed to obtain exact results for the full statistics of $\rho(r,N)$ for a random walk sequence [Refs. 185 and 217]. We are currently trying to extend these results for other correlated time series and also the equivalent quantity when the entries x_i's represent the eigenvalues of a random matrix. This is part of ongoing/future work.

8.1.2 Record statistics

As mentioned before, the record statistics is well understood for a time series $\{x_0, x_1, \cdots, x_N\}$ whose entries are i.i.d random variables. Few years back, in collaboration with R. M. Ziff (Michigan, USA), we were able to solve exactly the record statistics for a random walk sequence [Ref. 129], for arbitrary jump distribution $\phi(\eta)$. Using a renewal property of random walks, we showed that the all statistics of records (such as record number, record ages etc.) are completely *universal*, i.e., independent of the jump distribution $\phi(\eta)$ (and holds even for Lévy walks) for any N!

More precisely, we considered a random walk sequence starting at $x_0 = 0$. A record happens at step i, if $x_i > \max(x_0 = 0, x_1, \ldots, x_{i-1})$. Let R_N denote the number of records in step N. Clearly, R_N is a random variable. What is the distribution of R_N? We showed [129] that the distribution of R_N is given by the universal formula, valid for any N,

$$\text{Prob.}[R_N = M] = \binom{2N - M + 1}{N} 2^{-2N+M-1} \ , \ M \leq N + 1 \ . \qquad (36)$$

It does not depend on the jump distribution $\phi(\eta)$ and is universal for all N. *This was a totally unexpected and amazing result.* The origin of the universality goes back to the so called Sparre Andersen theorem [129]. The moments are also naturally universal and can be computed for all N. In particular, for large N, the mean and the variance behave as

$$\begin{aligned} \langle R_N \rangle &\approx \frac{2}{\sqrt{\pi}} \sqrt{N} \\ \langle R_N^2 \rangle - \langle R_N \rangle^2 &\approx 2\left(1 - \frac{2}{\pi}\right) N \end{aligned} \qquad (37)$$

Thus the mean and fluctuations both scale as $\sqrt{N}$ and indeed the distribution approaches a scaling form for large N [129]

$$\text{Prob}(R_N = M) \sim \frac{1}{\sqrt{N}} g_0\left(\frac{M}{\sqrt{N}}\right) \ , \ g_0(x) = \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{4}} \ , \ x > 0 \ . \qquad (38)$$

We also showed [129] that the age statistics of the records is universal for all N. Evidently, the mean age of a *typical* record grows, for large N, as $\langle l \rangle \sim N/\langle M \rangle \approx \sqrt{\pi N/4} \approx 0.8862\sqrt{N}$. We also computed the *extreme* age statistics, i.e., ages of the records that have respectively the *shortest* and the *longest* duration. These extreme statistics are also universal. While the mean longevity of the record with the shortest age grows, for large N, as $\langle l_{\min} \rangle \approx \sqrt{N/\pi} \approx 0.5642\sqrt{N}$, that of the longest age grows faster, $\langle l_{\max} \rangle \approx c\,N$ where c is a nontrivial universal constant

$$c = 2\int_0^\infty dy\, \log\left[1 + \frac{1}{2\sqrt{\pi}}\Gamma(-1/2, y)\right] = 0.626508\ldots \qquad (39)$$

where $\Gamma(-1/2, y) = \int_y^\infty dx\, x^{-3/2}\, e^{-x}$.

In collaboration with G. Schehr (LPTMS), G. Wergen (Cologne), C. Godreche (IPhT), we have extended these results for the simple random walk in several directions. For example, we obtained exact results for the record statistics for a random walk in presence of a constant drift [Ref. 171], record statistics for multiple number of random walkers [Ref. 170], record statistics for arbitrary renewal processes [Refs. 138, 191] etc. We developed exact analytical techniques that were very useful in other problems as well. For example, in collaboration with D. Mukamel (Weizmann), David's Ph.D student A. Bar and G. Schehr, we analysed the statistics of longest domains in a class of truncated long-range Ising models in one dimension that exhibits mixed-order phase transition [see Ref. 221].

In [Ref. 224], we studied the statistics of increments in record values in a time series $\{x_0 = 0, x_1, x_2, \ldots, x_n\}$ generated by the positions of a random walk (discrete time, continuous space) of duration n steps. For arbitrary jump length distribution, including Lévy flights, we show that the distribution of the record increment becomes *stationary*, i.e., independent of n for large n, and compute it explicitly for a wide class of jump distributions. In addition, we compute exactly the probability $Q(n)$ that the record increments decrease monotonically up to step n. Remarkably, $Q(n)$ is universal (i.e., independent of the jump distribution) for each n, and is given by the exact result

$$Q(n) = e\sqrt{\frac{2}{\pi}}\, K_{n+1/2}(1)\frac{2^{-n}}{n!} = \sum_{j=0}^{n} \binom{n+j}{n} \frac{2^{-n-j}}{(n-j)!}\,, \qquad (40)$$

where $K_\nu(x)$ is the Bessel function of index ν. For instance, $Q(1) = 1$, $Q(2) = 7/8$, $Q(3) = 37/48$, etc. For large n, we find that $Q(n)$ behaves as

$$Q(n) \sim \frac{\mathcal{A}}{\sqrt{n}}\,,\ \mathcal{A} = \frac{e}{\sqrt{\pi}} = 1.53362\ldots\,. \qquad (41)$$

Our exact results, published recently [**Phys. Rev. Lett.**, 117, 010601 (2016)], then provide a benchmark for record increment statistics in a wide variety of problems where RW time series is used as a basic model.

Currently, we are working on several aspects of record statistics for stochastic processes (such as statistics of record increments, records for absolute values of random walks, forward and backward records etc.) and it is one of my principal focus area for future studies. We have recently written an extensive review article on record statistics [Ref. 237]: C. Godreche, S. N. Majumdar, G. Schehr, "Record statistics of a strongly correlated time series: random walks and Lévy flights", **topical review** in J. Phys. A: Math. Theor., v-50, 333001 (2017).

Recently, myself and G. Schehr have signed a book contract with Oxford University Press to write a state of the art book on "Records and Extremes in Stochastic Processes" (scheduled to be finished in 2019).

Finally, as a practical application of record statistics, I collaborated with N. Berkowitz and Y. Edery from the environmental science department at Weizmann Institute (Israel) and also with A. Kostinski (Michigan, USA). Experiments often have the instrumental precision, i.e., an instrument can detect the occurrence of a record only up to a certain precision. In other words, the criterion for a record to happen at step i is, if $x_i - \delta > \max(x_0, x_1, x_2, \ldots, x_N)$, where δ denotes the instrument precision. How does a finite δ affect the record statistics? Also, by measuring record statistics of a sequence, can one infer an estimate of the instrumental error δ. We have studied the record statistics for i.i.d as well as for a random walk sequence [178], by taking into acount this additional δ factor. For instance, for random walks, we have shown that the mean number of records still grows universally as $\langle R_N \rangle \sim A(\delta)\sqrt{N}$ for large N, but with a prefactor that depends explicitly on δ. We computed $A(\delta)$ analytically for all jump distributions in a random walk sequence [178]. Currently, we are analysing several experimental data in view of our results and this is also an ongoing project.

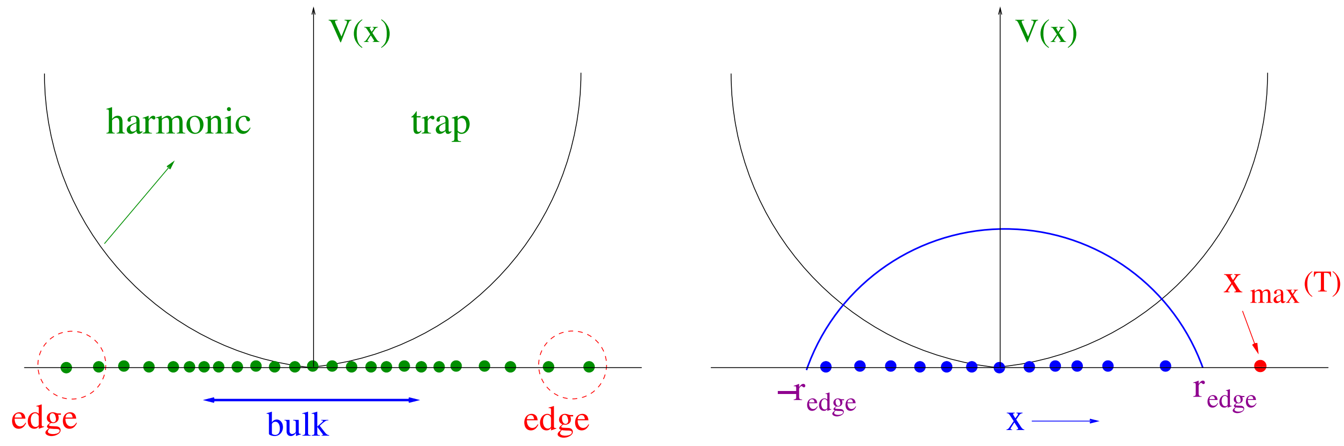
8.2 Random matrix theory and its diverse applications

Since its early days, random matrix theory (RMT) has been a very successful tool in analyzing the statistical properties of a broad range of collective systems in physics, mathematics, biology and statistics. Surprisingly, new applications continue to emerge and depending on the type of the application, one is forced to ask new types of questions and study new observables within the random matrix theory. Over the last few years, I have been involved in studying some of these new observables and we have developed new analytical tools using Coulomb gas for studying such new questions in RMT. This is also very much an area of ongoing/future work. Below, I outline some of these current/future projects.

8.2.1 Free fermions in a harmonic trap, random matrix theory and determinantal point processes: application to cold atom physics

Very recently, in collaboration with my Ph.D student R. Marino (now a postdoc at Weizmann Institute), D.S. Dean (Bordeaux), P. Le Doussal (ENS, Paris) and G. Schehr (LPTMS, Orsay), we have found an entirely new application of the random matrix theory (RMT) and its generalisations—in a system of non-interacting fermions trapped in a confining potential. This study is motivated by the recent cold atom experiments (laser trapping and cooling of dilute Bose and Fermi gases). In such a typical experimental set-up, bosons and fermions are trapped in a confining potential. Experimentalists have developed techniques whereby they can generate traps in one, two, or three dimensions and can change the shape of the confining potential (generically the traps are harmonic). In addition, the temperature and also the interaction between atoms can be tuned. Hence, one can explore very low temperature qunatum and statistical properties of these many-body systems. In particular, by tuning the interaction to zero, one can explore *purely* quantum fluctuatuions arising due to the quantum statistics (Bosons or Fermions) of these systems. For example, for bosonic systems, Bose-Einstein condensation has been observed experimentally. Similarly, for fermions, non-trivial quantum fluctuations occur (leading to interesting collective properties) simply due to the Pauli exclusion principle (and even in absence of any direct interaction between the fermions).

In a series of recent papers [Refs. 192, 203, 206, 218, 225, 227], we have studied non-interacting fermions in a confining trap, in one or higher dimensions and in presence of a nonzero temperature, and have found a host of novel ananlytical results (direct predictions for future experiments). This spectacular advance was possible due to (i) an exact mapping between the non-interacting fermions in one dimension at zero temperature and the eigenvalues of a complex Hermitian Gaussian random matrix (GUE) and generalisation of the ideas from RMT and the associated determinantal point processes to higher dimensions and finite



(2a) (2b)

Figure 8: (a) The confining potential induces a sharp edge to the density of fermions (b) The rightmost fermions position at finite temperature is denoted by $x_{\max}(T)$.

temperature that allowed us to compute any m-point correlation function exactly. Recent developments on quantum Fermi gas microscopes have opened the possibility of probing and verifying these results on quantum correlations, both spatial and temporal, in non-interacting Fermi gas. In addition, we have also computed exactly the number variance (i.e., the variance of the number of fermions in a symmetric interval $[-L, L]$ around the harmonic trap center in 1-d at $T = 0$) as well as the entanglement entropy (the later in collaboration with P. Calabrese (SISSA, Italy)) of the interval $[-L, L]$ at $T = 0$ [Ref. 203]. Thus, apart from making exact predictions for experiments, we believe that our recent results have brought together three different fields of research: (a) RMT (b) physics of cold atoms and (c) traditional many-body condensed matter theory. The key features of our results are highlighted below.

Universal correlations for free fermions in a confining trap:

In these cold atoms systems, the confining trap breaks the translational invariance. The physics in the bulk near the trap center (where the fermions do not feel the curvature of the confining trap) can be understood using the traditional theories of quantum many-body systems such as the local density approximation (LDA). However, away from the trap center, the fermions start feeling the curvature induced by the confining trap. As a result the average density profile of the fermions vanishes beyond a certain distance from the trap center—thus creating a *sharp edge* (see Fig. 2a). Near this edge, the density is small (few fermions) and consequently, quantum and thermal fluctuations play a more dominant role than in the bulk. Consequently, the traditional theories such as LDA break down in this 'edge' region. One needs new methods to describe this edge physics, and this is where the random matrix theory (RMT) provides a new starting point.

To provide a concrete example, we consider, the simplest instance of a system of non-interacting fermions confined in a harmonic trap in one space dimension, with potential $V(x) = \frac{1}{2}m\omega^2x^2$ (see Fig. 2a). At zero temperature, $T = 0$, the ground state wave function $\Psi_0(x_1, \ldots, x_N)$ of the N-body non interacting fermions can be constructed from the Slater determinant of the single particle eigenfunctions of the harmonic oscillator. The quantum zero point fluctuations is characterized by $|\Psi_0(x_1,\ldots,x_N)|^2$ which provides the joint probability distribution of the positions of N fermions at $T=0$. Recently, we evaluated this Slater determinant explicitly [see Ref. 192] and it reads

$$|\Psi_0(x_1,\ldots,x_N)|^2 \sim \prod_{i<j}(x_i-x_j)^2\, e^{-\alpha^2\sum_i x_i^2}$$

where $\alpha = \sqrt{m\omega/\hbar}$ denotes the inverse length scale associated with the harmonic trap. Remarkably, the right hand side of the above equation is also exactly the joint distribution of N eigenvalues of an $N\times N$ complex Gaussian Hermitian random matrix (called the Gaussian Unitary Ensemble, GUE, in the RMT literature). Thus we showed that the positions of N fermions in a $1d$ harmonic trap at zero temperature have the same statistical properties as the eigenvalues of a GUE matrix [192]

$$\{\alpha\, x_1, \alpha\, x_2\, \ldots, \alpha\, x_N\} \equiv_{\text{in law}} \{\lambda_1, \lambda_2\, \ldots, \lambda_N\} \tag{42}$$

where λ_i's denote the eigenvalues of a standard GUE matrix.

RMT has been studied extensively over the last century and many beautiful theoretical tools have been developed to study the spectral properties of random matrices. Thus the non-interacting fermions (in $1d$ and at zero temperature) provide a new application for these RMT results. Hence, the statistics of several observables in this fermionic system at $T-0$ and in $d=1$, can be computed exactly using the RMT techniques.

For instance, RMT predicts that the average density of fermions (or GUE eigenvalues), approaches, for large N, to the celebrated Wigner semi-circular form

$$\rho_N(x) = \frac{1}{N}\sum_{i=1}^{N}\langle\delta(x-x_i)\rangle \to \frac{\alpha^2}{\pi\, N}\sqrt{\frac{2N}{\alpha^2}-x^2},$$

with finite support over $x\in[-\sqrt{2N/\alpha},+\sqrt{2N/\alpha}]$ with two sharp edges at a distance $r_{\text{edge}} = \sqrt{2Nalpha}$ from the origin (see Fig. 3). In addition, these fermions form a determinantal point process (DPP). DPP refers to the fact that any m-point correlation function of the positions of fermions can be expressed as a determinant of an $m\times m$ matrix whose (i,j)-th entry $K_N(x_i,x_j)$ (known as Kernel) does not depend on m. Thus the Kernel $K_N(x,y)$ is central object in DPP, and if one knows this kernel, one knows, in principle, all correlation functions and other higher order observables, such as the hole probability (the probability that an interval contains no fermions), the number statistics (the distribution of the number of fermions in a given interval), extremal statistics, such as the distribution of the position of the rightmost fermion etc. In particular, one of our striking and recent predictions [see Ref. 206] using RMT is that the right-most fermion in 1D harmonic trap is distributed via the celebrated Tracy-Widom (TW) distribution (see Fig. 3), that was already discussed in the previous subsection. More precisely, at $T=0$,

$$x_{\max}(T=0) - r_{\text{edge}} = w_N\, \xi \tag{43}$$

where $r_{\text{edge}} = \frac{\sqrt{2N}}{\alpha}$, $w_N = \frac{N^{-1/6}}{\alpha\sqrt{2}}$ and $\xi\to$ Tracy-Widom GUE variable. **The exact correspondence between fermions and RMT thus provides one of the most direct and simplest physical settings where TW distribution emerges.**

It is then natural to ask what happens to this system of non-interacting fermions in a trap at finite temperature or in higher dimensions. In these cases, unfortunately, the beautiful direct

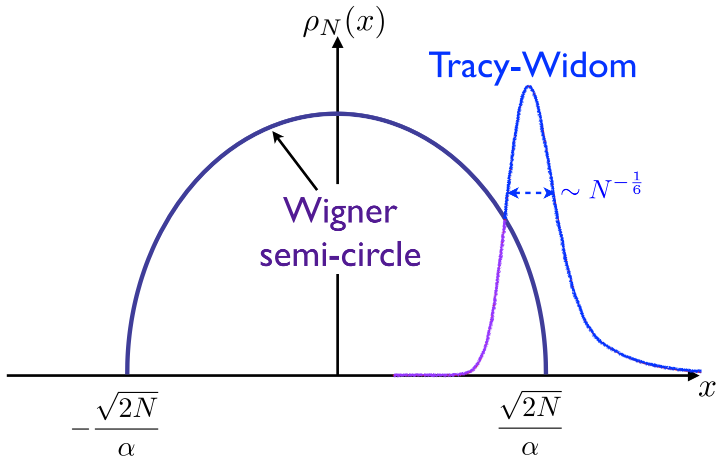


Fig. 3: Sketch of the celebrated Tracy-Widom (TW) distribution which describes the statistics of the largest eigenvalue (at the edge of the Wigner semi-circle) of random matrices belonging to the Gaussian Unitary Ensemble (GUE). Thanks to the connection between the GUE and $1d$ non-interacting fermions in a harmonic trap, this TW distribution also describes the quantum fluctuations of the rightmost fermion in the ground state of this fermionic system. This illustrates the richness of the physical edge properties of trapped Fermi gases.

connection to GUE eigenvalues no longer holds and one needs to develop new techniques. Recently we were able to obtain exact solutions [Refs. 21 and 33] to this problem (both in higgher dimensions and at finite temperature) by exploiting the fact that these fermions in higher dimensions (and at finite temperature) still form a determinantal point process (DPP), and we were able to compute the temperature and dimension dependent kernel $K_N(x, y)$ for large N. These gave us access to the correlation functions at finite temperature and in arbitrary dimensions. In addition, the fermions at finite temperature and higher dimensions form a new type of DPP that never appeared before in the physics or mathematics literature. In addition, we have shown that these kernels (both in the bulk as well as at the edge) are *universal*, i.e., independent of the precise shape of the confining potential as long as the potential is smooth, has a single minimum and grows faster than linearly at large distances.

In the 1-d finite T case, we found [Ref. 206] that the universal kernel for the fermions is amazingly the same as the one that has appeared in recent exact solution of the Kardar-Parisi-Zhang (KPZ) interface growth model in (1+1)-dimensions at finite time t. In fact, we showed that finite temp. in fermions is related to the inverse of the time t in the KPZ equation. This exact relation led to many beautiful subsequent results in both models explored in Ref. [225]. For example, we showed that while at $T = 0$, the position of the rightmost fermion is distributed via Tracy-Widom, ast temperature T increases and for $T >> N^{1/3}$, the distribution of $x_{\max}(T)$ crossses over to a Gumbel distribution. More precisely, the $T = 0$ temperature result in Eq. (43) get modified by

$$x_{\max}(T) - r_{\rm edge} = a_N(T) + b_N(T)\,\gamma \tag{44}$$

where $r_{\rm edge} = \frac{\sqrt{2N}}{\alpha}$, $a_N(T) = \frac{\alpha}{2\sqrt{2N}}\frac{T}{\hbar\omega}\ln\left(\frac{T^3}{4\pi N(\hbar\omega)^3}\right)$, $b_N(T) = \frac{\alpha}{\sqrt{2N}}\frac{T}{\hbar\omega}$ and $\gamma \to$ **Gumbel** variable.

To summarize, because of the recent convergence between experimental techniques and theoretical advances, the problem of fermions in a trap with or without interactions is becoming a very exciting and topical area of research, both experimentally and theoretically, and we can expect many new results emerging in the near future. Evidently this is the time to develop collaboration between theory and experiments on these topics, which is our future goal. On the theory side we are beginning to see that this problem is connected to several other interesting and fast developing areas of theoretical physics and mathematics, such as RMT and KPZ. In fact, non-interacting fermion problems appear in a wide variety of problems in statistical and mathematical physics. Cold atom systems provide a unique and novel physical realization of these theoretical models.

Number Variance and Entanglement entropy:

Another related problem, also motivated from the recent developments in cold atom physics, is to understand how the number variance and the entanglement entropy of a domain $\mathcal{D}$ vary with the domain size, for free fermions confined in a trap. This problem has created a lot of recent interest, and was studied so far numerically in one dimension at $T = 0$ for free fermions in a harmonic trap (for an interval $[-L, L]$ around the trap center) and very interesting behavior of both observables were found as a function of the interval size L [Vicari 2012, Eisler 2013 etc.]. But all these results were mostly numerical. Exploiting the exact connection to RMT mentioned in the previous subsection, recently (in Refs. 192 and 203), we were able to compute both the number variance and the entanglement entropy analytically for large N. The key features are highlighted below.

One important observable in the context of cold atom experiments is the number of fermions N_L in the ground state ($T = 0$) within a symmetric interval $[-L, +L]$ around the center of harmonic trap. The variance of N_L, denoted by $V_N(L)$, characterizes the quantum fluctuations in the ground state of this many-body system. The variance $V_N(L)$ as a function of the box size L turns out to be highly nontrivial even for the simplest possible many-body quantum system, namely 1d spinless fermions in a harmonic trap. In this case, it was numerically found that $V_N(L)$ has a rather rich non-monotonic dependence on L – it first increases with L and then drops rather dramatically (with oscillations on a smaller scale) when L exceeds some threshold value (see Fig. 10). Analytically deriving this dependence on L is thus a challenging problem.

We used the mapping of the fermion problem to the problem of eigenvalues of a $N \times N$ GUE matrix mentioned before. Under this mapping, the number of Fermions in the ground state in a harmonic trap in the box $[-L, L]$ is exactly equivalent to the classical statistics of the number of eigenvlues N_L of a GUE random matrix in the interval $[-L, L]$. We then solved exactly this random matrix problem using Coulomb gas method for large N (where the interval size is scaled by $\sqrt{N}$, so that the edge of the semi-circle is $\sqrt{2}$ and we have set $m = \omega = \hbar = 1$ for convenience). We computed not just the variance, but the full distribution of N_L for large N. Our result for the variance explains the drastic drop-off effect in Fig. 10 and it reads [Ref. 192]

$$V_N(L) \sim \begin{cases} \frac{1}{\pi^2} \ln\left(NL(2-L^2)^{\frac{3}{2}}\right) + O(1)\,, & N^{-1} \ll L < \sqrt{2} - O(N^{-2/3}) \\ \tilde{V}_2(s), & L = \sqrt{2} + \frac{s}{\sqrt{2}} N^{-\frac{2}{3}} \\ \exp\left[-2N\phi(L)\right]\,, & L > \sqrt{2} \end{cases} \tag{45}$$

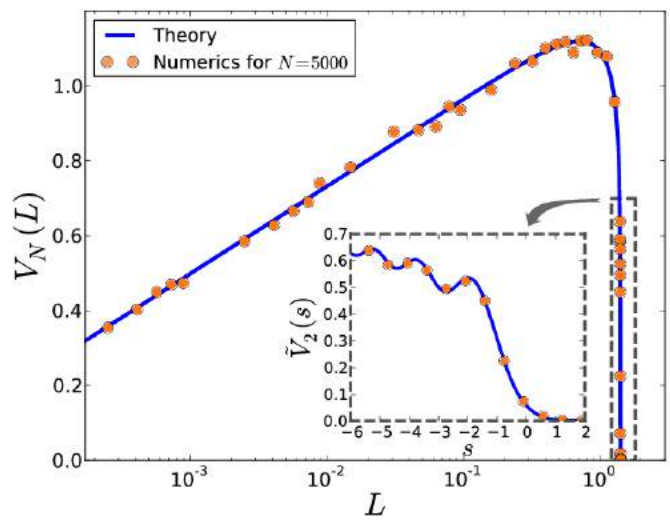


Figure 10: Number variance $V_N(L)$ as a function of L. Theoretical result Eq. (45) in solid blue line. **Inset**: edge scaling behavior of the variance around $L \sim \sqrt{2}$ (described by the scaling function $\tilde{V}_2(s)$, in (47), with $N = 5000$ and $\beta = 2$ (and averaged over 30000 matrices).

where the rate function $\phi(L)$ is given explicitly as [7]

$$\phi(L) = \frac{L}{2}\sqrt{L^2-2} + \ln\left(\frac{(L-\sqrt{L^2-2})}{\sqrt{2}}.\right) \tag{46}$$

The scaling function $\tilde{V}_2(s)$ is given by [192]

$$\tilde{V}_2(s) = 2\int_s^{\infty} dx\ \ K_{\mathrm{Ai}}(x,x) - 2\iint_{[s,\infty]^2} dx\,dy\,[K_{\mathrm{Ai}}(x,y)]^2 \tag{47}$$

where $K_{\mathrm{Ai}}(x,y)$ is the Airy kernel defined for $x \neq y$ as

$$K_{\mathrm{Ai}}(x,y) = \frac{\mathrm{Ai}(x)\mathrm{Ai}'(y) - \mathrm{Ai}(y)\mathrm{Ai}'(x)}{x-y} \tag{48}$$

with $\mathrm{Ai}(x)$ being the Airy function. At coinciding points, $K_{\mathrm{Ai}}(x,x) = (\mathrm{Ai}'(x))^2 - x\mathrm{Ai}^2(x)$. This scaling function has the asymptotic behavior

$$\tilde{V}_\beta(s) \sim \begin{cases} \frac{3}{\beta\pi^2}\ln|s|\ ,\ \ s \to -\infty \\ \exp\left(-\frac{2\beta}{3}s^{3/2}\right)\ ,\ \ s \to \infty\ . \end{cases} \tag{49}$$

Physically, it is not difficult to understand qualitatively this nontrivial result. The eigenvalues, on an average, are distributed via the Wigner semi-circular density, $\rho_{\mathrm{sc}}(\lambda) = (1/\pi)\sqrt{2-\lambda^2}$. The spacing between successive eigenvalues scales as $\sim 1/N$ in the bulk (where they are densely packed near $\lambda = 0$), and as $\sim N^{-2/3}$ near the edges $\pm\sqrt{2}$ (where they are sparse). Now consider the interval $[-L, L]$ and we want to count the number of eigenvalues in this interval. When the box $[-L, L]$ lies entirely inside the semicircle $[-\sqrt{2}, \sqrt{2}]$ (i.e., $L < \sqrt{2}$), there are still fluctuations because the eigenvalues can go in and out of the box $[-L, L]$, while staying within the semi-circle. As L increases, the spacing between eigenvalues becomes large (towards the edge) thus making available more space for fluctuations. This leads to an increase in the variance as $L \to \sqrt{2}$ from below. However, when L is very close to the edge $\sqrt{2}$ of the semi-circle, the fluctuation starts decreasing as most of the eigenvalues are now happy to be inside the semi-circle. In fact, the oscillations seen in Fig. 10 on scale $O(N^{-2/3})$ corresponds to the cases everytime L crosses a new eigenvalue near the edge. In the extreme case when $L > \sqrt{2}$, the eigenvalues are tightly sitting inside the semi-circle and they hardly fluctuate. On an average $\langle N_L \rangle = N$ for $L > \sqrt{2}$. To cause any fluctuation in N_L, on needs to pull out a single charge (or a few charges) from the semi-circle to just outside the box. This is an exponentially rare event and the energy cost for pulling out a single charge is precisely $N\,\phi(L)$, thus explaining the extreme right tail behavior of the variance in Eq. (45). Currently, we are extending these computations to finite temperature and higher dimensions.

The other interesting observable is the Rényi entanglement entropy S_q of the interval $[-L, L]$ at zero temperature. Consider the many-body fermionic system in its ground state, so that the density matrix of the full system in this pure state is simply, $\hat{\rho} = |\Psi_0\rangle\langle\Psi_0|$. Consider now the subsystem $A \equiv [-L, L]$ and let $\hat{\rho}_A = \mathrm{Tr}_{\bar{A}}\hat{\rho}$ denote the reduced density matrix of the subsystem A, obtained by tracing out the complementary subsystem $\bar{A}$ (so that A and $\bar{A}$ tgether consitute the full real line). The Rényi entanglement entropy of the subsystem A, parametrized by $q \geq 1$, is then defined as

$$S_q = \frac{1}{1-q} \ln \mathrm{Tr}\, \hat{\rho}_A^q\,. \tag{50}$$

In the limit $q \to 1$, this reduces to the standard von Neumann entropy, $S_1 = -\mathrm{Tr}[\hat{\rho}_A \ln \hat{\rho}_A]$. For free fermions in a harmonic trap in one dimension, the Rényi entropy was recently studied numerically [Vicari 2012, Eisler 2013]. Exploiting the connection to RMT, we were able to compute the Rényi entropy analytically for all L, and large N [Ref. 203]. For instance, we showed that for all L such that $\sqrt{2} - L << O(N^{-2/3})$, there is an exact relationship between the Rényi entropy and the number variance $V_N(L)$ discussed above

$$S_q = \frac{\pi^2}{6}\left(1 + \frac{1}{q}\right) V_N(L)\,. \tag{51}$$

In particular, it turns out that this relation is universal. Many other interesting results were also derived in [203]. Currently, using our results on fermions at finite temperature and higher dimensions (mentioned in the previous subsection), we are trying to entend the computation of the Rényi entropy for this system at finite temperature, as well as to higher dimensions.

8.2.2 Index problem: number fluctuations in spectral statistics

There is a well known problem called the Index problem in RMT that is closely related to the number fluctuations of fermions in a symmetric interval $[-L, L]$ around the harmonic trap. In fact, the index problem, stated below, just corresponds to calculating the number of eigenvalues in the semi-infinite iinterval $[0, \infty]$. Consider a standard $N \times N$ Gaussian random matrix with dyson index β, whose N eigenvalues are real and are distributed via the joint distribution in Eq. (14). A natural question to ask: how many of these N eigenvalues are positive (or equivalently negative)? Let $N_+ = \sum_{i=1}^{N} \theta(\lambda_i)$ denote the number of positive eigenvalues in a given sample. Clearly, N_+ is a random variable that fluctuates from sample to sample. What can we say about the statistics of N_+? Clearly, the mean value, is trivially $\langle N_+ \rangle = N/2$ by symmetry. But what about the fluctuations of N_+?

So, why are we interested in this index distribution? This question naturally arises in the study of the stability patterns associated with a multidimensional potential landscape $V(x_1, x_2, \ldots, x_N)$ [Wales, 2004]. For instance, in the context of glassy systems, the point $\{x_i\}$ represents a configuration of the system and $V(\{x_i\})$ is just the energy of the configuration [cavagna et. al, 2000]. Similarly, in the context of disordered systems or spin glasses, $V(\{x_i\})$ may represent the free energy landscape. In the context of string theory, V may represent the potential associated with a moduli space [Douglas, 2003]. Typically such an N-dimensional landscape has many stationary points (minima, maxima and saddles) with complex stability patterns that play an important role both in statics and dynamics of such systems [Wales, 2004]. The stability of a stationary point of this N-dimensional landscape is decided by the N real eigenvalues of the $(N \times N)$ Hessian matrix $M_{i,j} = [\partial^2 V/\partial x_i \partial x_j]$ which is symmetric. If all the eigenvalues are positive (negative), the stationary point is a local minimum (local maximum). If some, but not all, are positive then the stationary point is a saddle. The number of positive eigenvalues (the index), $0 \leq \mathcal{N}_+ \leq N$, is then a key object that determines in how many directions the stationary point is stable. Given a random potential V, the entries of the Hessian matrix at a stationary point are usually correlated. However, in many situations, important insights can be obtained by ignoring these correlations and just assuming the entries of the Hessian matrix are just independent Gaussian variables. This then leads to the study of the statistics of index for a GOE matrix. This toy model, called the random Hessian model (RHM), has been studied extensively in the context of disordered systems [Cavagna et. al. 2000], landscape based string theory [Aazami & Easther, 2006] and also in quantum cosmology [Mersini-Houghton, 2005]. Although in RHM $\beta = 1$, it is quite natural to study the index distribution for other Gaussian ensembles, namely for GUE ($\beta = 2$) and GSE ($\beta = 4$).

For the GOE ($\beta = 1$), the statistics of $\mathcal{N}_+$ was studied by Cavagna et. al. [Vaagna et. al., 2000] using supersymmetric replica method and some additional approximations and they argued that around its mean value $N/2$, the random variable $\mathcal{N}_+$ has *typical* fluctuations of $O(\sqrt{\ln N})$ for large N. Moreover, the distribution of these typical fluctuations is Gaussian. In other words, over a region of width $\sqrt{\ln N}$, the distribution for large N is given by [Cavagna et. al., 2000]

$$\mathcal{P}(\mathcal{N}_+, N) \approx \exp\left[-\frac{\pi^2}{2\ln(N)} \left(\mathcal{N}_+ - N/2\right)^2\right] \tag{52}$$

implying that for $\beta = 1$, $\langle (\mathcal{N}_+ - N/2)^2 \rangle \approx \ln(N)/\pi^2$ for large N.

On the other hand, this Gaussian form does not describe the *atypically* large fluctuations of $\mathcal{N}_+$. For example, in the extreme limit when $\mathcal{N}_+ = N$, the probability that all eigenvalues are positive $\mathcal{P}(\mathcal{N}_+ = N, N)$ was computed by D.S. Dean and myself for large N and for all β [100],

$$\mathcal{P}(\mathcal{N}_+ = N, N) \approx \exp\left[-\beta\theta N^2\right]; \quad \theta = \frac{1}{4}\ln(3). \tag{53}$$

These two rather different forms of the distribution $\mathcal{P}(\mathcal{N}_+, N)$ in the two limits, namely in the vicinity of $\mathcal{N}_+ = N/2$ (over a scale of $\sqrt{\ln N}$) (as in (52)) and when $\mathcal{N}_+ = N$ (as in (53)) raises an interesting question: what is the form of the distribution $\mathcal{P}(\mathcal{N}_+, N)$ for intermediate values of $N/2 << \mathcal{N}_+ < N$? In other words, how does one interpolate between the limits of *typically* small and *atypically* large fluctuations. To answer this question, it is natural to set $\mathcal{N}_+ = cN$ where the intensive variable $0 \leq c \leq 1$ denotes the fraction of positive eigenvalues and study the large N limit of the distribution $\mathcal{P}(cN, N)$ with c fixed. Again, due to the Gaussian symmetry, $\mathcal{P}(cN, N) = \mathcal{P}((1-c)N, N)$ and it is sufficient to restrict c in the range $1/2 \leq c \leq 1$.

In Refs. [142, 156], using Coulomb gas method, we computed the large N limit of the distribution $\mathcal{P}(cN, N)$ in the full range $0 \leq c \leq 1$ for all $\beta > 0$ and showed that

$$\mathcal{P}(cN, N) \approx \exp\left[-\beta N^2 \Phi(c)\right] \tag{54}$$

where the rate function $\Phi(c) = \Phi(1-c)$, independent of β, was computed explicitly for all $1/2 \leq c \leq 1$. The fact that the logarithm of the probability is $\sim O(N^2)$ for fixed c is quite natural, as it represents the free energy of an associated Coulomb fluid of N charges (eigenvalues). The Coulomb energy of N charges clearly scales as $\sim O(N^2)$. In the limit $c \to 1$, we get $\Phi(1) = \theta = \ln(3)/4$ in agreement with Ref. [100]. The distribution is thus highly non-Gaussian near its tails. In the opposite limit $c \to 1/2$, *we find a marginally quadratic behavior, modulated by a logarithmic singularity*

$$\Phi(c) \simeq -\frac{\pi^2}{2}\frac{(c-1/2)^2}{\ln(c-1/2)}. \tag{55}$$

Setting $c = \mathcal{N}_+/N$ and substituting this form in (54), we find that in the vicinity of $\mathcal{N}_+ = N/2$ and over a scale of $\sqrt{\ln N}$, indeed one recovers the Gaussian distribution

$$\mathcal{P}(\mathcal{N}_+, N) \approx \exp\left[-\frac{\beta\,\pi^2}{2\ln(N)}\left(\mathcal{N}_+ - N/2\right)^2\right] \tag{56}$$

thus proving that the variance $\langle (N_+ - N/2)^2\rangle \approx \ln(N)/\beta\pi^2$ for large N and for all β. For $\beta = 1$, this perfectly agrees with the results of Cavagna et. al. Furthermore, we demonstrated that the logarithmic singularity in the rate function, rather *unusual*, actually reflects a *phase transition* in the underlying Coulomb gas.

Thus, to leading order for large N, we find that the variance of N_+, for large N behaves as

$$\Delta(N) = \langle (N_+ - N/2)^2\rangle \approx \frac{1}{\beta\pi^2}\,\ln N + O(1) \tag{57}$$

The subleading constant term in Eq. (57) is still unknown for arbitrary β. However, for $\beta = 2$, we were able to compute it exactly using a special determinantal formula [156]

$$\Delta(N) = \frac{1}{2\pi^2}\,\ln N + \frac{1}{2\pi^2}(\gamma_E + 1 + 3\,\ln 2) + O(1/N) \tag{58}$$

where $\gamma_E = 0.5772\ldots$ is the Euler constant.

Evidently, the Coulomb gas method has proved to be extremely useful in solving a number of problems related to the statistics of the number of eigenvalues. This is a rather general method and not restricted only to Gaussian random matrices. For instance, in recent works, we have computed the distribution of the number of eigenavlues in a given interval for other random matrix ensembles, such as for the Wishart (Laguerre) ensemble [167] and for the Cauchy ensemble [187, 227].

8.2.3 Entanglement entropy in bipartite quantum systems (with applications in quantum information and communication)

One of the nice applications of the random matrix theory is to understand the statistical properties of the entanglement between two quantum systems. Consider a composite bipartite quantum system consisting of two subparts A and B (the system and a heat bath for instance). One can think of A and B as two Hilbert spaces with dimensions N and M respectively. All measurements are done on the subpart A and hence the reduced density matrix of the subpart A (after integrating out the B degrees of freedom) plays a very crucial role. In particular, the knowledge of the spectrum (eigenvalues $\{\lambda_i\}$ with $i = 1, 2, \ldots, N$) of the reduced density matrix ρ_A is important and carries all physical informations. For example, one crucial quantity that has played a major role (in the context of quantum computation) is the entanglement between A and B. Entanglement entropy is simply a quantitative measure of the quantum correlations between A and B and a standard definition is the von-Neumann entropy $S = -\sum_i \lambda_i \log(\lambda_i)$. Its value lies between 0 and $\log N$. The bigger the value of S, the stronger is the quantum correlation. A classic problem is when the composite system is in a pure state that is randomly chosen (with uniform probability) from all possible allowed pure states. In this case, the eigenvalues $\{\lambda_i\}$ of ρ_A are also random and hence the entropy S is a random variable and it is important to understand its statistical properties. Previous studies concerned mostly with the mean entropy which was shown to be very close to its maximal value, $\langle S \rangle \simeq \log(N)$ for large N (with $M \geq N$) and based on these studies, it was concluded that the random state is an almost maximal entropy state. With my former Ph.D student Celine Nadal and in collaboration with M. Vergassola (Pasteur Institute), we were able to compute the full probability distribution of S in the large N limit (Phys. Rev. Lett., 104, 110501 (2010)), using a Coloumb gas technique in statistical physics. We found that as one varies S, there are three different regimes $0 < S < S_1$, $S_1 < S < S_2$ and $S > S_2$ where the distribution $P(S, N)$ has three different mathematical forms. At the two critical points S_1 and S_2, the function $P(S, N)$ is weakly nonanalytic. These two critical points correspond to two different phase transitions in the underlying Coloumb gas problem. An outcome of this analytical study is in fact to show that although the mean entropy is close to its maximal value, the distribution $P(S, N)$ drops very sharply as $S \to \log N$, implying that the probability that S is actually $\log N$ is indeed very small. Thus the conclusion based on the study of average (and not the full probability distribution) can often be misleading.

I have also worked on other *extremal* observables in this problem, e.g. in collaboration with O. Bohigas and A. Lakshminarayan (see [120]), we have computed exactly the full distribution of the minimum eigenvalue $\lambda_{\min} = \min(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and in the process we have proved an earlier conjecture by Znidaric on the exact average value of $\lambda_{\min}$. I have contributed a chapter on this subject in a recent monograph on random matrix theory (ed. by G. Akemann, J. Baik and P. Di Francesco and forwarded by F.J. Dyson), published by Oxford University Press (2011).

Currently we are collaborating with T. Seligman (UNAM, Mexico) on some other aspects of

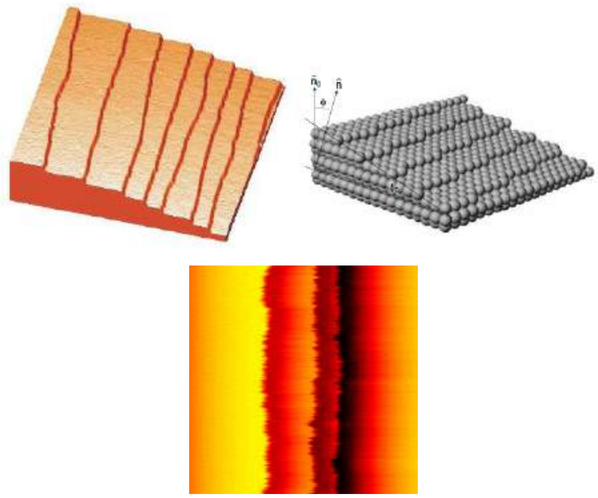


Figure 11: Experimental STM images of a vicinal surface showing step edges on a Cu(111) surface (from the group of E. Williams (Maryland)).

this system with a particular application in quantum computation. Quantum information and computation is a rapidly growing field of research and it gives us the opportunity to contribute in this field by using our theoretical tools developed in the context of random matrix theory.

Another beautiful application of this problem that we are currently working on is in wireless communications (the so called multi-antanae channel problems). *In collaboration with the random matrix group at Bristol University, UK (J. Keating, N. Snaith, F. Mezzadri etc.) and colleagues at Toshiba Research Laboratories at Bristol, we are in the process of finalising a proposal for a EU International Training Network (ITN) programme to train Ph.D students and postdocs on this modern application of random matrix theory.*

8.2.4 Nonintersecting brownian motions and its connections to Yang-Mills gauge theory in two dimensions

Nonintersecting Brownian motions, a model originally proposed by de Gennes in the context of mutually avoiding flux lines and polymers and then studied by M.E. Fisher in the context of commensurate-incommensurate phase transitions (dubbed as 'vicious' walkers by Fisher), appear in multiple problems and have been studied by both physicists, mathematicians and computer scientists alike. This system also appears in realistic experimental systems of fluctuating step edges on crystals across its vicinal surface. (see Fig. 11) [*this figure is from the group of E. Williams at University of Maryland, USA*].

Recent interests in this problem concern mainly on extreme value questions as well as its connection to classical random matrix theory. A simple system is the so called half-watermelons: N Brownian motions start close to the origin at time $t = 0$, stay on the positive side of the origin (hard wall at the origin) and do not intersect each other and then come back to the origin simultaneously after a fixed time, say $t = 1$. If one plots the space-time trajectories

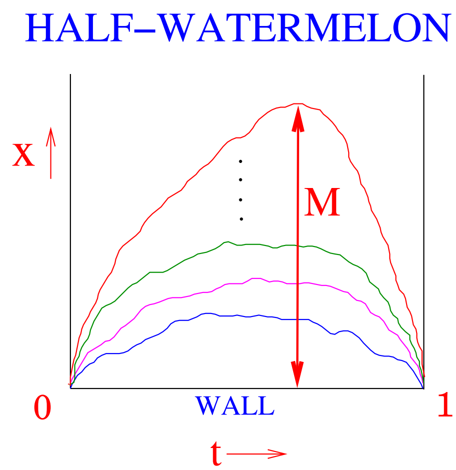


Figure 12: A schematic figure of N nonintersecting Brownian excursions over time interval $t \in [0, 1]$. The walkers all start near the origin at $t = 0$, stay on the positive half space and nonintersecting over $t \in [0, 1]$ and come back near the origin at $t = 1$. Thus generates a half-watermelon configuration. The height M denotes the maximum distance travelled in time $[0, 1]$ by the topmost particle.

(space on the y axis and time on the x axis), they look like half of a watermelon, hence the name (see Fig. 12).

Let M denote the *maximal* distance travelled by the rightmost walker till $t = 1$ (see Fig. 12). This is also called the height of the half-watermelon. There have been many studies of this random variable M. In particular, there was a rather indirect result (conjecture) due to Jahansson that asymptotically for large N, the probability density $P(M, N)$, appropriately centered and scaled, should have the Tracy-Widom form that describes the largest eigenvalue of a Gaussian orthogonal matrix.

In collaboration with A. Comtet (LPTMS), J. Randon-Furling (my former Ph.D student) and G. Schehr (LPTMS), we derived an exact formula [Ref. 130] for this distribution $P(M, N)$ for all M and N, using Fermionic path integral techniques (Phys. Rev. Lett., 101, 150601 (2008)). The cumulative distribution $F_N(L) = \text{Prob.}[M \le L]$ reads [130]

$$F_N(L) = \frac{B_N}{L^{2N^2+N}} \sum_{n_i=1,2\ldots} \prod_{i=1}^{N} n_i^2 \prod_{j<k} (n_j^2 - n_k^2)^2 \exp\left[-\frac{\pi^2}{2L^2}\sum_i n_i^2\right], \tag{59}$$

where the amplitude B_N is given by [130]

$$B_N = \frac{\pi^{2N^2+N}\, 2^{N/2-N^2}}{\prod_{j=0}^{N-1} \Gamma(j+2)\Gamma(j+3/2)}. \tag{60}$$

The formula for $F_N(L)$ above however involved an infinite discrete sum over N integer variables $(n_1, n_2, \ldots, n_N)$. Taking the asymptotics $N \to \infty$ and M large in this formula was highly nontrivial and it wasn't easy to prove Johansson's conjecture directly from this explicit formula. However, in collaboration with A. Comtet (LPTMS), P.J. Forrester (Melbourne) and G. Schehr (LPTMS), we realized that [Refs. 154, 175] exactly the same discrete sum, except for an important multiplicative factor, also appears in a completely different context: namely, in the partition function of the Yang-Mills gauge theory in two dimensions on a sphere with the gauge group $Sp(2N)$ that was studied by Gross, Witten, Douglas, Kazakov among others. This discrete sum was analysed by Gross and others using some beautiful properties of orthogonal polynomials.

We can then adopt these results from the large N gauge theory to our problem. Using them, we were able to show [154, 175] that the probability density $P(M, N)$ in the large N limit has a non-analytic point at $M = \sqrt{2N}$ across which the functional form of the distribution changes. In fact, this leads precisely to a *third-order* phase transition at this critical point. In the context of gauge theory, this 3-rd order phase transition corresponds to the so called Douglas-Kazakov transition between the 'strong' and the 'weak' coupling phases of the gauge theory. Thus, the left large deviation function in the maximal height distribution in the vicious walker problem indeed describes the free energy of the strong coupling phase of the gauge theory. In contrast, the right large deviation corresponds to the weak coupling phase. This 3-rd order transition is quite ubiquitous [for other examples, see our recent review in Ref. 188].

In addition, close to this critical point, on a scale $M = \sqrt{2N} + O(N^{-1/6})$, we were able to show that the scaled distribution $\log[F_N(L)]$ satisfies a second order nonlinear Painleve equation and hence is exactly the Tracy-Widom distribution of the GOE random matrix theory. This then proves Johansson's conjecture. Our work [154, 175], for the first time, makes this beautiful link between two seemingly different problems: nonintersecting Brownian motion model of de Gennes and the two-dimensional Yang-Mills gauge theory! In addition, it provides a rare example of a solvable extreme value problem in a strongly correlated system.

Nonintersecting Brownian motions is a very interesting collective system where many interesting questions, in particular extreme value questions, can be probed analytically. In 2012, we obtained an ANR grant 'WALKMAT' to study precisely various aspects of this vicious walker problem. We recruited our postdoc A. Kundu at LPTMS in 2012, who has been working on this subject. In a recent work [196], we have studied the extremal statistics of these vicious random walkers, but not over a fixed time interval $[0, 1]$, rather over a random stopping time when the first walker crosses the origin. We obtained analytically the full distribution of the height of the topmost and the bottommost walker. Currently, we are studying the order statistics, i.e., the distribution of the maximal distance travelled by the k-th topmost walker. In addition, with Kundu, Comtet and Schehr (LPTMS), and partly with M. V. Tamm (Moscow) we have studied other related problems for Brownian motions in one dimension. For example, we have exactly computed the statistics of the number of common and distinct sites visited by N independent random walkers in d dimensions [Refs. 172, 180] and also the statistics of the winding numbers of a single Brownian walker on a ring [Ref. 197].

In collaboration with J. Grela and G. Schehr, we have recently studied nonintersecting Brownian bridges in a flat-to-flat geometry (starting initially at uniformly separated positions and arriving at a uniformly separated poisition at some future time). This problem has interesting connections to other ensembles of RMT, such as the Borodin-Muttailib ensemble, as well as to Chern-Simons model (J. Grela, S. N. Majumdar, G. Schehr, "Non-intersecting Brownian bridges in the flat-to-flat geometry", J. Stat. Phys. v-183, 1 (2021).

Our work on the maximal height distribution [130] was reviewed in the 'research highlights' section of Nature Physics (vol-4, page 829, November 2008 issue).

8.2.5 Transport in mesoscopic cavities: distribution of conductance and shot noise power

Another beautiful application of the random matrix theory is in understanding the statistical properties of the conductance through a mesoscopic cavity such as a quantum dot. Ballistic transport of electrons through a quantum dot has been studied intensively both theoretically and experimentally over the last two decades. In the simplest setting one considers a single cavity of submicron dimension (e.g., a quantum dot) with two identical leads (each supporting N channels) connecting it to two separate electron reservoirs. An electron, injected through one lead, gets scattered in the cavity and leaves by either of the two leads. The transport of electrons through such an open quantum system is encoded in the $2N \times 2N$ unitary scattering matrix

$$S = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix}$$

connecting the incoming and outgoing electron wavefunctions, where r, t are $N \times N$ reflection and transmission matrices from the left and r', t' from the right. Several experimentally measurable transport observables such as the conductance (i.e., the time averaged current) and the shot noise power (that describes the current fluctuations associated with the granularity of electronic charge e) can be expressed in terms of N transmission eigenvalues T_i's of the $N \times N$ matrix $T = tt^\dagger$. For example, the dimensionless conductance $G = \mathrm{Tr}(tt^\dagger) = \sum_{i=1}^{N} T_i$ and the shot noise power $P = \mathrm{Tr}[tt^\dagger(1 - tt^\dagger)] = \sum_{i=1}^{N} T_i(1 - T_i)$. The eigenvalue $0 \leq T_i \leq 1$ has a simple physical interpretation as the probability that an electron gets transmitted through the i-th channel.

Over the past two decades, the random matrix theory (RMT) has been successfully used to model the transport through such a cavity. Within this RMT approach, one draws S at random from one of Dyson's Circular Ensembles, according to the symmetries of the system under consideration. This, in turn, induces a probability measure over the transmission eigenvalues and hence on the conductance and the shot noise power. Earlier studies concetrated on computing the mean and the variance of the conductance G. The fact that the variance of G becomes independent of N for large N was dubbed the famous 'universal conductance fluctuations'. In collaboration with P. Vivo (then at ICTP, now at LPTMS) and O. Bohigas (LPTMS), we were able to compute the full probability distribution of the conductance $P(G, N)$ in the limit of large N, using again a Coloumb gas technique (Phys. Rev. Lett., 101, 216809 (2008) and Phys. Rev. B, 81, 104202 (2010)). We found again that there are two phase transitions in the underlying Coloumb gas which leads to non-analytic behaviour of $P(G, N)$ at two critical values $G = N/4$ and $G = 3N/4$. The distribution has a Gaussian form in the middle $N/4 < G < 3N/4$, flanked on both sides (respectively for $G < N/4$ and $G > 3N/4$) by power law tails. This is completely new finding as the phase transition is somewhat unexpected here. Our results have been verified by extensive numerical simulations and we hope it may be posible to test our theoretical predictions experimentally as well.

In collaboration with P. Vivo (LPTMS) and K. Damle & V. Tripathi (Tata, India), we then applied similar random matrix approach to study the distribution of the Andreev conductance G_{NS} of normal-superconductor (NS) junctions with multiple transverse modes N. Transport across an NS junction is particularly interesting because an electron incident from the normal side can be reflected as a hole, with the injection of a Cooper pair into the superconducting condensate. This is the so called Andreev conductance. In the limit of large number of transverse modes N, we could compute the distribution of the Andreev conductance using Coulomb gas method [162]. The probability distribution $P(G_{NS}, N)$ displays a Gaussian behavior near the average value $\langle G_{NS} \rangle = (2-\sqrt{2})\, N$ and asymmetric power-law tails in the two limits of very small and very large G_{NS}. In addition, we found a novel third regime sandwiched between the central Gaussian peak and the power law tail for large G_{NS}. Weakly non-analytic points separate these four regimes—these were shown to be consequences of three phase transitions in an associated Coulomb gas problem [162].

In collaboration with P. Vivo (LPTMS) and K. Mallick (Saclay), we are currently working on some other aspects of this transport problem, e.g., computing the distribution of concurrence and amplified concurrence in a simple two-channel system. For this problem, Beenaker had computed the mean concurrence numerically. We have been able to compute analytically not just the mean, but the full distribution of concurrence and amplified concurrence.

Another interesting scattering problem in chaotic cavities concern the distribution of the so called Wigner time-delay in chaotic cavities, using the RMT approach. When an electron passes through a cavity, it undergoes scattering and the incident wave packet of the electron experiences a time-delay, a concept first introduced by Wigner. In collaboration with C. Texier (LPTMS), we have studied the distribution of the Wigner time-delay. Using Coulomb gas approach, we computed the distribution of Wigner-time delay in the limit of large number of channels [Ref. 181]. We showed that the existence of a power law tail in the distribution originates from narrow resonance contributions, related to a (second order) freezing transition in the Coulomb gas [181]. We are now working on various extensions of this project, such as computing the joint distribution of conductance and the Wigner time-delay. A. Grabsch will start his Ph.D in 2015 at LPTMS jointly with C. Texier and myself and we have several future projects in this area.

8.2.6 Matrix integrals and the associated fluid dynamics

This is a very recent project that I have started in collaboration with J.-P. Bouchaud and M. Potters (CFM). We recruited a common Ph. D student, J. Bun (LPTMS+CFM) in 2014, who has just started his Ph. D thesis on this subject. Very briefly, we are interested in the large N expansion of the celebrated Harischandra-Itzykson-Zuber matrix integral (Harischandra, 1957, Itzykson & Zuber, 1980). This matrix integral has found several applications in many different fields, including random matrix theory, disordered systems and qunatum gravity. But our motivation to study this problem came from finance. This integral is defined as

$$I_\beta(A,B) = \int_{G(N)} \mathcal{D}V \, \exp\left[\frac{\beta N}{2}\, \mathrm{Tr}\left(A\, V\, B\, V^\dagger\right)\right] \tag{61}$$

where the integral is over the flat Haar measure of the compact group $V \in G(N) = O(N)$, $U(N)$ or $Sp(N)$ in N dimensions and A and B are arbitrary $N \times N$ symmetric (respectively Hermitian or symplectic) matrices. The parameter β is the standard Dyson index and with $\beta = 1$, 2 and 4 respectively for the three groups. It turns out that only in the unitary case ($\beta = 2$), the HCIZ integral has an explicit formal expression in terms of a determinant.

The main problem is to determine how the integral I_β behave in the large N limit. On general grounds, one would expect that for large N, that $I_\beta(A, B) \sim \exp[N^2 F_2(\rho_A(x), \rho_B(x))]$ where $\rho_A(x)$ and $\rho_B(x)$ denote the asymptotic densities of the eigenvalues of A and B respectively and F_2 is the free energy that is a functional of these two densities. The question is: how to compute this 'free energy' $F_2(\rho_A, \rho_B)$. This has remained an open problem for a long time. In a classic paper, Matytsin (1994) mapped this problem (for $\beta = 2$) to a fluid dynamics problem. However, this mapping was rather formal and far from transparent. In addition, there was no explicit calculation of this free energy even for some representative cases of A and B.

In [198], we were able to map this problem to a evolving gas (fluid) of particles in one dimension undergoing Dyson Brownian motion and we showed, that for all β (*and not just for* $\beta = 2$), the free energy $F_2(\rho_A, \rho_B)$ is proportional precisely to the propagator of this system of Dyson Brownian gas from the initial density $\rho_A(x)$ at a *fictitious* time $t = 0$ to the final density $\rho_B(x)$ at time $t = 1$. Furthermore, by analysing this Dyson gas propagator via an instanton approach, we were able to derive the free energy F_2 explicitly for all β (for $\beta = 2$ our result reproduces Matytsin's result, but in a much more physically transparent way). In addition, we could also evaluate this free energy F_2 for certain choices of ρ_A and ρ_B.

This recent success [198] has motivated us to study related problems in finance (where the matrices are the so called Wishart matrices). Once again, one will have a Dyson Brownian gas, but this time in presence of a hard wall at the origin. Our student J. Bun is currently working on this problem. This is a very promising project which is just starting and we hope to obtain many more interesting results in this direction in near future.

8.3 Extreme value statistics and persistence/first-passage properties for various stochastic processes

As mentioned earlier, extreme value statistics concerns the statistics of the maximum or minimum of a set of random variables (either independent or correlated). This subject has found many applications in recent times in a variety of problems in physics, computer science, biology and climate studies. Over the past few years, I have worked extensively on this subject, mostly focussing on developing a general theory for the extreme statistics of strongly correlated variables and also in finding new applications. Below, I mention some of the new applications that we have found recently and on problems that I am currently working on.

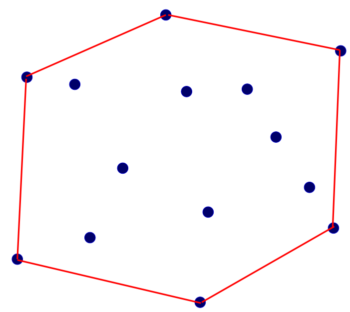


Figure 13: A realization of a $N = 14$ points on a plane and the unique convex hull associated with these set of points is shown in red.

8.3.1 Random convex hulls and extreme value statistics: application to ecology and epidemic spread in animals

With my former Ph.D student J. Randon-Furling and A. Comtet (LPTMS), we got interested in the statistics of convex hulls of a set of planar Brownian motions. This problem was motivated from an application to ecology as explained later. Subsequently, we found that this question about convex hull can be addressed for any general two dimensional stochastic process and has several important practical applications such as in estimating the area of the spread of an epidemic in animal populations etc. We found a beautiful, somewhat unexpected link between this problem and the extreme value statistics, using the so called Cauchy's formula in integral geometry. Before explaining this connection, let us first discuss the general basic problem.

The problem can be stated very simply as follows. Suppose we have a set of points on a plane. How do we characterize the *shape* of such a set of points? One natural way is to draw the convex hull of this set, i.e., the minimal convex polygon that contains all the points of this set (see Fig. 13). Some points will be in the interior of this hull and some on the perimeter. Clearly, if we choose a different set of points, the associated convex hull will also be different. For a given realization of the points, one can study several geometrical observables associated with the convex hull such as the perimeter, the area or the number of vertices on this convex hull. So, the general problem that has interested mathematicians and computer scientists (but so far there have been hardly any work by physicists) is: given a random set of points (chosen either *independently* or from a *correlated* distribution), what can we say about the statistics of the perimeter or the area of the convex hull? A lot of previous studies in mathematics and statistics were concentrated in finding the statistics of the perimeter and the area in the case of independent and identically distributed set of points. There was no general method available. For each choice of the distribution, a separate method was used.

Also, these previous methods do not work when the points are *correlated*. A natural example is when the points are the vertices of a random walk of T steps in two dimensions. This is a problem of considerbale practical interest, in particular in ecology and animal conservation. Suppose that we want to construct a reserved forest for some animals. For this purpose, ecologists typically observe the trajectory of an animal that roames around over space in search of food (usually they put a GPS on the animal and get a picture of the trajectory). Roughly speaking, the space or the territory over which an animal moves daily is called the **home range** of the animal. Now, how to quantify this two-dimensional home range from a given trajectory?

A popular method that is used routinely by ecologists is to construct the convex hull of the trajectory and use this as the estimate for the home range. Thus the question of the perimeter and the area of the convex hull of a random walk trajectory is of prime interest, because in many cases the animal trajectory can be simply modelled by a random walk (especially during the foraging period when they are searching for food). However, animals typically do not live in isolation, but rather in herds as a group. Suppose we have a population of n animals and we want to construct a reserved forest for them. How much area should one reserve?

For this one needs to estimate the size of the home range of n animals, over which they can freely roam around. The area of this home range should depend on the population size n and evidently it should increase with n. So, it is rather important in ecology to know how the perimeter/area of the home range of n animals grows with the population size n. Since a good measure of the home range is the convex hull, this then leads to the problem of studying the convex hull of n random walks where n is the number of animals. So, a natural quesion is: how does the perimeter or the area of the convex hull of n random walks depend on the animal population n? As a simple zero-th order model, one can assume that the animals (random walkers) are independent. But even in this simple case, the convex hull of the union of the trajectories of n random walkers is a nontrivial object. As n increases, the convex hull of the union changes its shape in a nontrivial non-local manner. So, how does the mean perimeter and the mean area of the convex hull of n random walkers each of T steps increase with n? This is the fundamental question. In Fig. 14, we show the convex hull of $n = 10$ independent Brownian motions whose trajectories are numerically generated.

So, how should one proceed to solve this problem? Numerically, it is relatively easy to generate the convex hulls (using standard algorithms available in the literature such as the Graham Scan algorithm), but analytically it looks like a formidable problem. However, we managed to solve this problem analytically after discovering an old, but not so well known, formula due to Cauchy (1832) which gives an explicit expression for the perimeter and the area of any closed convex curve in two dimensions, in terms of the so called support function. Our main idea was to adapt these formulae to *random* convex curves, i.e., for the random convex polygon generated from the trajectory of a two dimensional stochastic process. We came up with a very general formula, valid for arbitrary 2-d stochastic process, that expresses the mean perimeter and the mean area of the convex hull of the 2-d process in terms of the statistics of the extreme of the corresponding 1-dimensional component process [see Refs. 141, 147]. This very general mapping from 2-d process to 1-d process then allowed us to solve exactly, not just the n independent Brownian motions problem mentioned above, but also for a number of other stochastic processes in 2-d where the vertices are *correlated*. In addition, our approach reproduced all known mathematical results for i.i.d vertices, in a general and unified manner. Below, we just mention some of our main new results.

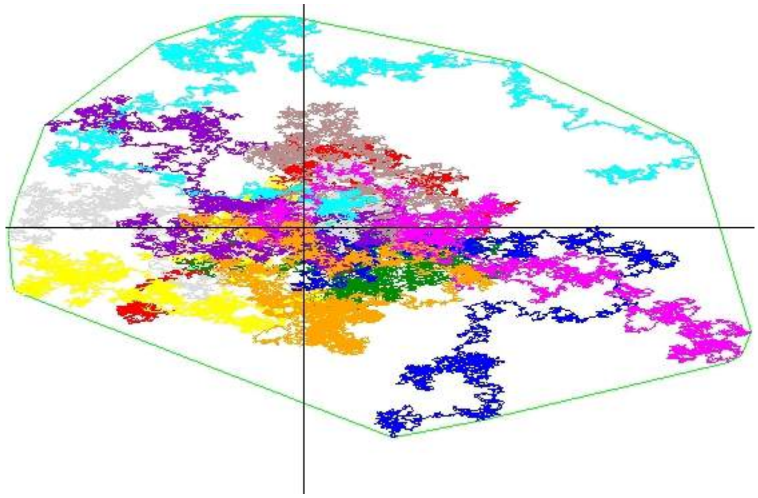


Figure 14: A realization of the convex hull of $n = 10$ independent Brownian motions each of the same duration.

For example, for n independent Brownian motions each of T steps, we found that the mean perimeter and the mean area behave respectively as

$$\langle L_n \rangle = \alpha_n \sqrt{T} \quad (62)$$

$$\langle A_n \rangle = \beta_n T \quad (63)$$

where we were able to derive exact expressions for the prefactors α_n and β_n. Note that the time dependence $\sqrt{T}$ for the perimeter and T for the area follows trivially from the diffusive scaling, but the dependence on the population size n, through the prefactors α_n and β_n, is highly nontrivial. Our exact results give [141, 147]

$$\alpha_n = 4\sqrt{2\pi}\, n \int_0^\infty du\ u\ e^{-u^2}\ [\mathrm{erf}(u)]^{n-1} \quad (64)$$

$$\beta_n = 4\sqrt{\pi}\, n \int_0^\infty du\ u\ [\mathrm{erf}(u)]^{n-1} \left(ue^{-u^2} - h(u) \right) \quad (65)$$

where $h(u) = \frac{1}{2\sqrt{\pi}} \int_0^1 \frac{e^{-u^2/t}\ dt}{\sqrt{t(1-t)}}$ and $\mathrm{erf}(u) = (2/\sqrt{\pi}) \int_0^u e^{-t^2}\, dt$ is the standard error function. For instance, $\alpha_1 = \sqrt{8\pi} = 5,013..$, $\alpha_2 = 4\sqrt{\pi} = 7,089..$, $\alpha_3 = 24\, \frac{\tan^{-1}\left(1/\sqrt{2}\right)}{\sqrt{\pi}} = 8,333..$ etc., while $\beta_1 = \frac{\pi}{2} = 1,570..$, $\beta_2 = \pi = 3,141..$, $\beta_3 = \pi + 3 - \sqrt{3} = 4,409...$ We also derived similar exact formulae for n Brownian *bridges*, i.e., closed trajectories [141, 147]. From these exact formulae for any n, one can then derive the large n asymptotics. Surprisingly, for increasing n (as the animal population increases), the prefactors increase very slowly, $\alpha_n \sim \sqrt{\log n}$ and $\beta_n \sim \log n$. This is rather good news for conservationists as it indicates that one doesn't have to increase the home range by too much even if the animal population increases, say, by 100 fold.

Incidentally, the details of this approach involve calculating the maximum distance travelled by a group of n one-dimensional random walks, thus providing a direct link to extreme value statistics problem. In fact, one can view this convex hull problem as a very nice and original application of the extreme value statistics.

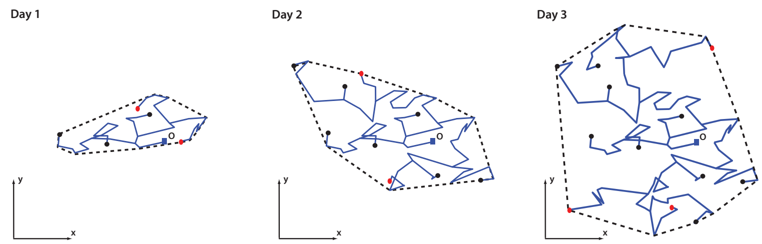


Figure 15: The snapshots of the trajectories of an assembly of infected individuals at three different times (schematic), starting from a single infected at the origin O at time $t = 0$. The blue lines show the trajectories of infected animals. Individuals that are still infected at a given time t are displayed as red dots, while those already recovered (or equivalent died) are shown as black dots. The convex hull enclosing the trajectories (shown as a dashed line) is a measure of geographical area covered by the spreading epidemic. As the epidemic grows in space, the associated convex hull also grows in time.

Since our result, exploiting Cauchy's formula and a mapping to the 1-d extreme value problem, provides a very general way to compute the mean perimeter and the mean area for *any* 2-d stochastic processes, it was natural to apply this method to other problems. This project has been very successful so far. For instance, one interesting question arises in the context of an epidemic spread in an animal population. An animal, after being infected by a disease, can diffuse and infect other animals and also can recover/die. During the outbreak stage of an epidemic, it is important to identify the infected region in space where the animals already have the disease. This infected regime is usually estimated by field epidemiologists by a convex hull around the infected region. Hence it is important to know how the epidemic (charcterized by the associated convex hull around the infected regime) grows with time T. A popular model for the epidemic spread is the branching Brownian motion (BBM), where in a small time interval dT, a particle branches into two (i.e., infects an additional animal) with probability $b\,dT$, dies (or recovers) with probability $a\,dT$ and with the remaining probability $1-(a+b)\,dT$ it diffuses with diffusion constant D. For $b > a$, the epidemics explodes. For $b < a$, the epidemics becomes extinct with time (supercritical phase). In the critical case, the epidemics grows slowly with time T (subcritical phase). A schematic picture of the growing convex hull is shown in Fig. 15.

In collaboration with A. Rosso (LPTMS), and A. Zoia & E. Dumonteil (CEA, Saclay), we solved this problem of the convex hull of 2-d branching Brownian motion exactly [PNAS, 110, 4239 (2013)] (Ref. 177). We showed that in the supercritical phase, the mean perimeter increases linearly with T for large T, $\langle L(T)\rangle \sim v\,T$ where we computed the velocity v exactly. The mean area increases as $\sim T^2$ for $b > a$. In contrast, at the critical point $b = a$, we found [177]

$$\langle L(T)\rangle \xrightarrow[T\to\infty]{} 2\,\pi\,\sqrt{\frac{6\,D}{a}} + O\left(T^{-1/2}\right) \tag{66}$$

$$\langle A(T)\rangle \xrightarrow[T\to\infty]{} \frac{24\,\pi\,D}{5\,a}\,\ln T + O(1) \tag{67}$$

Also, in collaboration with A. Rosso (LPTMS) and A. Reymbaut (former M.Sc student at LPTMS), we studied the convex hull of a 2-d non-Markov process, namely, the random acceleration process: $d^2\vec{r}/dt^2 = \vec{\eta}(t)$ where $\vec{\eta}(t)$ is a two-dimensional white noise $\langle \eta(t)\eta(t')\rangle = 2\,\delta(t-t')$. This problem arises in the context of describing the shape of a two-dimensional semi-flexible polymer chain. We have obtained exact results for this problem. We showed that the mean perimeter and the mean area of the convex hull of the random acceleration process grows, for all T, as [161]

$$\begin{aligned} \langle L(T)\rangle &= \frac{3\pi}{2}\,T^{3/2} && (68)\\ \langle A(T)\rangle &= \frac{5\pi}{192}\sqrt{\frac{3}{2}}\,T^{3} && (69)\end{aligned}$$

More recently, in collaboration with O. Benichou (UPMC) and his Ph.D student M. Chupeau, we applied this general method in a series of papers [204, 212, 214] to study the convex hull of a Brownian motion confined in the semi-infinite plane and obtained several exact results. We consider a single Brownian motion of duration T and diffusion constant D, starting at a distance d from a reflecting wall (see Fig. 5a). Let $L(d,T)$ denote the perimeter of the convex hull. This is a random variable, fluctuating from ampl to sample. We were interested in computing the mean perimeter $\langle L(d,T)\rangle$. We showed that, for any T, the mean perimeter has a scaling form

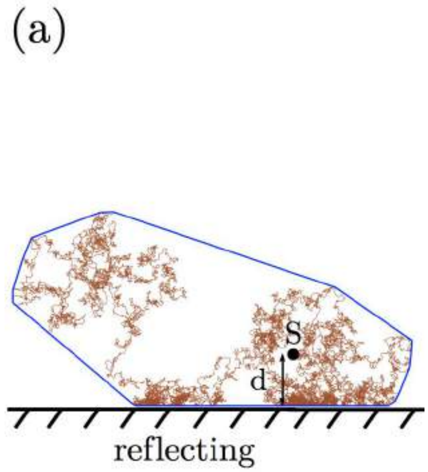
$$\langle L(d,T)\rangle = \sqrt{D\,T}\,f\left(\frac{d}{\sqrt{D\,T}}\right) \tag{70}$$

where the scaling function $f(x)$ is exactly computable. It has the asymptotic behaviors It has the asymptotic behaviors

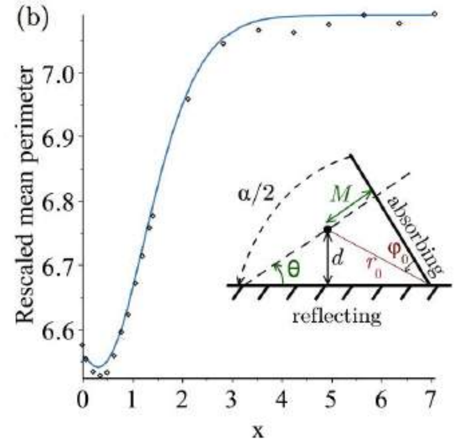
$$\begin{aligned} f(x) &\to \sqrt{16\,\pi} = 7.08982\ldots \quad \text{as} \quad x\to\infty \\ &= 2\sqrt{\pi}\,\mathrm{Si}(\pi) - \frac{8\sqrt{2\pi^3}}{3\Gamma(3/4)}\left(\frac{x}{\ln(1/x)}\right)^{3/2} && (71)\end{aligned}$$

where $Si(\pi) = \int_0^{\pi} \frac{\sin x}{x}\,dx = 1.85194\ldots$. The scaling function $f(x)$ thus a non-analytic behavior as $x \to 0$. Moreover, $f(x)$ displays a surprisingly non-monotonic behavior as a function of x (see Fig. 5b). Thus the mean perimeter of the convex hull of the trajectory becomes a minimum at $x = x^*$, i.e., there is an optimal starting distance $d^* = x^*\sqrt{D\,T}$ that makes the perimeter of the convex hull minimal, a rather surprising result. The non-monotonicity in this 2D case originates from the competition between two antagonistic effects due to the presence of the wall: reduction of the space accessible to the Brownian motion and an effective repulsion from the wall.

Another important question is as follows. Our approach allows us to estimate analytically only the first moment of the perimeter or the area of the convex hull of, say, a Brownian motion. What can one say about the higher moments, or even the full probability distribution of the perimeter and the area? In collaboration with A.K. Hartmann (Oldenburg, Germany) and his graduate student G. Claussen, we studied numerically the full distribution of the perimeter and the area of the convex hull of a random walk in two dimensions [Ref. 210]. To our knowledge, this was the first study of the full distribution of the perimeter and the area. We used a sophisticated large-deviation approach that allows us to study the distributions over a larger range of the support, where the probabilities $P(A)$ and $P(L)$ are as small as 10^{-300}. We analysed (open) random walks as well as (closed) Brownian bridges on the two-dimensional discrete grid as well as in the two-dimensional plane. The resulting distributions exhibit, for large time steps T, a universal scaling behavior (independent of the details of the jump distributions) as a function of $A/\sqrt{T}$ and $L/\sqrt{T}$, respecively. We are also able to obtain the rate functions characterizing the probability of large deviations, i.e,



(5a)



(5b)

Figure 16: (a) The trajectory of a walker starting at a distance d from the reflecting wall (b) the scaling function $f(x)$ has a non-monotonic behavior, with a minimum at an optimal scaled distance x^*.

describing rare events at the tails of these distributions. These results are summarized in Ref. 25. Furthermore, very recently, we were able to extend these large deviation sampling techniques to the case of multiple random walkers [Ref. 226].

The other interesting future project is to take into acount the interactions between different trajectories. For example, in case of insects like ants, the trail of one ant leaves the traces of pheromone which attracts other ants to it. It would be interesting to see how the statistics of convex hulls get affected in presence of interactions and memory effects. We have a future project with L. Giuglio (Bristol, UK) in these directions.

8.3.2 First-passage properties in polymer translocation process

Consider first the following general problem. Let $X(t)$ represents the position of a particle in one dimension undergoing a stochastic dynamics starting from the initial position $x_0 > 0$. Let the origin $X = 0$ represent an absorbing boundary or a trap. When the particle reaches the origin, it dies. A natural question is the so called 'persistence' or the survival probability: what is the probability $Q(x_0, t)$ that the particle survives up to time t. Its time derivative $F(x_0, t) = -dQ/dt$ represents the first-passage probability density, i.e., the probability that the particle reaches the origin for the first time between time t and $t+dt$. This persistence or the associated first-passage probability has been studied for many decades and for a general stochastic process $X(t)$, the computation of $Q(x_0, t)$ is extremely hard. It is known only for simple Markovian and some special types of non-Markov processes as has been discussed earlier. For many processes, the survival probability decays very slowly with time t as a power law, $Q(x_0, t) \sim t^{-\theta}$ where θ is called the persistence exponent. Considerable efforts, both theoretical and experimental, have been devoted over the last two decades in determining the persistence exponent θ for various stochastic processes, as we have discussed earlier.

Motivated by questions arising in the translocation of a polymer chain through a nanopore, we were interested in studying a slightly different question. Given that the particle has survived up to time t, what is the *conditional* probability density $P_+(x,t)$, normalized to unity, of finding the particle between x and $x+dx$ at time t? Clearly, due to the absorbing boundary condition at $x=0$, $P_+(x,t)$ must vanish as $x \to 0$. But how does it vanish? For processes whose persistence probability decays as a power law, $Q(x_0,t) \sim t^{-\theta}$, it turns out that $P_+(x,t)$ vanishes as $\sim x^{\phi}$ as $x \to 0$. The natural question is: what is this exponent ϕ? Is it related to the persistence exponent θ? If so, what is this relation?

In collaboration with A. Rosso (LPTMS, Orsay) and A. Zoia (CEA, Saclay), we studied this conditional probability $P_+(x,t)$ for a variety of Markovian and non-Markovian stochastic processes. In particular, we focussed on processes that are self-affine. A self-affine process is one where typically $X(t)$ grows with time as a power law, $X(t) \sim t^H$ where H is called the Hurst exponent of the process. The simplest example being the Brownian motion where clearly $H = 1/2$. For a generic self-affine process, we argued that the exponent ϕ is related to the persistence exponent θ via a very simple scaling relation $\phi = \theta/H$ [Phys. Rev. Lett., 102, 120602 (2009)]. This relation then has been tested and verified numerically for a variety of processes. In particular, for the so called fractional Brownian motion, which is a special self-affine Gaussian process with a correlator $\langle X(t_1)X(t_2)\rangle = [t_1^{2H} + t_2^{2H} - |t_1-t_2|^{2H}]$ and for which it is known exactly that $\theta = (1-H)$, our specific prediction is $\phi = (1-H)/H$. Note that for $H = 1/2$, the fractional Brownian motion reduces to the ordinary Brownian motion. In collaboration with K. Wiese (LPTENS) and A. Rosso, we have verified [160] this result for $H = 1/2 + \epsilon$ via a perturbation theory around the Markov process $H = 1/2$. Mathematicians such as A. Dembo (Stanford, USA) and F. Aurzada (Darmstadt, Germany) are now working to rigorously prove our conjecture $\phi = (1-H)/H$ for the fractional Brownian motion.

This result is of direct relevance for the polymer translocation process. Translocation simply means the passage of a DNA, RNA or a protein through a pore in the cell membrane and this process is key to understanding several biological functions of cells. Consider a polymer consisting of N monomers which is trying to pass through a pore, say from left to right. The number of monomers or the length of the part of the chain to the right of the pore at time t is denoted by $X(t)$ and is called the translocation coordinate. It starts at some initial value $X(t=0) = x_0$ and then fluctuates with time t and hence represents a stochastic process. If $X(t)$ hits 0 at some time, it means that the translocation has been unsuccessful. On the other hand, if $X(t)$ hits N (the length of the chain) before hitting 0, it means that the chain has gone through the pore from the left to the right and the translocation process has been successful. From several numerical simulations, it turns out that a good effective model that describes $X(t)$ quite well at a mesocopic level (ignoring microscopic details) is to assume that the $X(t)$ is a fractional Brownian motion with a Hurst exponent $0 < H < 1$. For an infinitely long chain ($N \to \infty$), the process $X(t)$ then takes place in the positive half line with an absorbing boundary at $X = 0$ and we see immediately why the question/result discussed in the previous paragraph is relevant for the translocation process of a long chain.

In contrast, when the chain length N is finite, a more relevant question is: what is the probability $q(x_0,N)$ that the translocation will be successful, i.e., what is the probability that the process, starting at x_0, will hit the level $X = N$ before hitting the level $X = 0$? For an ordinary Brownian motion, this is the classical Gambler's ruin probem and the answer is simply $q(x_0,N) = x_0/N$. Note that it vanishes linearly as $x_0 \to 0$. For a generic self-afine stochastic process, the computation of $q(x_0,N)$ is far from trivial. In collaboration with A. Rosso and A. Zoia, we showed that for a generic self-affine process with a Hurst exponent

H, $q(x_0, N)$ vanishes as a power law $q(x_0, N) \sim x_0^{\phi}$ as $x_0 \to 0$ where $\phi = \theta/H$ is the same exponent that is discussed in the previous paragraph [Phys. Rev. Lett. 104, 020602, (2010)]. The proof of this result also made use of the extreme value statistics.

Currently, we are extending this work in various other directions, for example, for non self-affine processes. In addition, we are trying to understand how to derive the effective stochastic process of the translocation cordinate $X(t)$ from a more microscopic starting point.

8.3.3 Stochastic search problems: diffusion with stochastic resetting

The optimal stochastic search appears as a classic problem in areas as diverse as computer science (e.g. searching for an element in an array) through biochemistry (e.g. a protein searching a binding site) to macrobiology (e.g. a predator seeking its prey) . It is also a teasing question in everyday life: how best does one search for lost keys? One general class of search strategies are termed *intermittent* which combine two processes: (i) periods of slow, local motion, termed *foraging* in which the target may be detected and (ii) periods of fast motion, termed *relocation*, during which the searcher relocates to a new territory. For example, if one is looking for the lost car key, one may first search in the garage for a while, and if she doesn't find the key, she may relocate to the kitchen or the bedroom and continue local searches there.

Similar *intermittent* strategy seems to work in *visual* search as well. When one searches for a specific face in a crowd, the eye starts at some initial position and then searches for the face in a given region. If the eye is unable to locate the face, it comes back to the initial point and 'restarts' the search process. In experiments in psychology, one attaches a camera to they eye of a patient and then the track of the eye is recorded. In order to have a quantitative understanding of this 'resetting' phenomenon in search and to understand why such a 'resetting' phenomenon may lead to an optimal search, we introduced a simple model [PRL, 106, 160601 (2011), Ref. 158] a few years back in collaboration with M.R. Evans (Edinburg, UK). For simplicity, I define the model here in one dimension, but generalisation to higher dimensions is straightforward.

Consider a single particle on a line, initially located at x_0. The particle moves via the following continuous time dynamics. In a small time interval dt, the position $x(t)$ of the particle at time t evolves via the rule

$$\begin{aligned} x(t+dt) &= x_0 \qquad \text{with probability} \quad r\,dt \\ &= x(t) + \eta(t)\,dt \quad \text{with probability} \quad 1 - r\,dt \end{aligned} \tag{72}$$

where $\eta(t)$ is a Gaussian white noise with zero mean and correlator, $\langle \eta(t)\eta(t') \rangle = 2\,D\,\delta(t-t')$. The rate $r \geq 0$ denotes the resetting rate with which the particular goes back to its starting point, but otherwise it diffuses with diffusion constant D (see Fig. 17). Thus, 'resetting' mimics the relocation move and 'diffusion' mimics foraging (local moves) in the context of intermittent search.

In absence of resettting ($r = 0$), the probability density of the particle position is simply

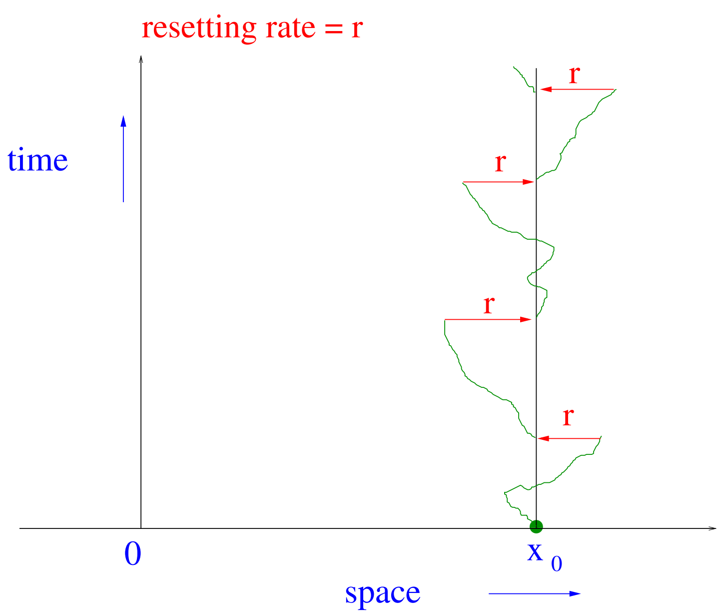


Figure 17: diffusion with resetting rate r to the initial position x_0 in 1-dimension.

diffusive:

$$p_{r=0}(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp[-(x-x_0)^2/4Dt]\,. \tag{73}$$

When the resetting is switched on ($r>0$), it turns out that $p_r(x,t)$ satisfies the Fokker-Planck equation [158]

$$\partial_t p_r = D\,\partial_x^2 p_r - r\,p_r(x,t) + r\,\delta(x-x_0) \tag{74}$$

which allows for a *stationary* solution

$$p_r(x,t\to\infty) = \frac{\alpha_0}{2}\,\exp[-\alpha_0\,|x-x_0|] \quad \text{where} \quad \alpha_0 = \sqrt{r/D}\,. \tag{75}$$

Thus the stationary solution has a cusp at $x=x_0$. This actually represents a *nonequilibrium steady state*, since the *detailed balance* is violated by the resetting move and the steady state carries a nonzero probability current in the configuration space. A similar calculation in higher dimensions gives [193]

$$p_r(\vec{x},t\to\infty) = \frac{(\alpha_0)^d}{(2\pi)^d}\,\left[\alpha_0\,|\vec{x}-\vec{x}_0|\right]^\nu\,K_\nu\left(\alpha_0\,|\vec{x}-\vec{x}_0|\right) \tag{76}$$

where $\nu = d/2-1$, $\alpha_0=\sqrt{r/D}$ and $K_\nu(z)$ is the modified Bessel function.

To estimate the efficiency of this search strategy "diffusion with stochastic resetting', we considered first a fixed target at the origin (say in $d=1$) and the searcher starts at x_0 and undergoes diffusion with stochastic resetting. The convenient quantity to compute is the mean first passage time (MFPT) to the target, i.e., the mean time to find the target. Note that in $d=1$, in absence of resetting ($r=0$), this MFPT diverges. However, in presence of resetting $r>0$, the MFPT $\bar{T}$ becomes finite. This was computed exactly by solving a backward Fokker-Planck equation for the survival probability [158] and it gives

$$\bar{T}(r,x_0) = \frac{1}{r}\left[\exp\left(\sqrt{r/D}\,x_0\right)-1\right]\,. \tag{77}$$

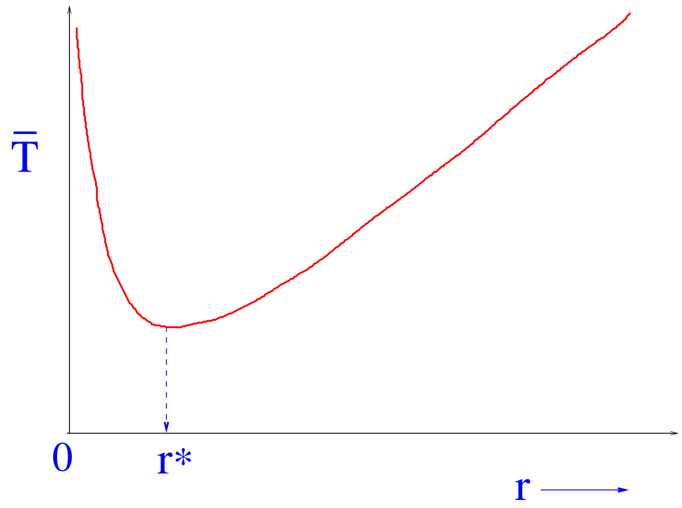


Figure 18: mean first-passage time as a function of the resetting rate r, for fixed $x_0 = 1$ and $D = 1$.

As a function of the resetting rate r, with fixed x_0, $\bar{T}$ is nonmonotonic, with a minimum at r^* and diverging at the two ends $r = 0$ and $r \to \infty$ (see Fig. 18). When $r \to 0$, $\bar{T}$ diverges as it corresponds to normal diffusion: there are trajectories which take the searcher to positive inifinity. In contrast, when $r \to \infty$, the particle is almost constantly reset to x_0 and hence never gets the opportunity to reach the origin. In between these two extreme limits, $\bar{T}$ has a minimum (this corresponds to the optimal reset rate that minimizes the mean search time) at $r = r^*$ given by [158]

$$r^* = \gamma^2 \frac{D}{x_0^2} \quad \text{where} \quad \gamma - 2\left(1 - e^{-\gamma}\right) = 0 \to \gamma = 1.59362\ldots \tag{78}$$

Similar optimal results have been derived in higher dimensions [193].

In a recent work [Ref. 193], we have generalised this model to higher dimensions. We have shown that there is always an optimal resetting rate that minimizes the search time in all dimensions and we could compute it explicitly in all dimensions [Ref. 193].

In another recent work [211], we studied the dynamics of approach to the nonequilibrium steady state in models of diffusion with stochastic resetting. We found an unusual relaxation mechanism in these systems. We showed that as time progresses, an inner core region around the resetting point reaches the steady state, while the region outside the core is still transient. The boundaries of the core region grow with time as power laws at late times with new exponents. Alternatively, at a fixed spatial point, the system undergoes a dynamical transition from the transient to the steady state at a characteristic space dependent timescale $t^*(x)$. We calculated analytically in several examples the large deviation function associated with this spatio-temporal fluctuation and show that generically it has a second order discontinuity at a pair of critical points characterizing the edges of the inner core. These singularities act as separatrices between typical and atypical trajectories. Our results were verified in the numerical simulations of several models, such as simple diffusion and fluctuating one-dimensional interfaces.

In another recent work [202], we generalised the continuous-time diffusion with stochastic resetting model to the case where the searcher undergoes a discrete-time Markov jump process in one dimension, starting at $x_0 \geq 0$, where successive jumps η's are drawn independently from an arbitrary jump distribution $f(\eta)$. In addition, with a probability $0 \leq r \leq 1$, the position of the searcher is reset to its initial position x_0. The efficiency of the search strategy is again characterized by the mean time to nd the target, i.e., the mean first passage time (MFPT) to the origin. For arbitrary jump distribution $f(\eta)$, initial position x_0 and resetting probability r, we were able to compute analytically the MFPT. For the heavy-tailed Levy stable jump distribution characterized by the Levy index $0 < \mu \leq 2$, we showed that, for any given x_0, the MFPT has a global minimum in the (μ, r) plane at $(\mu^*(x_0), r^*(x_0))$. We found a remarkable first-order phase transition as x_0 crosses a critical value x_0^*, at which the optimal parameters change discontinuously. Our analytical results were verified in numerical simulations.

This result indeed then proves that the simple 'resetting' strategy does indeed make the search of a target more efficient and moreover, our analytical calculation provides us with a precise estimate of the optimal resetting rate r^* in all dimensions. Over the last few years, we have worked extensively on various ramifications of this simple search stratgey that may improve further the search efficiency: for example, (i) what happens when the resetting is switched on only when the particle deviates from x_0 by a maximum thresold Δ [164], (ii) how does one compute the MFPT in presence of a space dependent resetting rate $r(x)$ [164], (iii) comparison between this nonequilibrium search via resetting and an equilibrium search where the particle moves in a confining potential [179], (iv) resetting in presence of partial adsorption [176] etc. We also generalised to the case when there is a single target, but there are *multiple* searchers [158, 193].

In all these models above, the resetting is always done at the initial point. Motivated by animals searching for food, where they keep track of already visited sites and the frontiers of the already explored region, we studied a simple, solvable model [216], where the resetting is done to the current frontier of the explored region. In the simplest 1-d version of the model, we considered a random walker on a lattice, where at each time step the walker resets to the maximum of the already visited positions (to the rightmost visited site) with a probability r, and with probability $(1-r)$ it undergoes a symmetric random walk. For $r = 0$, it reduces to a standard random walk, where the typical distance grows as $\sqrt{n}$ for large time n. When the resetting probability $r > 0$ is switched on, we found that both the average maximum and the average position grow ballistically for large n, with a common speed $v(r)$ given by [216]

$$v(r) = \frac{r(1-r)}{r - 2r^2 + \sqrt{r(2-r)}} \,. \tag{79}$$

Moreover, the fluctuations around the average maximum and the average position both grow diffusively, with a common diffusion constant $D(r)$ that was also computed explicitly. We also show that the probability distribution of the difference between the maximum and the location of the walker, becomes stationary as $n \to \infty$. However, the approach to this stationary distribution is accompanied by a dynamical phase transition, characterized by a weakly singular large deviation function. We also show that $r = 0$ is a special 'critical' point, for which the growth laws are different from the $r \to 0$ case. We calculated the exact crossover functions that interpolate between the critical $(r = 0)$ and the off-critical $r \to 0$ behavior, for finite but large n.

In the single particle setting, our model on diffusion with stochastic resetting generates, via a very simple mechanism, a simple yet nontrivial nonequilibrium stationary state (NESS). This led to a natural question: Can one generate such nontrivial NESS in an extended system? With this motivation, we studied with S. Gupta (postdoc, LPTMS) and G. Schehr (LPTMS), a fluctuating $(1+1)$-dimensional interface subjected to resetting dynamics. We considered, for example, a fluctuating surface undergoing Edwards-Wilkinson or KPZ dynamics, but at times (distributed exponentially) the interface goes back to its initial *flat* configuration. In a recent Letter [PRL, 112, 220601 (2014), Ref. (190)], we showed that indeed the interface, at long times, approaches a new NESS which we were able to characterize completely. For example, for the KPZ interface, we showed that the height distribution at a fixed point in space in this NESS is related to the Tracy-Widom distribution of random matrices belonging to the GOE class [190].

While the 'optimal resetting' paradigm, discussed above, has been tested and verified in a large number of theoretical and numerical studies, there was no experimental verification so far. In a recent paper, in collaboration with the group of S. Ciliberrto at ENS-Lyon, we provided the first experimental verification of the 'optimal resetting' paradigm, using optical laser traps. The goal of the experiment was not just to mimic the theoretical models, but it led to designing an experiment with a realistic resetting protocol, which was rather challenging. Moreover, our experimental protocol led us, in turn, to study new models that exhibit interesting, rich and novel phenomena associated to resetting, namely metastability and phase transition in the MFPT as a function of resetting rate/period.

"**Optimal mean first-passage time for a Brownian searcher subjected to resetting: experimental and theoretical results**", by B. Besga, A. Bovon, A Petrosyan, S. N. Majumdar, S. Ciliberto, Phys. Rev. Research, v-2, 032029 (R) (2020).

In all the analysis of diferent models mentioned above, we found that there is indeed a very nice and general connection between the reset problem and extreme value statistics. For example, the survival probability of the target very precisely depends on the maximum of the stochastic process that describes the motion of the searcher . This is actually a very general result and holds for arbitrary stochastic moves of the searcher–diffusion with resetting being just a special case. Thus the diffusion with reset represents another very nice application of extreme value statistics of correlated variables. Currently, we are in the process of further generalizing this model and its variants in several directions and 'diffusion with stochastic resetting' has become a rather hot area of research given the number of papers published on this subject over the last few years. Very recently, we have wriiten an exhaustive review on this subject:

M. R. Evans, S. N. Majumdar, and G. Schehr, "Stochastic resetting and applications", J. Phys. A. : Math. Theor. v-53, 193001 (2020).

8.3.4 Branching Brownian motion: extremal and gap statistics

Branching processes are prototypical models of systems where new particles are generated at every time step – these include models of evolution, epidemic spreads and nuclear reactions amongst others. An important model in this class is the Branching Brownian motion (BBM).

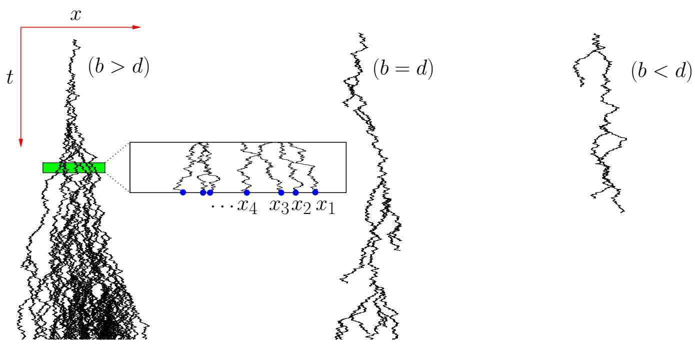


Figure 19: A realization of the dynamics of branching Brownian motion with death in the supercritical regime ($b > d$) (left), at the critical point ($b = d$) and in the subcritical regime ($b < d$). The particles are numbered sequentially from right to left as shown in the inset.

BBM is a paradigmatic model of branching processes with wide applications and has been studied extensively in both mathematics and physics literature. Recently, in collaboration with our postdoc K. Ramola (LPTMS) and G. Schehr (LPTMS), we have studied analytically the statistics of the position of the rightmost particle and also the statistics of gaps between the positions of successive particles near the tip of the branching process, in one dimensional BBM. We obtained numerous exact results (summarized below). The first of these results were published in [**PRL**, 112, 210602 (2014)] (Ref. 189). Subsequently, further results on the details of the computations appeared in Ref. [205] and also the exact distribution of span of the branching process was published in Ref. [208].

In a one dimensional BBM, the process starts with a single particle at the origin $x = 0$ at time $t = 0$. The dynamics proceeds in continuous time according to the following rules. In a small time interval Δt, each particle performs one of the three following microscopic moves: (i) it splits into two independent particles with probability $b\Delta t$, (ii) it dies with probability $d\Delta t$ and (iii) with the remaining probability $1 - (b + d)\Delta t$ it performs a Brownian motion moving by a stochastic distance $\Delta x(t) = \eta(t)\Delta t$. Here $\eta(t)$ is a Gaussian white noise with zero mean and delta-correlations with

$$\langle \eta(t) \rangle = 0, \quad \langle \eta(t_1)\eta(t_2) \rangle = 2D\delta(t_1 - t_2) \tag{80}$$

where D is the diffusion constant. The delta function in the correlator (80) can be interpreted in the following sense: when $t_1 \neq t_2$, the noise is uncorrelated. In contrast, when $t_1 = t_2 = t$, the variance $\langle \eta^2(t) \rangle = 2D/\Delta t$.

A realization of the dynamics of such a process is shown in Fig. (19). Depending on the parameters b and d, the average number of particles at time t in the system exhibits different asymptotic behaviors. For $b < d$, the *subcritical* phase, the process dies and on an average there are no particles at late times. For $b > d$, the *supercritical* phase, the process is explosive and the average number of particles grows exponentially with time t, $\langle n(t) \rangle = e^{(b-d)\,t}$. In the borderline $b = d$ case, the system is critical, where on an average there is exactly one particle in the system at all times, $\langle n(t) \rangle = 1$. This critical case is relevant to several physical and biological systems with stable population distributions.

In one dimension, the positions of the particles at a particular time t represent a set of random variables that are naturally ordered according to their positions on the line with

$x_1(t) > x_2(t) > x_3(t)$.... (see Fig. (19). It is then interesting to study their order statistics, where one is concerned with the distribution of $x_k(t)$, which denotes the position of the k-th rightmost particle. An equally interesting quantity is the spacing between consecutive particles, $g_k(t) = x_k(t) - x_{k+1}(t)$ as well as the density of the particles near the tip of the branching process. These questions related to the extremes in this one-dimensional BBM, in particular in the supercritical phase ($b > d$), have been studied extensively over the last few decades [McKean 1975, Bramson 1978, Sawyer 1979, Brunet-Derrida 2009–].

Indeed BBM is a useful toy model to study the broader question of extreme value statistics (EVS) of correlated random variables, a field that has been growing in prominence in recent years. Several important properties sensitive to rare events can be characterized by EVS in a wide variety of disordered systems, as mentioned before. Although probability distributions functions (PDFs) of the extreme values of uncorrelated variables are well understood, the computation of extreme and near-extreme value distributions for strongly correlated variables constitute important open problems in this field, in which myself and my collaborators have made very important contributions over the last two decades. Random walks and Brownian motion have recently proved to be useful laboratories where several exact results concerning EVS of correlated variables can be obtained. In this context BBM represents a useful model where the relevant random variables (the particle positions at time t) are strongly correlated, and yet exact results concerning the extremes can be obtained.

In the supercritical regime ($b > d$), the statistics of the position of the rightmost particle $x_1(t)$ has been studied for a long time [Mckean 75, Bramson 78]. In particular, for the case $d = 0$, the cumulative distribution of $x_1(t)$ is known to be governed by the Fisher-Kolmogorov-Petrovsky-Piscounov (FKPP) equation. This equation exhibits a travelling front solution: the average position of the rightmost particle increases linearly with time $\langle x_1(t)\rangle \sim vt$ with a constant velocity v while the width of the front remains of $\mathcal{O}(1)$ at late times. Very recently, Brunet and Derrida studied (still for $d = 0$) the order statistics, i.e., the statistics of the positions of the second, third, etc $x_2(t), x_3(t)\ldots$. They found that, while $x_k(t) \sim vt$ at late times, with the same speed v for all k, the distributions of the gaps $g_k(t)$ become independent of t for large t, while retaining a non-trivial k-dependence. They also computed the PDF of the first gap $g_1(t)$ numerically to very high precision and also provided an argument for the observed exponentially decaying tail. Several natural questions remained outstanding. For instance, can one calculate the gap distributions for arbitrary k for $d = 0$ as well as for arbitrary b and d?

Let us then summarize our main exact results. We were able to compute the order and the gap statistics of BBM at the critical point $b = d$ at a fixed time t, by conditioning the process to have a given number of particles at time t [Refs. 189 and 205] . This method of conditioning allowed us to circumvent the technical difficulties arising from the inherent non-linearities of the problem and provides exact results for arbitrary b and d. Upon conditioning the system to have exactly n particles at time t, we derived an exact backward Fokker-Planck (BFP) equation for the joint distributions of the ordered positions of the n particles at time t. These equations are still nonlinear, but the equation for the probability distribution in the n-particle sector involves only $m < n$ particle sectors. As a result, these equations can, in principle, be solved recursively for all n and the asymptotic results at late times for any fixed n can be extracted explicitly. We find that at large times, and for all b and d, the PDFs of the positions x_k's behave diffusively, $P(x_k, t \to \infty|n) \to \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{{x_k}^2}{4Dt}\right)$, with $k = 1, 2, \ldots$.

Note that for $b > d$, this diffusive behavior is in contrast with the case without conditioning on the particle number where it is ballistic. However, as in the case without conditioning, the PDFs of the gaps $g_k(t)$ become stationary in the long time limit. We showed that they are characterised by an exponential tail $p(g_k|n) \sim \exp[-\sqrt{\frac{|b-d|}{2D}}\; g_k]$ for large gaps in the subcritical ($b < d$) and supercritical ($b > d$) phases, and a power law tail $p(g_k) \sim 8\left(\frac{D}{b}\right) g_k{}^{-3}$ at the critical point ($b = d$), independently of n and k.

In Ref. 208, we have extended our method to study analytically the spatial extent of the one dimensional BBM. We studied the correlation between the rightmost ($X_{\max} \geq 0$) and leftmost ($X_{\min} \leq 0$) visited sites up to time t. As before, at each time step the existing particles in the system either diffuse (with diffusion constant D), die (with rate d) or split into two particles (with rate b). We focused on the regime $b \leq d$ where these two extreme values $X_{\max}$ and $X_{\min}$ are strongly correlated. We showed that at large time t, the joint probability distribution function (PDF) of the two extreme points becomes stationary $P(X, Y, t \to \infty) \to p(X, Y)$. Our exact results for $p(X, Y)$ demonstrate that the correlation between $X_{\max}$ and $X_{\min}$ is nonzero, even in the stationary state. From this joint PDF, we compute exactly the stationary PDF $p(\zeta)$ of the (dimensionless) span $\zeta = (X_{\max} - X_{\min})/\sqrt{D/b}$, which is the distance between the rightmost and leftmost visited sites. This span distribution is characterized by a linear behavior $p(\zeta) \sim \frac{1}{2}(1 + \Delta)\,\zeta$ for small spans, with $\Delta = \left(\frac{d}{b} - 1\right)$. In the critical case ($\Delta = 0$) this distribution has a non-trivial power law tail $p(\zeta) \sim 8\pi\sqrt{3}/\zeta^3$ for large spans. On the other hand, in the subcritical case ($\Delta > 0$), we show that the span distribution decays exponentially as $p(\zeta) \sim (A^2/2)\zeta \exp\left(-\sqrt{\Delta}\;\zeta\right)$ for large spans, where A is a non-trivial function of Δ which we computed exactly.

8.4 Statistical physics problems with biological applications

I have been involved in a number of projects where we use simple statistical physics models to address and understand some biological problems. Two such problems, namely the polymer translocation process through a nonopore and also the convex hull problem with ecological application have already been described in the previous subsection. Below I discuss two other projects with biological applications.

8.4.1 Switching and growth for microbial populations in catastrophic responsive environments

In collaboration with P. Visco (Paris 7), R. Allen and M.R. Evans (both at the unversity of Edinburgh, UK), we have studied the biological phenomenon of phase variation or stochastic switching between alternative states of gene expression in microbes in presence of a catastrophic environement, via a simple model [Biophysical Journal, 98, 1099 (2010), Ref. (144)]. We use this model to assess whether such switching is a good strategy for growth of bacteria in environments with occasional catastrophic events (for example the usage of antibiotics which may wipe out a bacterial population). We find that the switching is can be advantageous, but only when the envronment is responsive to the microbial population. In our model, the microbes switch randomly betwen two phenotypic states, with different groth rates. The enviroment undergoes sudden catastrophes, the probability of which depends on the composition of the population. We derive a simple analytical result for the population growth rate. For a responsive environment, two alternative strategies emerge. In the no-swtiching strategy, the population maximizes its instantaneous growth rate, regardless of catastrophes. In the switching strategy, the microbial switching rate is tuned to minimize the environmental response. Which of the two strategies is most favourable depends on the parameters of the model. Previous studies had shown that microbial switching can be favourable when the environment changes in an unresponsice fashion between different states. In our work, we demonstrated that an alternative role for phase variation in allowing microbes to maximize their growth in catastrophic responsive environments.

Our study suggests a numer of avenues for further research some of which are in the process of current investigation. Our theoretical model was analytically tractable with a particular choice of the catastrophic rate and also a particular distribution of the strength of the catastrophe when it happens. How robust are these choices? We are currently studying the robustness of the results to changes in the choice of catastrophic distributions as well as the dependence of the rate of catastrophes on the population fitness.

8.4.2 Dynamics of DNA bubbles during thermal denaturation

As a Weston visiting professor at the Weizmann Institute, I visit the group of D. Mukamel every year. We have initiated a collboration on the very interesting topic of the dynamics of a DNA loop (or a bubble) below its melting transition. There have been tremendous interest, both experimental and theoretical, in the phenomenon of thermal denaturation of DNA where, beyond a critical temperature T_c, the two strands of a DNA molecule become fully separated. Below the critical temperature T_c, the strands are only partially unbound via the formation of fluctuating loops or bubbles where the two strands are locally separated. The size distribution of such bubbles at thermal equilibrium has been an object of intense studies over the past few decades.

With increasingly sophisticated single-molecule experimental techniques such as optical tweezers, magnetic traps and fluoresecence spectroscopy, it has been possible to study, not just the equilibrium properties of the bubble size distribution but also the dynamics of the bubbles: as time t evolves, the size $x(t)$ of a single bubble (measured by the number of broken covalent bonds in the bubble) fluctuates, via the breaking or closing of bonds at its two ends due to thermal fluctuations.

We are currently studying, both analytically and numerically, two aspects of the size distribution of a single DNA bubble during its temporal evolution in the low temperature $T \leq T_c$ regime. As mentioned before, the bubble size $x(t)$ fluctuates in time t due to thermal fluctuations. A bubble, however, has a finite life-time t_f at which the size $x(t)$ becomes zero for the first time, i.e., the bubble closes. The time evolution of the bubble size can be modelled via a stochastic process $x(t)$ evolving via a simple Langevin dynamics of a particle moving in an external potential in presence of a thermal noise. Within this simple picture, t_f is just the first-passage time of the process, starting from an initial position x_0 chosen from the equilibrium distributin of bubble size. Given this simple model of a particle in a potential, we would like to study the statistics of two pertinent observables: (i) the time-averaged size of a bubble (till its life-time t_f): $\overline{x} = \frac{\int_0^{t_f} x(t)\, dt}{t_f}$ (ii) the maximum bubble size M till its life-time t_f.

Thus the novel aspect of this project is to take into acount the fact that a bubble only has a finite life-time t_f which itself is a random variable that fluctuates from one history of evolution to another. Theoretical studies of the distributions of the time-averaged size and the maximal size are nontrivial because the life-time t_f itself is a random variable. However, we believe that it should be possible to develop a path-integral approach to study this problem analytically, especially when the motion of the bubble span can be approximated by a simple Markovian stochastic process. Furthermore, we also wish to extend these studies by taking into acount (a) the interactions between different bubbles and (b) the fact that, in priciple, a given bubble can also split into two bubbles. These two effects can in principle affect the distribution of the time-averaged size $\overline{x}$ and the maximal size M which we wish to explore.

8.4.3 Statics and dynamics of active particles

Very recently, in collaboration with Indian colleagues A. Dhar and A. Kundu (ICTS, Bangalore) and U. Basu and S. Sabhapandit (RRI, Bangalore) (part of our joint Indo-French research program), we have studied the static and dynamical properties of non-interacting active particles. I have also ongoing collaborations on this and related subjects with a number of colleagues internationally: A. Rosso and G. Schehr (LPTMS), M. R. Evans (Edinburgh, UK), D. S. Dean (Bordeaux), P. Le Doussal (ENS, Paris), A. Hartmann (Oldenburg, Germany), H. Schawe (Cergy-Pontoise) and G. Gradenigo (Rome). We had two postdocs U. Basu (currently a faculty at RRI, Bangalore) and T. Banerjee (currently a postdoc at Cambridge, UK) who worked on this subject. In addition, my current Ph.D student F. Mori is also working partly on some aspects of these active particles.

Active particles are quite common in nature (like E. Coli bacteria) and can also be fabricated in the laboratory (Janus particles). Unlike thermally driven diffusing molecules, the active particles consume energy directly from the environment and move in a **self-propelled** manner with a finite persistence length in random directions. Even though the collective behavior of such active particles have been studied widely in the last decades and many interesting properties have been found, we realised that even at an individual level the motion of an active particle is rather interesting, due to the fact that the effective driving noise has a finite memory. This makes the relevant stochastic process non-Markovian, and leads to many interesting features. For example, an active particle in a a confining potential leads to a non-Boltzmann stationary state. In addition, an active particle can have non-trivial and anomalous first-passage properties due to the non-Markovain nature of the driving noise. For the past two years, we have focussed on two simple models of active particles: (i) active Brownain motion in two dimensions and (ii) a run-and tumble particle in any dimension. We have developed a series of new analytical tools to study the (i) non-equilibrium steady state of such a particle in an external trap (ii) first-passage properties (sometimes surprisingly universal) (iii) unusual relaxation to the steady state (iv) characterzing the spatial extent covered by the trajectory of an active particle like E. Coli in two dimensions by studying the geometrical properties of the convex hull of the trajectory. This is a very hot area of current research and we are exploring many directions. Our results are summarized in Refs. [247], [256], [259], [262], [264], [271], [279], [281], [283], [288], [289], [290], [293], [297], [298], [306], [308], in the list of publications.

8.5 Nonequilibrium steady state problems

I have a long-standing interest in nonequilibrium steady state problems. Some of my earlier works on this topic have been briefly summarized in Sections 7.1 and 7.16. Here I describe the more recent projects. It can be divided into two subtopics describes below.

8.5.1 Long range correlations induced by a localised drive in interacting particle systems

During my regular visits to the Weizmann Institute since 2011 as a visiting Weston professor, I collaborated with D. Mukamel and his postdoc T. Sadhu (currently postdoc at LPTENS) on a rather interesting problem, which was briefly mentioned in section 7.16. Consider first a system of particles in thermal equilibrium with some equilibrium density profile. Now, let us switch on an external potential localised in space. Clearly, the system will re-equilibriate in presence of this new potential and the equilibrium density profile will change due to the additional potential. However, if the perturbing potential is localised in space, the density profile will also be perturbed only locally and as a function of the distance from the perturbed region, the perturbation in the density profile will decay exponentially fast. This is because the perturbed system is still an equilibrium system. What happens if instead of switching on an external local potential, we drive the system out of equilibrium by switching on a local drive that manifestly violates detailed balance. Does the perturbation in the density profile still decays exponentially fast away from from the perturbed region?

This is the question we were interested. We demonstrated [163] that under rather generic conditions a localised drive in an otherwise equilibrium system in dimensions higher than one, results in a steady state density profile with an *algebraically decaying tail.* This is done by first studying the case of non-interacting particles diffusing on a d-dimensional lattice with a directional drive along a single bond (like a battery). We then generalize the results to arbitrary localised configurations of driving bonds, and to the case of and to the case of particles with exclusion interaction.

In the case of non-interacting particles with a single driving bond, we showed that the density profile can be mapped onto the electrostatic potential generated by an electric dipole located at the driving bond, whose strength can be calculated self-consistently. Thus, for example, in $d = 2$ dimensions, the density profile decays as $1/r$ at distance r away from the driving bond, in all directions except the one perpendicular to the drive, where it decays as $1/r^2$. More interestingly, other localised configurations of driving bonds result in different power-law profiles. In this case the density profile can be determined by a linear superposition of the profiles generated by each driving bond. For example, when the electric dipoles corresponding to two driving bonds form a quadrupole, the density profile generically decays as $1/r^2$ while in some specific directions it decays as $1/r^4$, at large distances. The correspondence to the electrostatic problem still holds when *local* exclusion is switched on. The only difference is in the dipole strength which, unlike the noninteracting case, can not be determined self-consistently. In the interacting case, our results thus generalize the one dimensional situation studied by Bodineau, Derrida and Lebowitz (J. Stat. Phys. 140, 648 (2010)) and we show that for $d \geq 2$, the density profile decays algebraically away from the battery, unlike in the one dimensional case. Our results were summarized in Ref. [163].

Recently, we generalised these results to the case of interacting particles undergoing exclussion processes [Ref. 194, PRE, 90, 012109 (2014)]. We showed that the presence of a driven bond in an otherwise diffuusive lattice gas with simple exclusion interaction results in long-range density-density correlation in its stationary state. In dimensions $d > 1$, we showed that in the thermodynamic limit this correlation decays as $C(\mathbf{r}, \mathbf{s}) \sim (r^2 + s^2)^{-d}$ at large distances r and s, away from the drive, with $|\mathbf{r} - \mathbf{s}| \ll 1$. This is derived using an electrostatic analogy whereby $C(\mathbf{r}, \mathbf{s})$ is expressed as the potential due to a con guration of electrostatic charges distributed in $2d$-dimension. At bulk density $\rho = 1/2$, we showed that the potential is that of a localised quadrupolar charge. At other densities the same is correct in leading order in the strength of the drive and is argued numerically to be valid at higher orders.

The main conclusion we drew from these studies is the following. In equilibrium, the effect of a spatially localised perturbation is typically con ned around the perturbed region. Quite contrary to this, in a non-equilibrium stationary state often the entire system is affected. This appears to be a generic feature of non-equilibrium systems. To test this further, recently we studied such non-local response in the stationary state of a lattice gas with a shear drive at the boundary which keeps the system out of equilibrium [Ref. 201]. We showed that a perturbation in the form of a localised blockage at the boundary, induces algebraically decaying density and current profile. In two examples, non-interacting particles and particles with simple exclusion, we derived analytically the power-law tails of the profiles.

In another related problem, in collaboration with A. Kundu (now at ICTS, Bangalore), D. Mukamel (Weizmann) and David's postdoc J. Cividini (Weizmann), we studied recently the effect of localised drive in interacting particle systems, but this time the drive is not localised in space (as discussed previously), but rather on a specific 'tracer' particle [Refs. 219, 222]. The problem, its context and the results are summarized below.

Motion of single driven tracer particle in the pool of other non-driven interacting (hardcore interaction with the tracer particle and among others) particles have been studied in various contexts. In experimental studies, single driven tracer in quiescent media have been used to probe rheological properties of complex media such as DNA, polymers, granular media or colloidal crystals. Some practical examples of biased tracer are a charged impurity being driven by applied electric field or a colloidal particle being pulled by optical tweezer in presence of other colloid particles performing random motion. Because the tracer particle is subjected to drive, but not the others, the system develops a nontrivial density profile that evolves with time in an infinite system. In addition, the time-dependent correlations between particles induced by the localised drive is also interesting to understand. On the theoretical side, lattice models have been studied where the surrounding medium is a Symmetric Simple Exclusion Process (SSEP) and the tagged particle is a hard-core tracer driven towards a preferred direction. However, computing physical observables such as the time-dependent density profile and the correlations between particles (due to the drive on the tracer particle) is not easy in these lattice models. The difficulty can be traced to the 'exclusion' effect combining with the fact that the system is defined on a lattice. To circumvent this difficulty, we studied the same questions in a continuous-space model known as the random average process (RAP). It turns out that in RAP, the particles on a line can not overtake each other (thus preserving the 'exclusion' effect on lattice models) and hence many physical propertries are qualitatively similar in RAP and SSEP. But the advantage of RAP is that it is easier to solve analytically. We therefore studied the dynamics of a single driven tracer in the RAP process and obtained various exact results.

In RAP particles move on a one dimensional continuous line in contrast to SSEP where hardcore particles move on a lattice. We considered an infinite number of particles occupying an infinite line with density ρ_0. Without any loss of generality, we label the driven tracer particle as the 0th particle and then label other particles according to their positional order with respect to the tracer particle from $-\infty$ to ∞. We denote the positions of the particles at time t by $x_i(t) \in \mathbb{R}$ for $i \in \mathbb{Z}$. We consider that the particles initially are arranged according to the following fixed equispaced configuration

$$x_i(0) = \rho_0^{-1} i, \quad i = -\infty, ..., -1, 0, 1, ..., \infty. \tag{81}$$

The dynamics of the particles are given as follows. In an infinitesimal time interval t to $t+dt$, any particle (say i^{th}) other than the driven tracer particle (DTP), jumps from $x_i(t)$, either to the right or to the left with probability $dt/2$ and with probability $(1-dt)$ it stays at $x_i(t)$. The DTP jumps from $x_0(t)$ to the right with probability pdt, to the left with probability qdt and does not jump with probability $(1-(p+q)dt)$. The amount of jump, either to the right or to the left, made by any particle is a random fraction of the space available between the particle and its neighboring particle to the right or to the left. For example, the i^{th} particle jumps by $\eta_i^r[x_{i+1}(t) - x_i(t)]$ to the right and by $\eta_i^l[x_{i-1}(t) - x_i(t)]$ to the left. The random variables $\eta_i^{r,l}$ are independently chosen from the interval [0, 1] and each is distributed according to the same distribution $R(\eta)$, with moments $\mu_k = \int_{\eta=0}^1 \eta^k R(\eta) d\eta$. The time evolution of the positions $x_i(t)$can be written as,

$$x_i(t+dt) = x_i(t) \quad + \quad \sigma_r^i \eta_i \; [x_{i+1}(t) - x_i(t)] + \sigma_l^i \eta_i \; [x_{i-1}(t) - x_i(t)] \tag{82}$$

where the η variables are independent and identically distributed according to $R(\eta)$ and $\sigma_{r,l}^i$ for $i \neq 0$ are 1 with probability $\frac{dt}{2}$ and 0 otherwise. The random variable σ_r^0 is 1 with probability pdt and 0 with probability $1 - pdt$. Similarly, σ_l^0 is 1 with probability qdt and 0 with probability $1 - qdt$. Clearly, we see that all the particles are symmetrically moving except the 0-th particle, which moves asymmetrically. It turns out that this model can be solved exactly both for the averagae density profile at time t, as well as for the two-point correlation function between the positions of the i-th and the j-th particle [Ref. 222]. For example, the average position of the i-th particle at late time t has the scaling form

$$\langle x_i(t) \rangle = \rho_0^{-1} \; \sqrt{2\mu_1 t}\, \mathcal{Y}\left(\frac{i}{\sqrt{2\mu_1 t}} \right) \tag{83}$$

where the scaling function is given exactly by

$$\mathcal{Y}(x) = x + \frac{p-q}{p+q} \left[\frac{1}{\sqrt{\pi}} \, e^{-x^2} - |x| \, \mathrm{erfc}(|x|) \right] , \tag{84}$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} \, du$. Details of other results can be found in Ref. [222]. In addition, we also studied the same model (with a DTP) on a ring of finite size L and computed exactly the probability distribution of the gaps between successive particles in the steady state [see Ref. 219].

8.5.2 Condensation phenomena in nonequilibrium steady states

I have been interested for a long time in understanding and analysing the phenomenon of *condensation* in nonequilibrium steady states of interacting particle systems, such as in the zero range process or various mass transport models (discussed briefly in section 7.1). In these models, typically there is a continuous mass or a discrete number of particles at each site of a lattice. The dynamics consists of choosing a site at random and transporting a certain fraction of mass or the number of particles to a neighbouring site at a certain rate which depends on the occupation number of the chosen site. The system is closed with periodic boundary conditions and the dynamics conserves the total mass. The dynamics typically violates detailed balance. At long times, the system arrives at a nonequilibrium stationary state where the joint distribution of masses at different sites becomes time independent. For some choices of the rate of mass transport, the steady state may exhibit condensation pheonomenon whereby a single site carries a mass that is thermodynamically large, i.e., proportional to the system size. This is a rather ubiquitous pheonomena and is different from the traditional Bose-Einstein (BE) condensation in two ways: (i) unlike the BE case where the condensation happens in the momentum space, here *the condensation happens in real space* and (ii) unlike the BE case where the condensate always sits on the $\vec{k} = 0$ mode in the momentum space, here the condensate can occupy any lattice site with equal probability. In any given realization, the condensate occurs at a particular site chosen at random and in an infinitely large system, once the condensate forms at a site, it takes an infinite amount of time to melt and re-locate at another site. Thus, in the thermodynamic limit, there is a spontaneous translational symmetry breaking in these systems associated with the formation of a condensate.

In earlier works (in collaboration with M. R. Evans and others), we had established analytically the precise criteria for this condensation transition, namely, what kind of mass tranfer rates from a site to its neighbour may lead to the occurrence of a condensation transition in the steady state. Very recently, in collaboration with M. R. Evans and his postdoc J. Szavits-Nossan (both from Edniburgh University), we have shown that the presence of constraints can lead to novel type of condensation transition. Such constraints appear naturally in several systems, such as in polydisperse hard rods. These new results are summarized in a recent paper [Ref. 186, "Constraint driven condensation in large fluctuations of linear statistics", J. Szavits-Nossan, M. R. Evans, S. N. Majumdar, **PRL**, 112, 020602 (2014)]. A longer paper with the details of derivations have also appeared recently [Ref. 200].

9 Invited Talks in Conferences, Seminars and Short Visits

9.1 Invited Speaker at International Conferences & Workshops (since 1996):

1. Invited speaker in "Dynamics of Nonequilibrium Systems", Trieste, **Italy**, 1996.

2. Invited speaker in "Condensed Matter Theory-20", Pune, **India**, 1997.

3. Invited speaker in "Statistical Physics of Frustrated Systems", Trieste, **Italy**, 1997.

4. Invited speaker in "Workshop on Nonlinear Dynamics", Los Alamos, **USA**, 1998.

5. Invited series of lectures in the summer school "Temporal-Spatial Patterns", Enschede, **The Netherlands**, 1998.

6. Invited speaker in "Recent Developements in Theoretical Physics", Bombay, **India**, 1999.

7. Invited speaker in "Workshop on Nonequilibrium Systems", satellite Statphys meeting, Calcutta, **India**, 1999.

8. Invited speaker in "Dynamics of Nonequilibrium Systems", Trieste, **Italy**, 1999.

9. Invited speaker in "Nonequilibrium Dynamics", Porto, **Portugal**, 1999.

10. Invited speaker in "Recent Trends in Nonequilibrium Statistical Physics", Bangalore, **India**, 1999.

11. Invited speaker in "Fifth Claude Itzykson Meeting on Nonequilibrium Dynamics", Saclay, **France**, 2000.

12. Invited speaker in "India and Abroad: Perspectives in Condensed Matter Physics", S.N. Bose Center for Natural Sciences, Calcutta, **India**, 2001.

13. Invited speaker in "Statphys 21 Satellite Meeting: VII Latin American Workshop on nonlinear Phenomena", Cocoyoc, Morelos, **Mexico**, July, 2001.

14. Invited speaker in "Statphys-Kolkata IV", IACS (Indian Association for the Cultivation of Science) and S.N. Bose Center for Natural Sciences, Calcutta, **India**, 2002.

15. Invited speaker in "Journées Sur les Graphes en Physique", IHP, Paris, **France**, June, 2002.

16. Invited speaker in "50 Years of Theoretical Physics", Indian Association for the Cultivation of Science, Calcutta, **India**, January, 2003.

17. Invited speaker in "Geometry and Statistics of Random Growth", Paris, **France**, Jan-

April, 2003.

18. Invited speaker in “Arbres Aléatoires et Algorithmes”, Versailles, **France**, March, 2003.

19. Invited speaker in “Workshop on Non-Equilibrium Systems”, Center for Nonlinear Studies (CNLS), Los Alamos National Laboratory, **USA**, June, 2003.

20. Invited speaker in “Non-Equilibrium Statistical Physics in Low Dimensions and Reaction Diffusion Systems”, Max Planck Institute at Dresden, **Germany**, September 2003.

21. Invited series of lectures in “SERC school on Statistical Physics”, Tata Institute, Bombay, **India**, February 2004.

22. Invited speaker in “Optimization Algorithms and Disordered Quantum Systems”, Institute Henri Poincare, Paris, **France**, June 2004.

23. Invited speaker in “Workshop on Nonequilibrium Processes”, Korea Institute of Advanced Studies (KIAS), Seoul, **South Korea**, June 2004.

24. Invited speaker in “Statphys 22”, Indian Institute of Science (IISC), Bangalore, **India**, July 2004.

25. Invited series of lectures in “Workshop on the Principles of the Dynamics of Non-Equilibrium Systems”, Isaac Newton Institute, Cambridge University, Cambridge, **UK**, March 2006.

26. Invited speaker in “First-Passage and Extreme Value Problems in Random Processes”, Isaac Newton Institute, Cambridge University, Cambridge, **UK**, June 2006.

27. Invited series of lectures in “Les Houches Summer School on Complex Systems”, Les Houches, **FRANCE**, July 2006.

28. Invited speaker in “International Conference on the Interdisciplinary Advances in Statistical Physics”, Beijing, **China**, September 2006.

29. Invited speaker in “Workshop on Random Curves, Surfaces, and Transport”, Institute of Pure and Applied Mathematics (IPAM), UCLA, **USA**, April 2007.

30. Invited speaker in “International Workshop on Random Matrix Theory: From Fundamental Physics to Applications”, Krakow, **Poland**, May 2007.

31. Invited speaker in “Statistical Physics and Low Dimensional Systems”, Nancy, **France**, May 2007.

32. Invited series of lectures in “Summer school on Physics and Computer Science”, Bremen, **Germany**, June 2007.

33. Invited speaker in “International Conference on the Statistical Mechanics of Distributed Information Systems”, Mariehamn, **Finland**, July, 2007.

34. Invited speaker in “Physique Statistique et Traitement du Signal et de l’Image”, GDR Phenix & ISIS, ENS Lyon, **France**, November, 2007.

35. Invited speaker in “International Conference on Nonequilibrium Phenomena in Condensed Matter”, New Delhi **India**, February, 2008.

36. Invited speaker in “Statistical-mechanics and Quantum-Field-Theory Methods in Combinatorial Enumeration”, Isaac Newton Institute, Cambridge **UK**, April, 2008.

37. Invited speaker in the workshop “Physics of Distributed Information Systems”, Nordita, Stockholm **Sweden**, May, 2008.

38. Invited speaker in “International Conference on Random Matrices (ICRAM)”, Sousse **Tunisia**, June, 2008.

39. Invited series of lectures in “The Beg Rohu Summer School on Manifolds in Random Media, Random Matrices and Extreme Value Statistics”, Beg Rohu **France**, June, 2008.

40. Invited speaker in “NSPCS2008: Nonequilibrium Statistical Physics of Complex Systems”, Korea Institute for Advanced Studies (KIAS), Seoul **South Korea**, July, 2008.

41. Invited series of lectures in “Les Houches Summer School on Exact Methods in Low-dimensional Statistical Physics and Quantum Computing”, Les Houches, **France**, July, 2008.

42. Invited speaker in “IV Brunel Workshop on Random Matrix Theory”, Brunel, **UK**, December, 2008.

43. Invited speaker in “New paths in Random Walks–International Conference”, CIC (Curenavaca), **Mexico**, January, 2009.

44. Invited speaker in “Workshop on Statistical Mechanics BASM-II”, Bangalore, **India**, March, 2009.

45. Invited speaker in “Steady-states, Fluctuations and Dynamics of Nonequilibrium Systems”, Technion and Weizmann Institute, **Israel**, June, 2009.

46. Invited speaker in “Workshop on Random Matrices and their Applications in Physics and Number Theory”, IHP (Paris), **France**, June, 2009.

47. Invited speaker in “Nonequilibrium Physics from Classical to Quantum Low Dimensional Systems”, ICTP (Trieste), **Italy**, July, 2009.

48. Invited lectures at the international summer school “ Fundamental Problems in Statistical Physics XII”, La Foresta, Leuven, **Belgium**, September, 2009.

49. Invited talk at the “EPSRC Symposium Workshop on Non-equilibrium dynamics of spatially extended interacting particle systems (NEQ)”, University of Warwick, **UK**, January, 2010.

50. ‘Distinguished Colloquium’ at the “International Workshop on Non-equilibrium Statistical Physics” (NESP), Indian Institute of Technology, Kanpur, **India**, February, 2010.

51. Invited talk at the 4-th KIAS conference on “Nonequilibrium statistical physics of complex systems”, Korea Institute of Advanced Studies (KIAS), **South Korea**, July, 2010.

52. Invited talk at the 23-rd Marian Smoluchowski Symposium on Statistical Physics: “Random Matrices, Statistical Physics and Information Theory”, Krakow, **Poland**, September, 2010.

53. Second M.L. Mehta memorial lecture at Tata Institute, Bombay, **India**, January, 2011.

54. Invited speaker at ‘Rencontre de Physique Statistique’ held at ESPCI, Paris, **France**, January, 2011.

55. Weston Colloquium at the Weizmann Institute of Science, Rehovot, **Israel**, April, 2011.

56. Invited lectures at the Les Houches summer school on “Vicious Walkers and Random Matrices”, Les Houches, **France**, May, 2011.

57. **Plenary** speaker at the 7-th ‘Extreme Value Analysis’ (EVA 2011) held at Universite’ de Lyon, **France**, June, 2011.

58. Invited speaker at “Extreme Value Statistics in mathematics, Physics and Beyond” held at Lorentz Center, Leiden, **Netherlands**, July, 2011.

59. Joint Max-Planck and LAFNES-11 Colloquium, as part of the international conference “Large Fluctuations in Non-equilibrium Systems” held at Max Planck Institute for the Physics of Complex Systems, Dresden, **Germany**, July, 2011.

60. Invited talk at the international workshop on “Weak Chaos, Infinite Ergodic Theory, and Anomalous Dynamics” held at Max Planck Institute for the Physics of Complex Systems, Dresden, **Germany**, July-August, 2011.

61. Invited talk at the international conference on “ Discretization in mathematics and in Theoretical Physics”, Strasbourg, **France**, September, 2011.

62. Invited talk at the international conference on “Random Processes, Conformal Field Theory and Integrable Systems”, Moscow, **Russia**, September, 2011.

63. Invited talk at the international conference “GranMa 2011” (Grand Matrices Aleatoires), held at Institut Henri Poincare, Paris, **France**, October, 2011.

64. Invited talk at the international conference “Open Quantum Systems and Quantum Information Theory”, Université Paul Sabatier, Toulouse, **France**, November, 2011.

65. Invited talk at the “106th Statistical Mechanics Conference” held at Rutgers University, **USA**, December, 2011.

66. Subhramanyam Chandrasekhar lectures as part of the international conference "Random Matrix theory and applications", held at International Center for Theoretical Sciences (ICTS), Bangalore, **India**, January, 2012.

67. Invited speaker at the international conference "Search 2012: Search and Stochastic Phenomena in Complex Physical and Biological Systems" held at IFISC, Palma de Mallorca, **Spain**, May, 2012.

68. Invited speaker at the international conference "Nonequilibrium Statistical Physics of Complex Systems" (the 5-th KIAS conference on Statistical Physics) held at Korea Institute of Advanced Studies (KIAS), Seoul, **South Korea**, July, 2012.

69. Invited speaker at the international workshop "Autour des probabilites de persistance" held at the department of Mathematics at the University of Lille, **France**, September, 2012.

70. **Plenary** speaker at ICNP1 (First International Conference on Numerical Physics), held at the University of Sciences and Technology, Oran, **Algeria**, October, 2012.

71. Invited speaker at the international conference "Statistical Mechanics in Low Dimensions" (in honor of Henk HILHORST), held at LPT, Université Paris-Sud (Orsay), **France**, December, 2012.

72. Invited speaker at the VIII Brunel-Bielefeld workshop on "Random matrix Theory and Applications", held at Brunel University (London), **UK**, December, 2012.

73. Invited speaker at the "Workshop on quantum graphs and applications", held at the University of Bristol (Bristol), **UK**, December, 2012.

74. Invited speaker at the international conference "Diversity and Complexity: Realm of Today's Statistical Physics", held at the Saha Institute of Nuclear Physics (Kolkata), **India**, January, 2013.

75. Invited guest lectures on 'random walks' for the Master's course on 'Complex Systems' at King's College (London), **UK**, March, 2013.

76. **Keynote** speaker at the "38th Conference of the Middle European Cooperation in Statistical Physics-MECO38" held at ICTP (Trieste), **Italy**, March, 2013.

77. Invited lectures on 'models of nonequilibrium physics' at the 4-th RRI school on statistical physics held at RRI (Raman Research Institute) (Bangalore), **India**, April, 2013.

78. Invited lectures (9 lectures) on 'Random matrix Theory and its applications' at the Beg-Rohu summer school on 'Disordered Systems' (Beg-Rohu), **France**, June, 2013.

79. Invited speaker at the conference 'Rencontre Nicoise de Physique Theorique et de Probabilite', held at the University of Nice, **France**, June, 2013.

80. **Plenary** speaker at STATPHYS-25 held at Seoul, **South Korea**, July, 2013.

81. Invited lectures (4 lectures) on '3rd order phase transitions in random matrix madels' at the Bielefeld summer school on ' Randomness in Physics and Mathematics: From Quantum Chaos to Free Probability', held at the University of Beilefeld, **Germany**, August, 2013.

82. Invited speaker at the EPSRC symposium/workshop on "Models from Statistical Mechanics in Applied Sciences', held at the University of Warwick, **UK**, September, 2013.

83. Invited speaker at the international workshop, "Small systems far from equilibrium: order, correlations, and fluctuations", held at the Max-Planck-Institute for complex systems, Dresden, **Germany**, October, 2013.

84. Invited speaker at the international workshop, "Animal movement in confined space: from space use patterns to epidemic spread", held at the University of Bristol, **UK**, December, 2013.

85. Invited lectures at the "RRI & ICTS summer school in statistical physics", held at the Raman Research Institute, Bangalore **India**, April, 2014.

86. Invited lectures at the international workshop "Advances in Nonequilibrium Statistical Mechanics", held at the Galileo Galilei Institute (GGI), Florence, **Italy**, May-June, 2014.

87. Invited lecture series on random matrix theory at the international summer school "Spectral analysis for random matrices and applications", held at the Universidad de Los Andes, Bogota, **Colombia**, May, 2014.

88. Invited lectures at the international conference "Random Walks in Random Media", held at CIRM, Marseille, **France**, June, 2014.

89. Invited speaker at the international conference "Viewpoints on Emergent Phenomena in Non-equilibrium Systems", held at the Higgs Centre for Theoretical Physics, University of Edinburgh, **UK**, June, 2014.

90. Invited speaker at the international conference "Random Matrix Theory: Foundations and Applications", held at the Jagiellonian University, Krakow, **Poland**, July, 2014.

91. Invited speaker at the international conference "6-th Kias conference on "Nonequilibrium Statistical Physics of Complex Systems" held at the Korea Institute of Advanced Studies (KIAS), Seoul, **South Korea**, July, 2014.

92. Invited speaker at the international workshop "Persistence probabilities and related fields", held at the Technical University, Darmstadt, **Germany**, July, 2014.

93. Invited speaker at the international school on "Non-linear Dynamics, Dynamical Transitions and Instabilities in Classical and Quantum Systems", held at ICTP, Trieste, **Italy**, July, 2014.

94. Invited speaker at the international workshop on " Large Deviations in Statistical Physics", held at the University of Stellenbosch, Stellenbosh, *South Africa*, November, 2014.

95. Invited speaker at the international workshop on " Applications of Random Matrix Theory and Statistical Physics in Communications and Networks", held at the Institut Henri Poincare, Paris, *France*, November, 2014.

96. Invited speaker at the international workshop "Statistical Mechanics Day VII", held at the Weizmann Institute, **Israel**, November, 2014.

97. Invited speaker at the international workshop "Frontiers in Condensed Matter Physics", held at the Israel Academy of Sciences and Humanities, **Israel**, December, 2014.

98. Invited speaker at the international conference "Second ICTS Indian Statistical Physics Community Meeting 2015", IISC (Bangalore), **India**, February, 2015.

99. Invited speaker at the DPG (Deutsche Physikalische Gesellschaft e.V.) spring meeting, Berlin, **Germany**, March, 2015.

100. Invited speaker at the international workshop "Stochastic processes in random media", held at the National University of Singapore, **Singapore**, May, 2015.

101. Invited speaker at the summer school "Spring College on the Physics of Complex Systems" held at ICTP, Triste, **Italy**, May-June, 2015.

102. Invited speaker at the international conference "Science at ICTS", held at the International Center for Theoretical Sciences (ICTS), Bangalore, **India**, June, 2015.

103. Invited speaker at the international workshop "The dynamics of foraging", Max-Planck-Institute for complex systems, Dresden, **Germany**, October, 2015.

104. Invited speaker at the international workshop "NESP2015: workshop on non-equilibrium statistical physics", held at the International Center for Theoretical Sciences (ICTS), Bangalore, **India**, October-November, 2015.

105. Invited speaker at the international workshop "XI Brunel-Bielefeld workshop on random matrices", held at ZiF - Center for Interdisciplinary Research, Bielefeld University, Bielefeld, **Germany**, December, 2015.

106. Invited speaker at the international conference "17th. Annual U.C. Berkeley Statistical Mechanics Meeting", held at U.C. Berkeley college of chemistry, Berkeley, California, **USA**, January, 2016.

107. Invited speaker at the international workshop "New approaches to non-equilibrium and random systems: KPZ integrability, universality, applications and experiments", held at KITP, Univ. of California at Santa Barbara, **USA**, January, 2016.

108. Invited speaker at the conference "ISPC-3", held at ICTS, Bangalore, **India**, February, 2016.

109. Invited speaker at the conference "Optimal and random point configurations", held at Institut Henri Poincare (IHP), Paris, **France**, June, 2016.

110. Invited speaker at the conference “Nonequilibrium Statistical Physics of Complex Systems– 7-th KIAS meeting”, held at Korea Institute of Advanced Studies (KIAS), Seoul, **South Korea**, July, 2016.

111. Invited speaker at the conference “Statistical topology of random manifolds: theory and applications”, held at ICTP, Trieste, **Italy**, July, 2016.

112. Invited speaker at the conference “Entanglement and Non-equilibrium physics of pure and disordered systems”, held at ICTP, Trieste, **Italy**, July, 2016.

113. Invited speaker at the conference “Random geometry and Physics”, held at IHP, Paris, **France**, October, 2016.

114. Invited speaker at the conference “Statphys Kolkata IX”, held at Saha Institute of Nuclear Physics (SINP), Kolkata, **India**, December, 2016.

115. Invited speaker at the conference “String theory: past and present”, held at the International Centre for Theoretical Sciences (ICTS), Bangalore, **India**, January, 2017.

116. Invited speaker (and moderator) for the session “Extreme Value Statistics in Stochastic Processes”, as part of the international conference “Inhomogeneous Random Systems (IRS 2017)”, held at IHP, Paris, **France**, January, 2017.

117. Invited speaker at the “Spring School on Probability in Mathematics and Physics”, held at TU Darmstadt, **Germany**, March, 2017.

118. Invited minicourse on “Top eigenvalue of a random matrix: Tracy-Widom distribution and 3rd order phase transition”, held at the Interdisciplinary Institute (Poncelet lab at Moscow), **Russia**, May, 2017.

119. Invited speaker at the conference “From Field Theory to Non-Equilibrium”, held at the University of Nice-Sophia Antipolis, **France**, June, 2017.
120. Invited speaker at the international workshop on ‘Random Matrices’, held at the Park City Mathematical Institute (PCMI, Utah), **USA**, June, 2017.

121. Invited speaker at the international workshop on “Climate fluctuations and Non-equilibrium Statistical Mechanics: An Interdisciplinary Dialogue”, held at the Max-Planck-Institute for Complex Systems (Dresden), **Germany**, July, 2017.

122. Invited speaker at the international conference “Out of equilibrium dynamics in soft and condensed matter”, held at the International Institute of Physics (IIP) (Natal), **Brazil**, August, 2017.

123. Invited speaker at the international conference “The statistical physics cornucopia” (in honour of the 60’th birthday of Marc Mezard), held at the Theatre de Reine Blanche (Paris), **France**, September, 2017.

124. Invited speaker at the international workshop “Large Deviation Theory in Statistical Physics: Recent Advances and Future Challenges”, held at ICTS (Bangalore), **India**,

September, 2017.

125. Invited speaker at the international conference “Probabilistic techniques and Quantum Information Theory”, held at IHP, Paris, **France**, October, 2017.

126. Invited speaker at the international workshop “ Correlations, Fluctuations and anomalous transport in systems far from equilibrium”, held at SRITP, Weizmann Institute of Science, **Israel**, December, 2017.

127. Invited speaker at the international summer school “Statistical Field Theories (2018)” (10 lectures (1h each) on Random Matrix Theory) held at the Galileo Galilei Institute, Florence, **Italy**, February, 2018.

128. Invited speaker at the international conference “Frontiers of Statistical Physics”, held at the Indian Statistical Institute, Calcutta, **India**, February, 2018.

129. Invited speaker at the international conference “Emergent phenomena in classical and qunatum systems” (125 years of S.N. Bose), held at the S. N. Bose Centre for Basic Sciences, Calcutta, **India**, February, 2018.

130. Invited at the international workshop “Point Configurations in Geometry, Physics and Computer Science”, Semester program at ICERM, Brown University, Providence, **USA**, April, 2018.

131. Invited speaker at the international conference MECO43 (Middle European Cooperation in Statistical Physics), held at Krakow, **Poland**, May, 2018.

132. Invited speaker at the international workshop “Integrable Probability”, held at MIT (Boston), **USA**, June, 2018.

133. Invited speaker at the international conference “Randomness and Symmetry”, held at the University College of Dublin (UCD), **Ireland**, June, 2018.

134. Invited speaker at the international conference “Random Matrix Theory meets Theoretical Physics”, held at the Universite' Paris-Descartes, Paris, **France**, June, 2018.

135. Invited speaker at the conference “Nonequilibrium Statistical Physics of Complex Systems– 8-th KIAS meeting”, held at Korea Institute of Advanced Studies (KIAS), Seoul, **South Korea**, July, 2018.

136. Invited speaker at the conference “Probabilistic methods in statistical physics for extreme statistics and rare events”, held at Scuole Normale Superiore, Pisa, **Italy**, September, 2018.

137. Invited speaker at the conference “Modern aspects of Quantum physics”, held at the Ruder Boskovic Institute, Zagreb, **Croatia**, October, 2018.

138. Invited speaker at the conference “ XIV-th BrunelBielefeld Workshop on Random Matrix Theory and Applications”, held at Brunel University, London, **UK**, December, 2018.

139. Invited **Plenary** speaker at the conference " New directions in theoretical physics III", held at the Higgs Center, University of Edinburgh, **UK**, January, 2019.

140. Invited speaker at the international workshop "Universality in random structures: Interfaces, Matrices, Sandpiles", held at ICTS (Bangalore), **India**, January, 2019.

141. Invited speaker at the workshop "Statistical Physics and Nonlinear Dynamics", organized by the Queen Mary University (London) and held at the British Council (Paris), **France**, April, 2019.

142. Invited speaker at the international conference " Random Matrices and Random Graphs", held at CIRM (Marseille), **France**, April, 2019.

143. Invited speaker at the international workshop " New directions in Quantum information", held at Nordita (Stockholm), **Sweden**, April, 2019.

144. Invited talk (as the recipient of the EPS prize for Statistical and Nonlinear Physics (EPS-SNPD, 2019)) at the international conference "Statistical Physics of Complex Systems" held at Nordita (Stockholm), **Sweden**, May, 2019.

145. Invited speaker at the international ICTS-RRI summer school on Statistical Physics, held at ICTS (Bangalore), **India**, June 2019.

146. Invited speaker at the international conference "Statistical Physics Meets Movement Ecology", held at Bristol University (Bristol), **UK**, July, 2019.

147. Invited speaker at the international conference "32-nd Marian Smoluchowsky Symposium on Statistical Physics", held at Jagiellonian University (Krakow), **Poland**, September, 2019.

148. Invited speaker at the international conference "Fluctuations in Nonequilibrium Systems: Theory and applications " held at ICTS (Bangalore), **India**, March, 2020.

149. Invited speaker at the international workshop "Stochastic Processes under Constraints", held at Oberwolfach (partly virtual)), **Germany**, September, 2020.

150. Invited speaker at the APS march meeting (virtual) in the 'Kadanoff Prize Session', **USA**, March, 2021.

151. Invited speaker at the international workshop (virtual) "Random Matrix Theory and Networks", Max-Planck Institute, Dresden, **Germany**, June, 2021.

152. Invited speaker at the international conference (virtual) "Lattice Paths, Combinatorics and Interactions", CIRM, Marseille, **France**, June, 2021.

9.2 Invited Seminars and Colloquiums:

Over the last few years invited seminars/colloquiums were given at various places all over the world. Some of these places are: Tata Institute (Bombay, India), Indian Institute of Science (Bangalore, India), International Center for Theoretical Sciences (ICTS) (Bangalore, India), Bell Laboratories (Murray Hill, USA), Harvard University (USA), MIT (USA), Yale University (USA), Cornell University (USA), Rutgers University (USA), Pennsylvania State University (USA), University of Maryland at College Park (USA), Los Alamos National Laboratories (USA), NEC Research Institute-Princeton (USA), Brookhaven National Laboratories (USA), Indiana University (USA), University of California at Los Angeles (USA), University of California at Berkeley (USA), KITP (Santa Barbara, USA), Ecole Normale Supérieure (Paris, France), IPhT & SPEC (Saclay, France), ESPCI (Paris, France), Université Paris-Sud (Orsay, France), LPTHE & LPTMC (Jussieu, Paris, France), IHP (Paris, France), Université Cergy-Pontoise (France), Ecole Normale Supérieure (Lyon, France), Université de Marseille (Luminy, France), Oxford University (UK), Cambridge University (UK), University of Manchester (UK), University of Edinburgh (UK), Brunel University (UK), ICTP (Trieste, Italy), Max-Planck Institute (Dresden, Germany), University of Essen (Germany), University of Cologne (Germany), International University at Bremen (Germany), University of Gottingen (Germany), University College of Dublin (Ireland), University of Geneva (Switzerland), University of Twente (The Netherlands), Bilkent University (Ankara, Turkey) etc.

9.3 Short visits (1995-):

March/2020	Invited adjunct professor at International Centre for Theoretical Sciences (ICTS) (Bangalore, India) for 8 weeks.
November/2019	Invited VAJRA professor at Raman Research Institute (Bangalore, India) for 4 weeks.
May/2019	Invited Weston Professor at the Weizmann Institute (Rehovot, Israel) for 3 weeks.
March/2019	Invited visit for two weeks to the Tata Institute of Fundamental Research (Bombay, India) as an adjunct Professor.
January-February/2019	Invited visit for a month to Raman Research Institute (Bangalore, India) as a 'VAJRA' fellow (Visiting Adjunct Faculty awarded by the Science and research Board, Department of Science and Technology, Government of India).
April/2018	Invited visit to ICERM, Brown University, Providence, USA for three weeks during the workshop, "Point Configurations in Geometry, Physics and Computer Science".
March/2018	Visiting adjunct Professor at the Raman Research Institute, Bangalore, India, for two weeks.
Jan/2018	Visiting adjunct Weston Professor at the Weizmann Institute of Science, Rehovot, Israel for three weeks.

October/2017	Simon Visiting chair at the International Centre for Theoretical Sciences (ICTS), Bangalore, India for a month.
May/2017	Visiting adjunct Weston Professor at the Weizmann Institute of Science, Rehovot, Israel for three weeks.
Jan/2017	Visiting adjunct Professor at the Tata Institute, Bombay, India for two weeks.
Nov-Dec/2016	Visiting adjunct Professor at the Raman Research Institute, Bangalore, India for one month.
May/2016	Visiting Weston Professor at the Weizmann Institute, Rehovot, Israel for one month during May 2016.
January-February/2015	Visiting adjunct professor at the Tata Institute, Bombay and ICTS, Bangalore **India** for one month.
October/2014	Visiting Weston professor at the Weizmann Institute, Rehovot, **Israel** for three weeks.
September/2014	Visiting professor at the University of Kentucky, Lexington, **USA** for one week.
May-June/2014	Visiting scientist at the Galileo Galilei Institute, Florence, **India** for one month.
Jan/2014	Visiting professor at the Saha Institute of Nuclear Physics, Calcutta, **India** for one week.
Nov/2013	Visiting professor at the University of Calcutta, **India** for one month.
Jan-Feb/2013	Visiting adjunct professor at the Tata Institute, Bombay **India** for one month.
Jan-Feb/2012	Visiting adjunct professor at the Tata Institute, Bombay **India** for one month.
Feb-May/2011	Weston Visiting Professor at the Weizmann Institute, Rehovot **Israel** for 3 months.
Jan-Feb/2011	Visiting Professor at the Tata Institute, Bombay, **India** for 4 weeks.
June/2010	Visiting Professor at All Souls College, Oxford University **UK**.
Jan-Feb/2010	Visiting Professor at the Tata Institute, Bombay, **India** for 4 weeks.
December/09	Visiting Professor at the Edinburg University **UK** for one week.

October/09	Visiting Professor at the Weizmann Institute, Rehovot **Israel** for 2 weeks.
April/09	Visiting Professor at ICTP, Trieste **Italy**.
January/09	Visiting Professor at Tata Institute **India**.
January/08	Visiting Professor at Tata Institute **India**.
October/07	Visiting Professor at the University of Edinburgh **UK**.
January/07	Visiting Professor at Tata Institute **India**.
September/06	Visiting Professor at the Chinese Academy of Sciences, Beijing **China**.
March/06	Visiting Professor at Isaac Newton Institute, Cambridge **UK**.
January/06	Visiting Professor at Tata Institute **India**.
Nov/05	Visiting Professor at the University of Edinburgh, **UK**.
September/05	Visiting Professor at the International University of Bremen, **Germany**.
July/05	Visiting Professor at University of Gottingen, **Germany**.
March/05	Visiting adjunct Professor at Tata Institute **India**.
May/04	Visiting Professor at the Cambridge University, **UK**.
May/04	Visiting Professor at the University of Manchester, **UK**.
Feb/04	Visiting Professor at Tata Institute of Fundamental Research, Bombay **India**.
June/03	Visiting Professor at the Los Alamos National Laboratory, **USA**.
May/03	Visiting Professor at the University Of Maryland, College Park **USA**.

May/03	Visiting Professor at the Brandeis University **USA**.
Dec/03-Feb/04	Visiting Professor at Tata Institute of Fundamental Research, Bombay, **India**.
Nov/02	Visiting Professor at the Helsinki University of Technology, Helsinki, **Finland**.
Nov/01-Jan/02	Visiting Professor at Tata Institute of Fundamental Reasearch, Bombay, **India**.
Dec/00-Jan/01	Visiting Professor at Tata Institute of Fundamental Reasearch, Bombay, **India**.
July/00	Visiting Professor at the University of Manchester, **UK** for 3 days.
June/00	Visiting Professor at the University of Essen, **Germany** for 3 days.
Oct/99	Visiting Professor at S.N. Bose National Center for Basic Sciences, Calcutta, **India** for one week.
Aug/99	Visiting Scientist at ICTP, Trieste, **Italy** for two weeks.
Jun/99-Jul/99	Visiting Professor position at IPN, LPTMS, Université Paris-Sud Bâtiment 100, Centre Scientifique d'Orsay, **France**.
March/99-May/99	Visiting CR-1 position (chercheur associé) at CNRS, Lab. de Physique Quantique, Université Paul Sabatier, Toulouse, **France**.
Jun/98	Visiting Professor at the University of Twente, Enschede, **The Netherlands** for two weeks.
April/98	Visiting Scientist at Bell Labs, Lucent Technologies, **USA** for one week.
April/98	Visiting Scientist at Yale University, **USA** for one week.
July/97-Aug/97	Visiting scientist at ICTP, Trieste, **Italy**.
Oct/97-Dec/97	Visiting CR-1 position (chercheur associé) at CNRS, Lab. de Physique Quantique, Université Paul Sabatier, Toulouse, **France**.
Jul/96	Visiting Scientist at ICTP, Trieste, **Italy** for two weeks.
Oct/95-Dec/95	Visiting CR-1 position (chercheur associé) at CNRS, Lab. de Physique Quantique, Université Paul Sabatier, Toulouse, **France**.

Jul/95 Visiting Professor at the University of Manchester, **UK** for 3 days.

Jul/95 Visiting Professor at the University of Oxford, **UK** for 3 days.

9.4 Contributed talks and participation in conferences (partial list):

• Kathmandu Summer School on Field Theory and Condensed Matter Physics (Kathmandu, Nepal, 1990)

• Modern Field Theory Colloquium (Tata Institute, Bombay, 1990).

• Summer School and Workshop on Fractals (ICTP, Trieste, 1991).

• "25 years of Edward model": International Workshop on Polymers (Puri, India, 1992).

• Gordon Conference on "Aspects of Disorder in Condensed Matter Physics" , New England, USA (1993).

• "Dynamics of Ordering" (Queen's University, Kingston, Canada, 1993).

• 'Statphys Meeting' at Rutgers University, USA (1993).

• 'Statphys Meeting' at Rutgers University, USA (1994).

• APS March meeting (Pittsburgh, USA, 1994).

• 'Statphys Meeting' at Rutgers University, USA (1995).

• "Fourth Claude Itzykson Meeting on Biology and Physics", Saclay, France, 1999.

• "Statistical Physics of Glasses, Spin Glasses, Information Processing and Combinatorial Optimization", Les Houches, France (2005).

10 List of Publications

• **309** publications in reviewed journals, 4 conference proceedings, 9 invited reviews/book chapters.

• **Total no. of citations till 06/07/2021 : 16042/10924 (Google Scholar (GS)/ISI Web of Science (ISI))**

• **h-index: 66/54** (GS/ISI)

(*Two of my most cited papers, numbered below respectively (19) (with 920/628 citations (GS/ISI)) and (21) (with 569/348 citations (GS/ISI)) appear under the initials S. Majumdar and not S.N. Majumdar*)

In Journals (The number of citations (GS/ISI, date: 06/07/2021) of papers with more than 100 citations are indicated):

Journals	Numbers
Science	1
PNAS	1
Physical Review Letters	68
Physical Review Research	2
Advances in Physics	1
Physics Reports	1
Physical Review E Rapid	10
Physical Review A Rapid	1
Physical Review E	76
Physical Review A	7
Physical Review B	4
Europhysics Letters	8
Journal of Chemical Physics	2
Biophysical Journal	1
Journal of Physics-A: Math. Theor.	51
Journal of Physics-C	1
Journal of Statistical Physics	18
Journal of Statistical Mech.: Th. Exp.	39
Nuclear Phys. B	1
Physica A	5
Physica C	1
Annals of Physics	1
J. Math. Phys., Analysis and Geometry	1
Chaos, Solitons, and Fractals	1
Quantitative Finance	1
SciPost	2
Current Science	2
Markov Proc. Rel. Fields	1
Eur. J. Phys. E	1
Total	309

(1) S.N. Majumdar, "An Ideal Polymer Chain in Arbitrary Dimensions Near an Attractive Site", Physica A, v-169, 207 (1990).

(2) D. Dhar and S.N. Majumdar, "Abelian Sandpile Model on the Bethe Lattice", J. Phys. A: Math. Gen., v-23, 4333 (1990). [**citations: 193/129 (GS/ISI)**].

(3) S.N. Majumdar and D. Dhar, "Height Correlations in the Abelian Sandpile Model", J. Phys. A: Math. Gen., v-24, L357 (1991). [**citations: 175/124 (GS/ISI)**]

(4) S.N. Majumdar and M. Barma, "Tag Diffusion in Driven Systems, Growing Interfaces and Anomalous Fluctuations", Phys. Rev. B, v-44, 5306 (1991).

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