

1 Context and Motivation

In the work K. Polovnikov, S. Nechaev, and M. Tamm, Effective Hamiltonian of topologically stabilized polymer states. *Soft Matter*, 14(31), 6561–6570 (2018), authors constructed a fractal Laplacian in Euclidean space capable of describing fractional Brownian motion (fBm) with a Hurst exponent $H < 1/2$. This model relies exclusively on quadratic non-local interactions between all units, given by:

$$E = \sum_{i \neq j} a_{ij} (x_i - x_j)^2, \quad \text{where} \quad a_{ij} = s^{-\gamma} \quad (1)$$

and $s = |i - j|$ represents the chemical distance between monomers i and j . By tuning the interaction exponent γ , it is possible to continuously generate any desired sub-diffusive exponent $H < 1/2$.

2 Research Objectives

The central goal of this M2 project is to generalize this construction from Euclidean spaces to arbitrary complex networks defined by an adjacency matrix b_{kl} . Specifically, the first steps of this construction imply:

- **Construction of subdiffusive fractal Laplacians for tree-like graphs:** As an initial tractability step, construct and analyze fractal Laplacians describing subdiffusive (compact) fBm on regularly branching trees;
- **Construction of superdiffusive fractal Laplacians for tree-like graphs:** Next steps consists in construction of fractal Laplacians describing superdiffusive (expanded) fBm on regularly branching trees;
- **Spectral Properties:** Compute and study the eigenvalue spectra, density of states, and spectral dimensions of these new graph operators.

3 Application to Community Detection

In network science, the spectrum of standard graph Laplacians is widely used to identify graph partitions and detect communities where each discrete eigenvalue corresponds to a particular community. It is also known that Laplacians of “nonbacktracking random walks” (see F. Krzakala, C. Moore, E. Mossel, J. Neeman, A. Sly, L. Zdeborová, and P. Zhang, *PNAS*, 110(52), 20935–20940 (2013)) determine communities better. So, one could expect that the fractal Laplacians of “expanded” random walks on graphs could be useful for community detection.