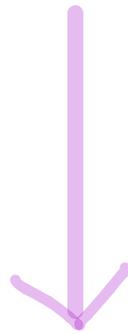
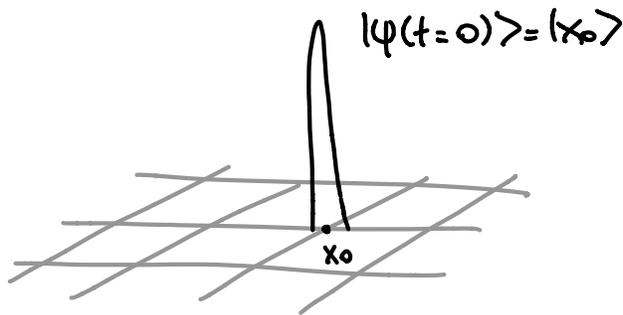


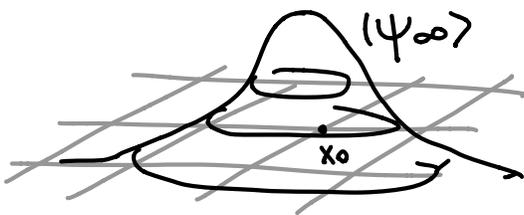
# Localization, Dissipation, Self-baths

## Single particle localization: the picture

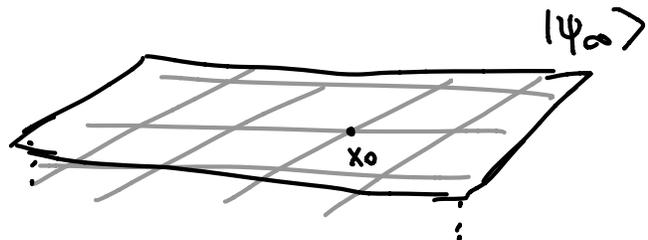


time evolution

$$|\psi(t)\rangle = e^{-itH} |\psi(0)\rangle \xrightarrow{t \rightarrow \infty} |\psi_\infty\rangle$$



LOCALIZED



DELOCALIZED

$$\text{IPR: } \frac{\sum_x |\langle \psi_\infty | x \rangle|^4}{\sum_x |\langle \psi_\infty | x \rangle|^2} \xrightarrow{N \rightarrow \infty} \mathcal{O}(1)$$

$$A_{x_0}(t) = \Theta(t) \langle x_0 | e^{-itH} | x_0 \rangle \sim \mathcal{O}(1)$$

$$\text{IPR: } \frac{\sum_x |\langle \psi_\infty | x \rangle|^4}{\sum_x |\langle \psi_\infty | x \rangle|^2} \xrightarrow{N \rightarrow \infty} 0$$

$$A_{x_0}(t) \approx A(t) e^{-\gamma t} + B(t) \xrightarrow{t \rightarrow \infty} 0$$

[cartoon pictures]

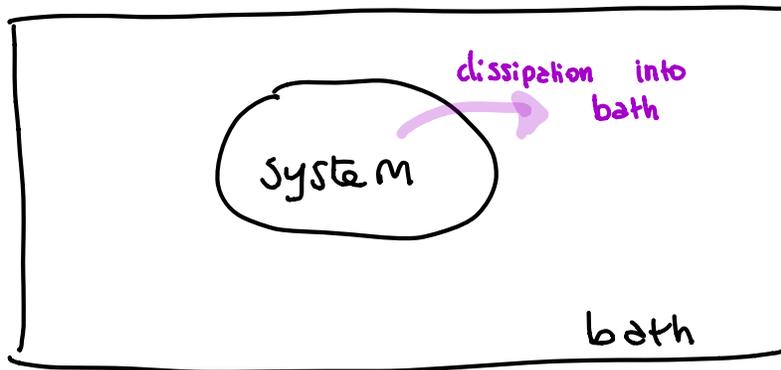
## Single-particle localization: criterion & interpretation

$$\lim_{\eta \downarrow 0} \lim_{N \rightarrow \infty} P\left(-\text{Imm} \sigma_x(E+i\eta) > 0\right) = 0 \iff \text{localization}$$

↑  
local self-energy

Interpretation:

- Decay rate ( $\rightarrow$  imaginary part of self-energy) is positive when there is dissipation.
- when couple a system (e.g. one particle) to a bath, there is dissipation  $\Rightarrow$  stat phys description of system.



- Anderson model has no bath: isolated system. Yet, in delocalized phase:

- add infinitesimal dissipation  $\eta$  (like symmetry-breaking field in ferromagnet):  $\Gamma = -\text{Imm} \sigma_x(E+i\eta) \sim \mathcal{O}(\eta)$
- take  $N \rightarrow \infty$ , then  $\eta \rightarrow 0$  and find  $\Gamma \sim \mathcal{O}(1)$ : the system self-sustains dissipation

- Means that the rest of system acts as effective bath for subsystems.

- When system localized, this breaks down!

▣ The many-body version: many interacting quantum particles, no bath, quantum mechanics (Unitary evolution)

Delocalized phase: if look at subsystem of whole system, the rest of system acts as a bath.

AT LONG TIME, CAN DESCRIBE THE SUBSYSTEM WITH STAT. MECH.,  
BOLTZMANN MEASURES

Many-body localization: picture breaks down.

CAN NOT DESCRIBE SUBSYSTEMS  
WITH BOLTZMANN MEASURE!

[ Underlying question: conjugate quantum dynamics with stat phys descriptions ]