

Conformal field theory approach to low-dimensional critical statistical models

In recent years, research has been devoted to understanding the universal properties of geometrical statistical models [1], where the main objects of interest are non-local structures such as the magnetic domains of the Ising model [2, 3, 4, 5]. This work has provided new insights into solutions of conformal field theories (CFTs) [6] and has deepened our understanding of how scale (conformal) invariance is realized at the quantum level. Building on this foundation, the proposed project aims to explore new situations in which conformal invariance is expected to appear in novel ways, specifically in statistical models with disorder and/or long-range interactions [8, 9, 10]. These systems are known to exhibit rich and often poorly understood critical behaviors, where standard tools of local field theory may fail or require significant generalizations.

A central objective of the project will be to investigate how quenched long-range disorder and interactions modify the geometrical and conformal properties of critical systems. In particular, the student will study the emergence (or breakdown) of conformal invariance in disordered environments, the role of rare events, and the possible appearance of new universality classes. Special attention will be devoted to the geometrical observables (clusters, interfaces, loops), whose scaling properties often provide direct access to underlying field-theoretical descriptions.

The project will also address the interplay between long-range interactions and conformal symmetry. Recent theoretical developments suggest that long-range models can interpolate between different universality classes. Understanding these connections remains an open challenge, both analytically and numerically.

From a methodological perspective, the work will combine advanced Monte Carlo simulations with modern theoretical tools from renormalization group theory and conformal field theory. The student will develop and implement efficient numerical algorithms to study large-scale systems, analyze finite-size scaling, and extract critical exponents and conformal data. On the analytical side, the project may involve conformal bootstrap techniques, perturbative renormalization group approaches, and comparisons with exactly solvable models when available.

Particular emphasis will be placed on identifying robust signatures of conformal invariance (or its violation), such as crossing probabilities, fractal dimensions, and correlation functions. The project may also explore connections with Schramm-Loewner evolution (SLE) and logarithmic conformal field theories, which naturally arise in disordered systems.

More broadly, this research is expected to contribute to a deeper understanding of universality in complex systems and to establish new bridges between statistical physics, quantum field theory, and probability theory.

During the PhD, the student will be introduced to key ideas of the renormalization group, conformal field theory, and Monte Carlo simulations in critical systems. They will acquire a strong background in both analytical and computational techniques, preparing them for research at the interface of theoretical physics and applied mathematics.

The PhD project will be jointly supervised by Marco Picco and Raoul Santachiara.

References

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