

Exploring the Structure and Dynamics of Financial Markets through Graph Ricci Curvature

Context

Financial markets are high-dimensional complex systems whose internal structure — sector organisation, contagion channels, systemic fragility — is encoded in the empirical correlation matrix of asset returns. Standard approaches to extracting this structure rely on aggressive sparsification via minimum spanning trees (MST), discarding dependence information and imposing topological assumptions the geometry can supply on its own. In (arXiv:2510.15942 2025), introducing ORC — a geometry-driven descriptor of network organisation via optimal transport [1,2] — to stock correlation networks, we established that these networks are ORC-spherical for the first time— every edge carries strictly positive curvature, a consequence of the market mode — and showed that Ricci flow develops a discrete neckpinch singularity with a self-selecting gap ratio $G \gg 1$, producing a five-level hierarchy of economically coherent clusters via the ARCUS surgery algorithm. This result opens a programme of open questions that form the basis of the proposed thesis.

$$\kappa_{ij} = 1 - W(m_i, m_j) / d(i, j)$$

Research Project Aim

This project builds directly on Srinivasan (2025) and pursues the open questions it raises. Is ORC-sphericity universal? The NASDAQ-100 is narrow and tech-heavy; we will test the S&P 500, Eurostoxx 600, and Nikkei 225, asking whether all-positive initial curvature is a fundamental property of market co-movement — forced by the market mode — or specific to particular index compositions, and extending the framework to mixed-curvature settings if exceptions arise. Does the neckpinch always develop, and does G signal crises? We will study how curvature distributions and hierarchical decompositions evolve through the 2008 crisis, the 2020 COVID shock, and the 2022 rate correction, asking whether the gap ratio G or the surgery population κ^- provides an early marker of market fragility, and connecting the neckpinch threshold to the spectral gap of the correlation matrix. What are the theoretical foundations? We will resolve whether Ricci flow on an ORC-spherical graph preserves positive curvature in subgraphs — a discrete analogue of Hamilton's theorem — and establish the NASDAQ-100 neckpinch as a large- N instance of a precisely understood geometric singularity. The companion paper deferred by Srinivasan (2026), comparing ARCUS against MST, PCA, and spectral clustering across multiple indices and characterising the latent cross-sector cluster at Level 3, forms the immediate empirical component of the project.

Methods and Techniques

The mathematical toolkit for this project spans several interconnected areas. Optimal transport underpins ORC via Wasserstein distances between lazy random walk measures on neighbouring nodes; the dynamical optimal transport framework of Benamou and Brenier is expected to provide theoretical traction on the continuous-time limit of the discrete flow and on the geometry of the neckpinch singularity. Random matrix theory provides null models for the near-degenerate ORC-spherical geometry of empirical correlation matrices — Marchenko–Pastur, Wishart, and spiked-covariance ensembles — allowing genuine geometric signal to be separated from finite-sample noise, and connects the neckpinch threshold to the spectral gap of the correlation matrix. Discrete differential geometry — Hamilton's Ricci flow, neckpinch singularities, and the positive curvature theorems of Hamilton and their discrete analogues developed by Lin, Lu and Yau — provides the geometric framework within which the empirical ARCUS findings will be interpreted and, where possible, proved. Numerical implementation will use the GraphRicciCurvature library for large-scale ORC computation across multiple indices and rolling time windows.

Key References

- [1] Ollivier, Y. Ricci curvature of Markov chains on metric spaces. *J. Funct. Anal.* 256 (2009).
- [2] Ni, C.C. et al. Community detection on networks with Ricci flow. *Scientific Reports* 9 (2019).
- [3] Srinivasan, B. Intrinsic Geometry of the Stock Market from Graph Ricci Flow. *arXiv:2510.15942* 2025