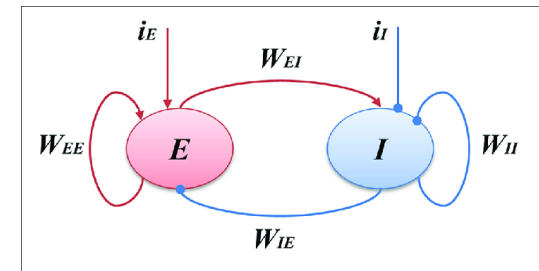
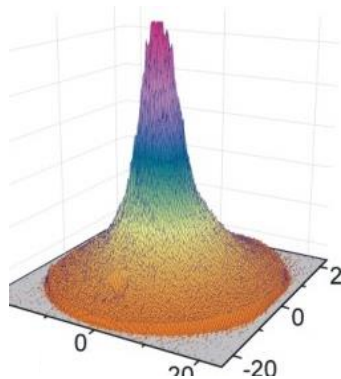


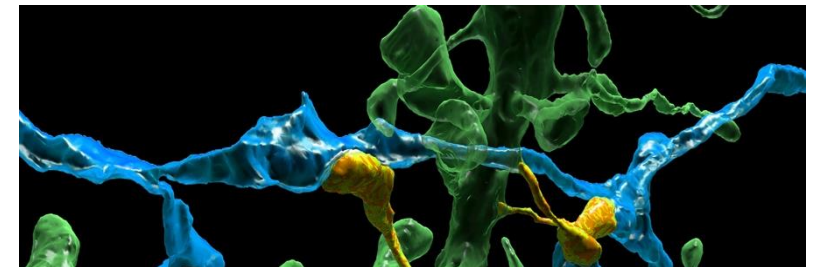
Physics without Newton's third law: phase transitions in non-reciprocal matter

IMPACT, Cargese, August 2024

Alex Edelman , Michel Fruchart , Ryo Hanai, Kyle Kawagoe, Sergei Shmakov, Cheyne Weis, Shuoguang Liu, Xioayuan Huang, Peter Littlewood and Vincenzo Vitelli



Shmakov, PBL Physical Review E 109, 024220 (2024)
Weis et al arXiv:2207.11667
Fruchart, Hanai, PBL, VV, Nature 592, 363-369 (2021)
Hanai, PBL, Physical Review Research 2, 033018 (2020)
Hanai, Edelman, Ohashi, PBL, Phys. Rev. Lett. 122, 185301 (2019)



University of
St Andrews

A Bose condensate viewed as a collection of synchronized oscillators



ROCKPORT

EST. 1971

Taktschläge B.
Largo
Larghetto
Adagio
Andante
Moderato 60-120
Allegro 120-168
Presto 168-200
Prestissimo 200-208

per minute
Largo 40-60
Larghetto 60-66
Adagio 66-75
Andante 75-108
Moderato 108-120
Allegro 120-168
Presto 168-200
Prestissimo 200-208



Christiaan Huygens 1629-95

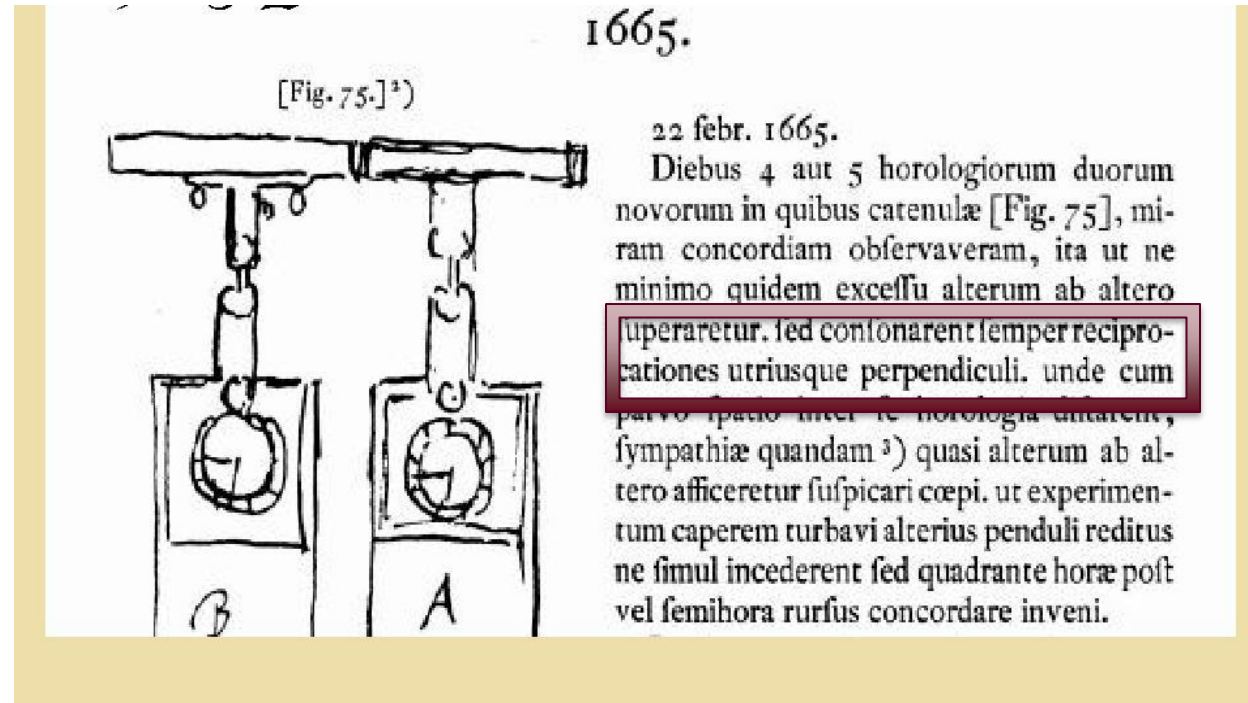
1656 – Patented the pendulum clock

1663 – Elected to Royal Society

1662-5 With Alexander Bruce, and sponsored by the Royal Society, constructed maritime pendulum clocks – periodically communicating by letter

Huygens Clocks

In early 1665, Huygens discovered "...an odd kind of sympathy perceived by him in these watches [two pendulum clocks] suspended by the side of each other."



Out of phase!

He deduced that effect came from "imperceptible movements" of the common frame supporting the clocks

Huygens clocks: the first experimental nonlinear dynamical system ?

Oscillators are non-linear (amplitude of oscillation not small)

Non-equilibrium – steady state established between balance of driving and dissipation

There exist periodic steady states that are stable to small external perturbation (“noise”)

By tuning parameters in the model (e.g. elastic vs. inertial coupling) one can make abrupt transitions from one state to another (e.g. in-phase to out-of-phase)

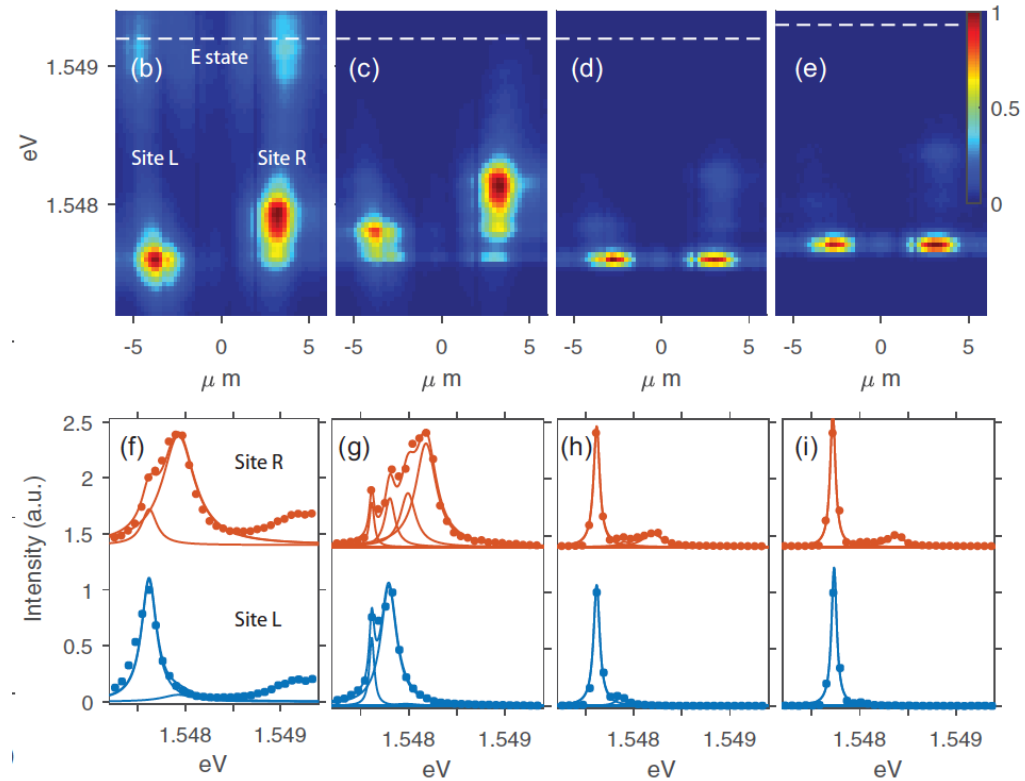
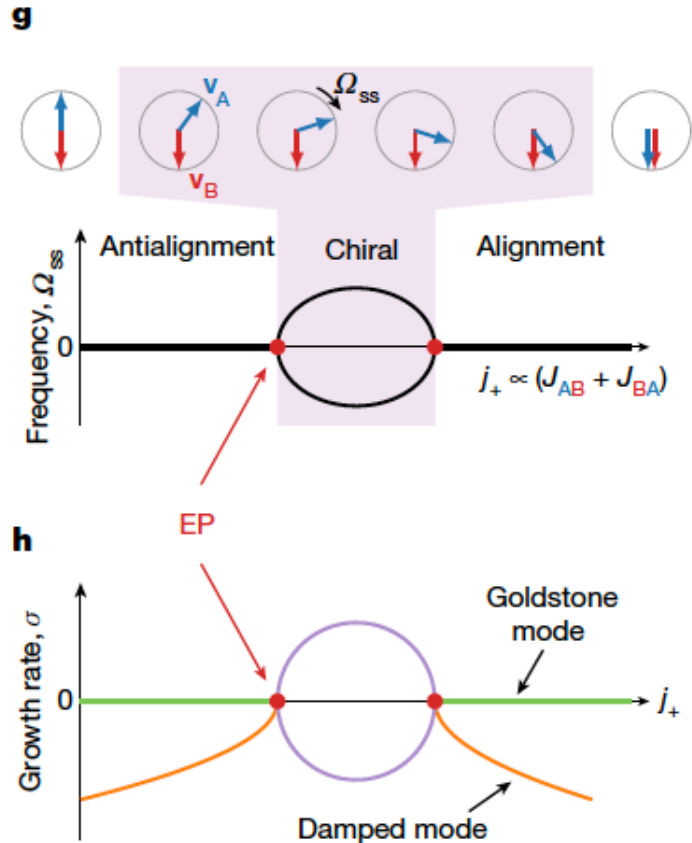
How does this transition happen?

Directly? viz. FM \rightarrow AF

Via disordered state? viz. FM \rightarrow PM \rightarrow AFM

Or something else?

Generically the transition from synchronized to antisynchronized occurs via an intermediate chiral state at an exceptional point.



Kim et al PRB 101.085302 (2020)

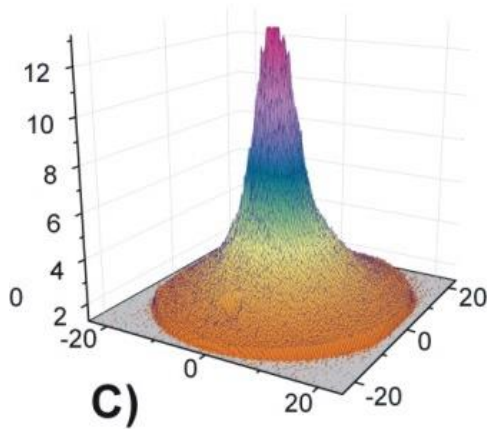
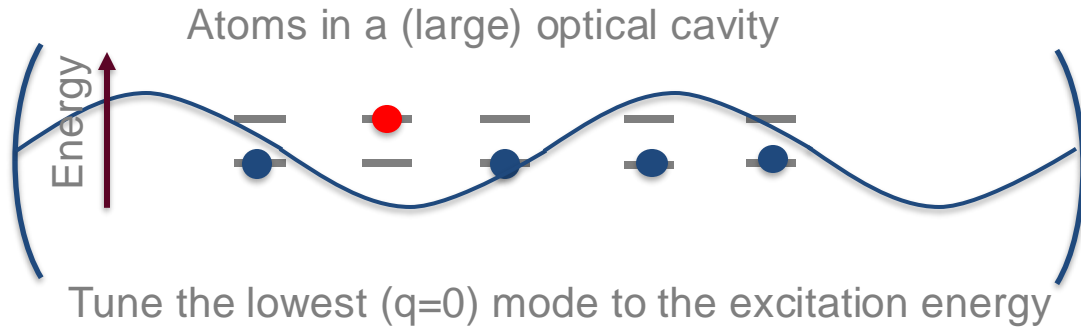
Fruchart et al. Nature 592, 363-369 (2021)

Three topics of this talk

- Are there 'phase transitions' between different kinds of synchronized states and how does one describe those transitions?
- Is there a synchronization equivalent of a critical point in a many-body system, and does it come decorated with 'universal' fluctuation dynamics?
- Is there a connection between synchronized quantum systems (e.g. a laser) and synchronization in classical models (e.g. flocks, Kuramoto models) ?

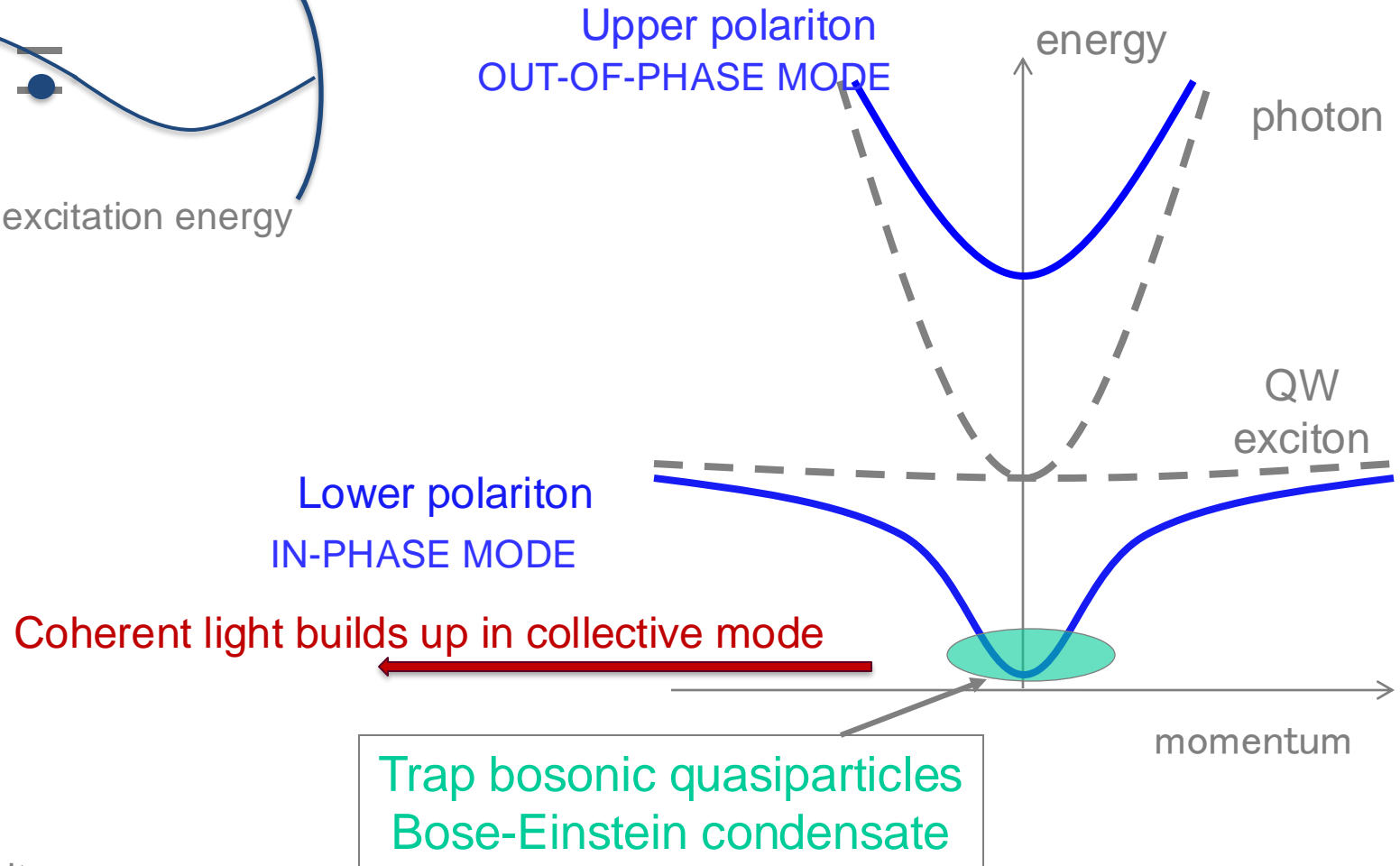
Polaritons

$$|\text{pol}\rangle = c_1|\text{exc}\rangle + c_2|\text{ph}\rangle$$



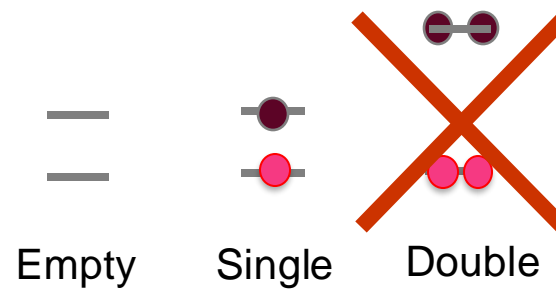
Momentum distribution of polaritons

Kasprzak et al Nature 2006



Polaritons and the Dicke Model - a.k.a Jaynes-Tavis-Cummings model

Excitons as “spins”



Spins are flipped by absorption/emission of photon



$$H = \omega \psi^\dagger \psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}} \sum_i [S_i^+ \psi + \psi^\dagger S_i^-]$$

$$N \sim [(\text{photon wavelength})/(\text{exciton radius})]^d \gg 1$$

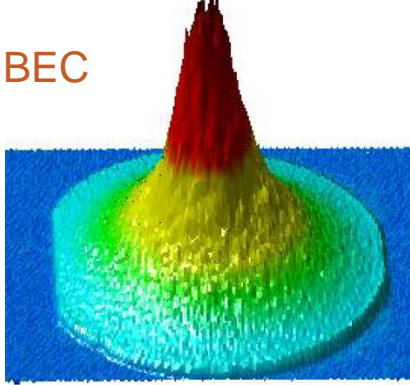
Mean field theory – i.e. BCS coherent state – expected to be good approximation

$$|\lambda, w_i\rangle = \exp \left[\lambda \psi^\dagger + \sum_i w_i S_i^+ \right] |0\rangle$$

QM ground state is coherent oscillation of excitons and photons **in phase**

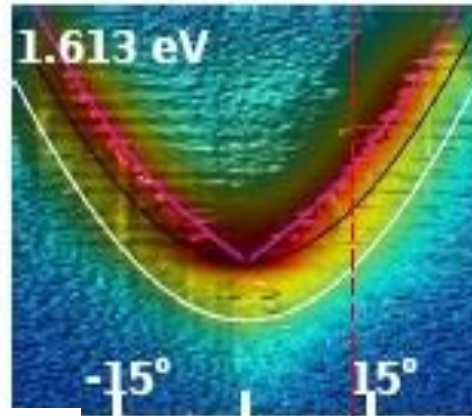
This coherent state persists when non-conserving terms are added:
dissipation (photons leak out) and pumping (excitons added)

BEC



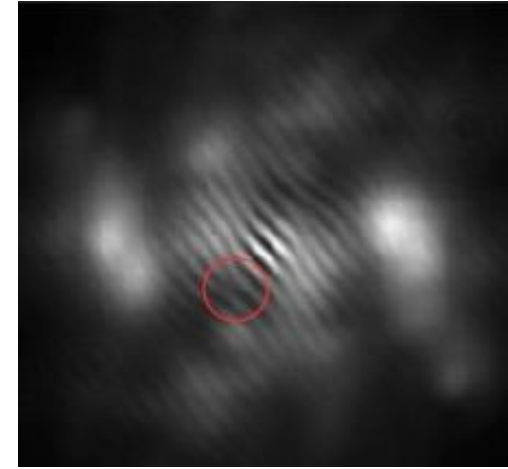
Kasprzak et al 2006

Bogoliubov spectrum



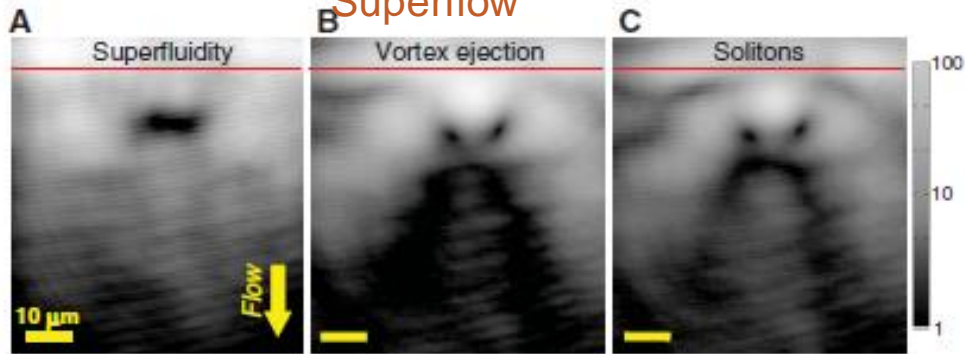
Funomiya et al, 2008

Vortices



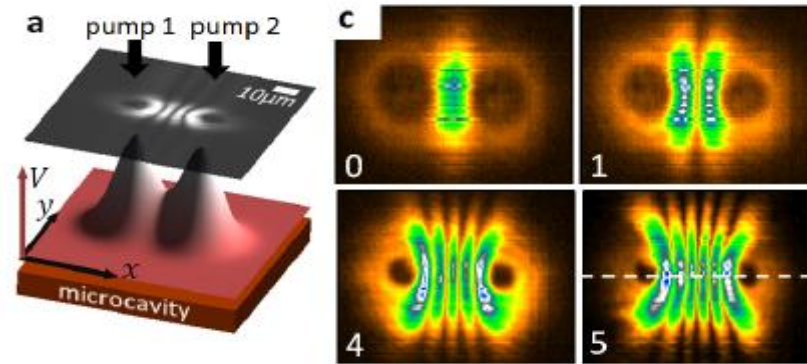
Lagoudakis et al 2008

Superflow



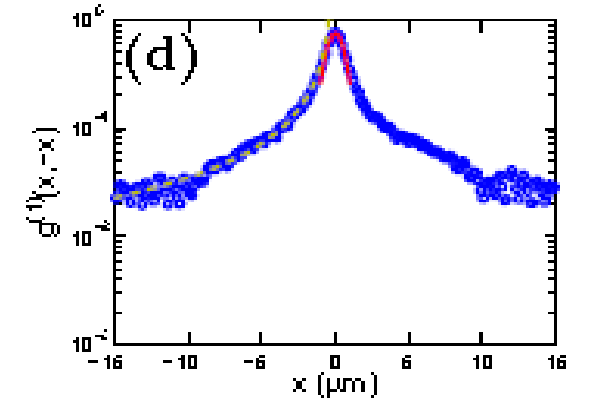
Amo et al, 2011

Coupled condensates



Tosi et al 2012

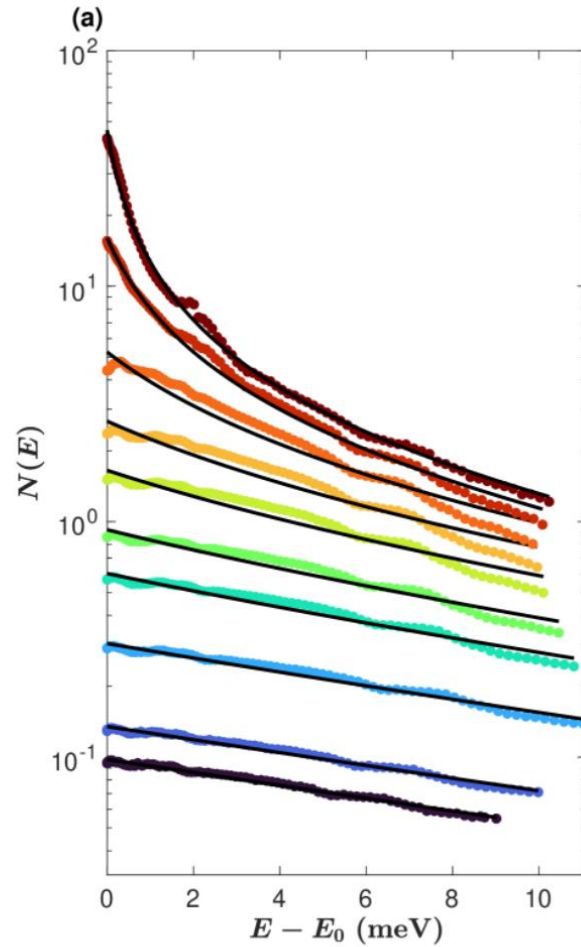
Power law correlations



Roumpou et al 2012

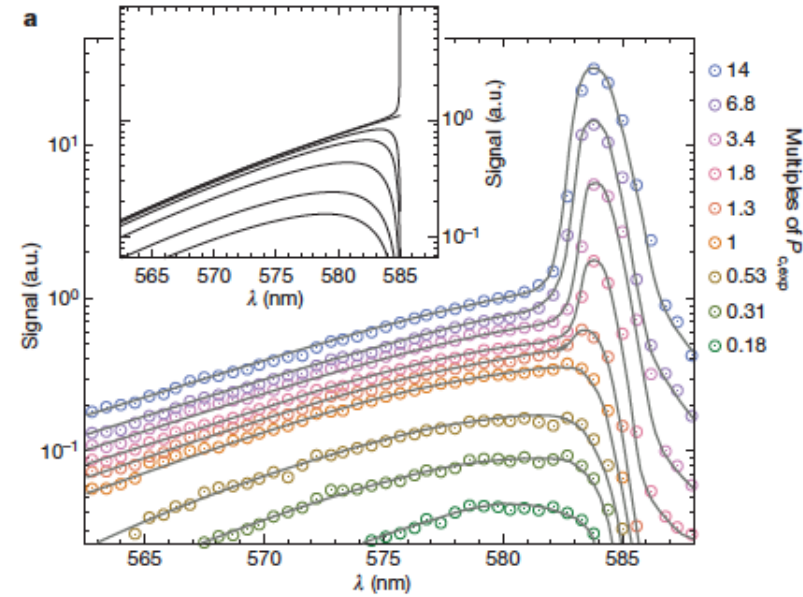
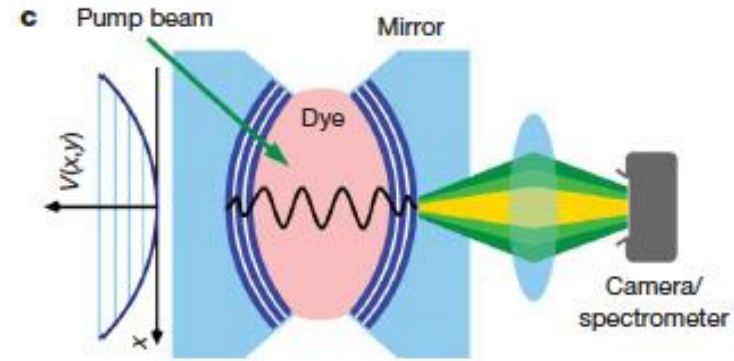
Fits to BEC populations

Polaritons in GaAs QW



Alnateh et al. arXiv:2406.13689v1

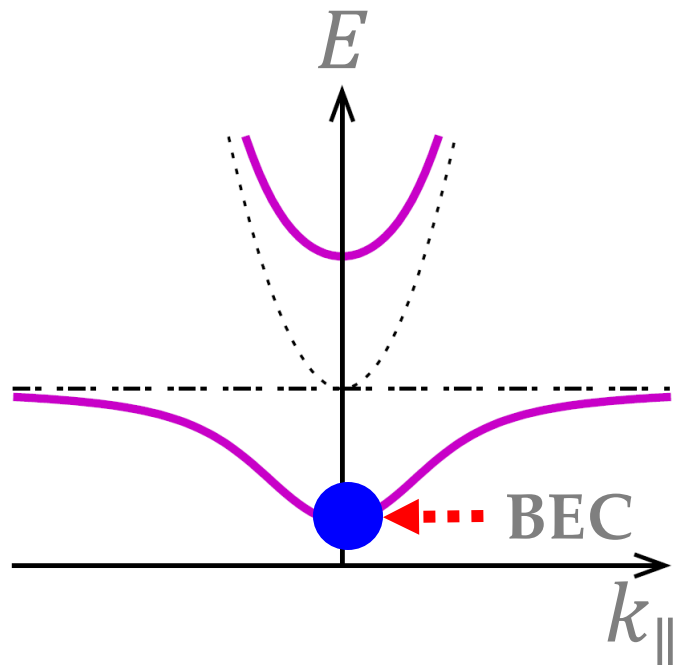
Photons in optical cavity



Klaers et al, Nature 468, 545 (2010)

Two component order parameter: mixed light and matter

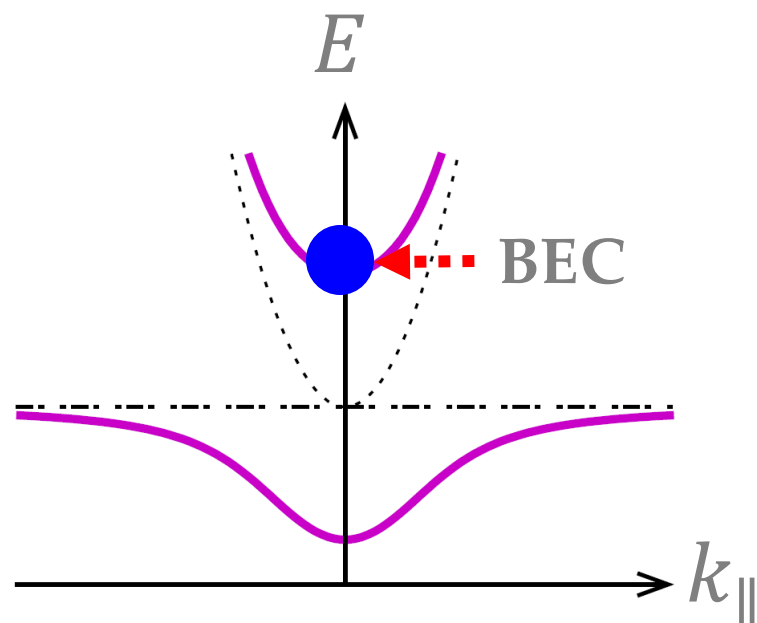
Non-equilibrium condensate is established as a steady-state solution with coherent oscillation (a.k.a. laser) – balancing excitation and decay



Conventional BEC in “in-phase” mode
Solution of non-linear 2x2 non-linear
Schrodinger equation with eigen-frequency at
(renormalized) **lower** polariton frequency

Two component order parameter: mixed light and matter

Non-equilibrium condensate is established as a steady-state solution with coherent oscillation (a.k.a. laser) – balancing excitation and decay

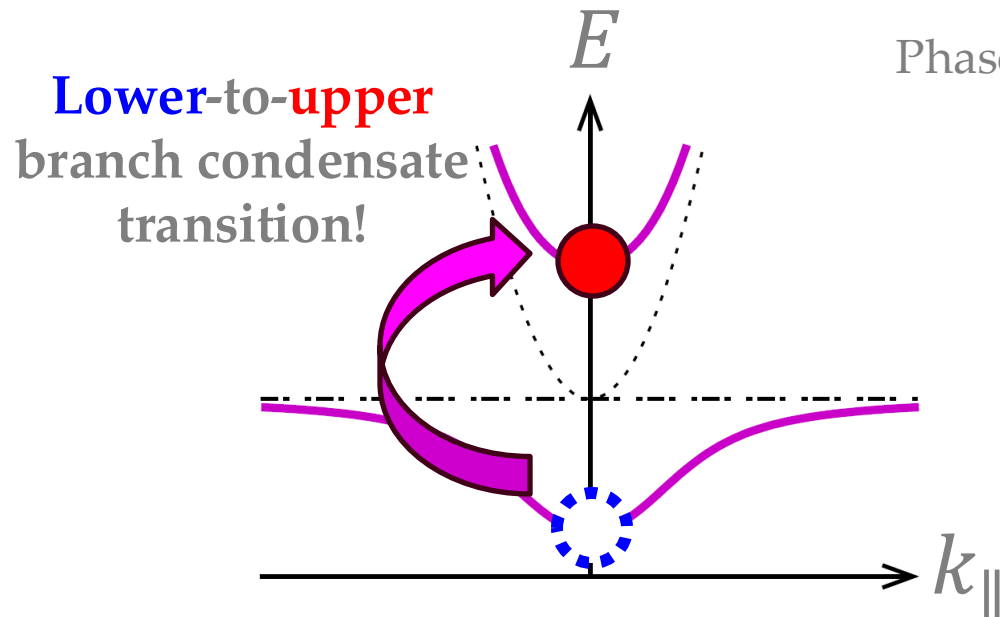


Non-equilibrium BEC in “out-of-phase” mode
Solution of non-linear 2x2 non-linear
Schrodinger equation with eigen-frequency at
(renormalized) **upper** polariton frequency

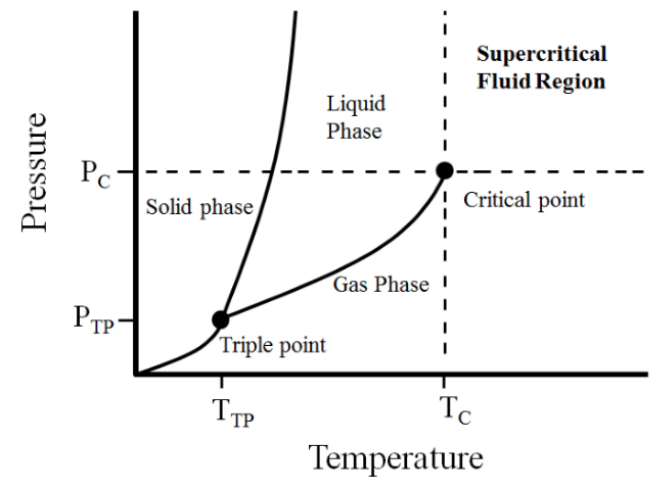
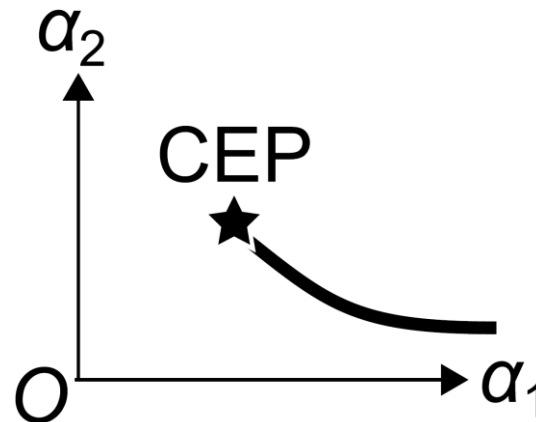
No symmetry distinction between upper and lower condensates

The transition can jump (first order-like) but can also coalesce if the parameters are such that the modes become degenerate: an **exceptional point**

Can only occur if the Hamiltonian is non-Hermitian

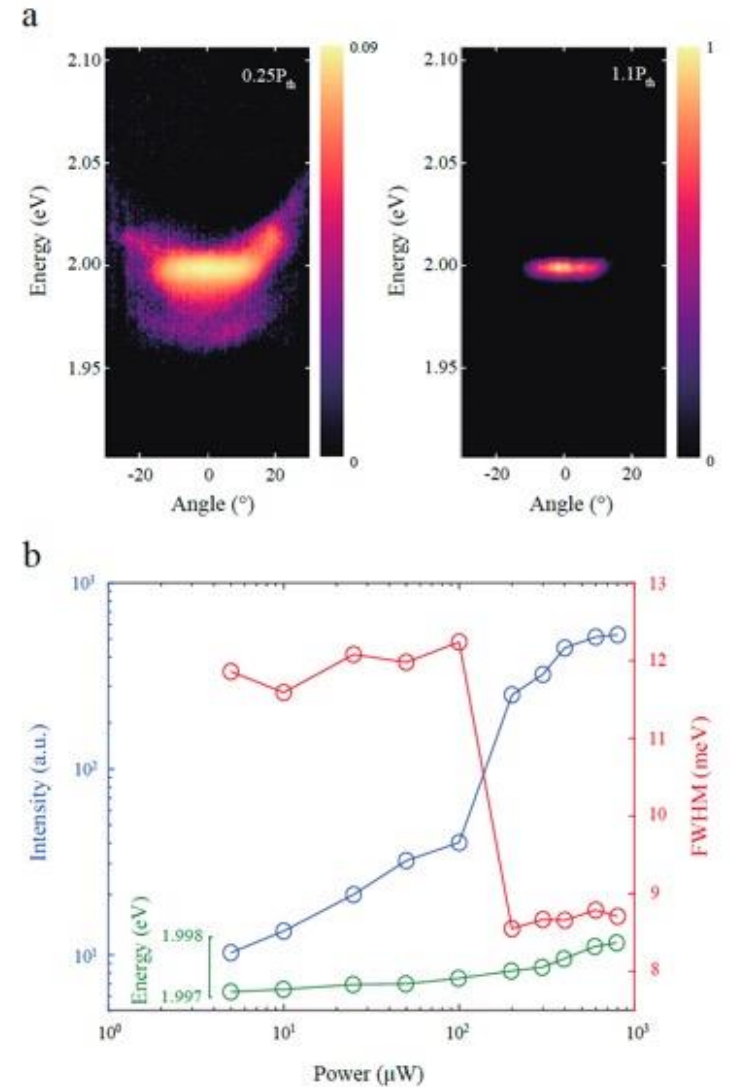
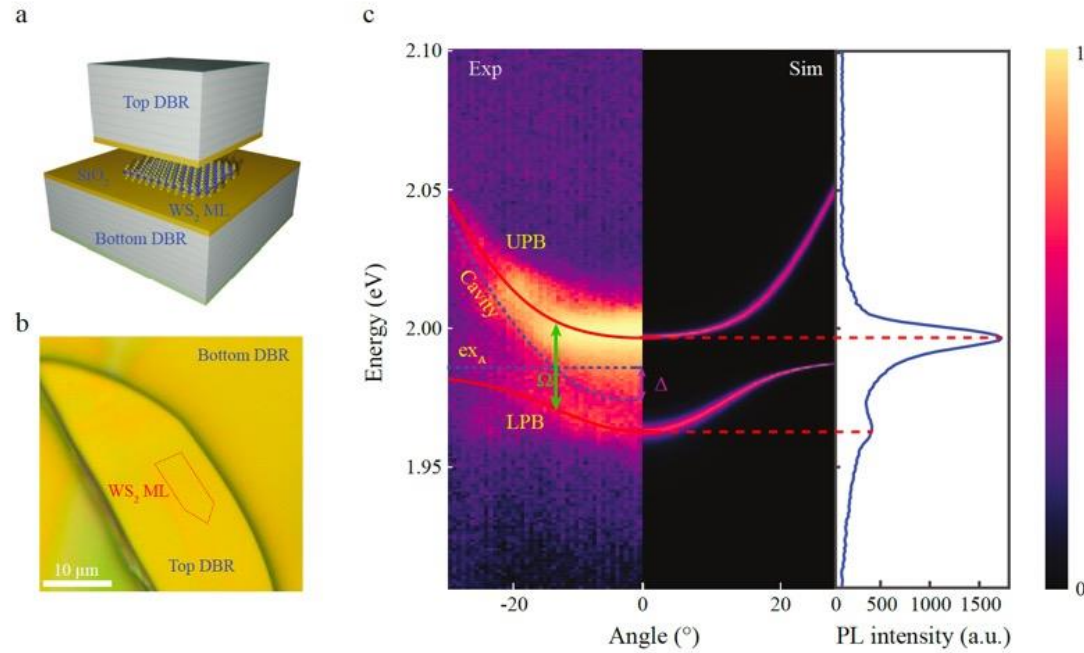


Phase diagram with two tuning parameters: e.g. pumping and loss



At the “critical exceptional point” CEP, “in-phase” and “out-of-phase” solutions are degenerate
Analogous to conventional critical point (e.g. liquid-gas) but there are no massive modes – very different critical dynamics

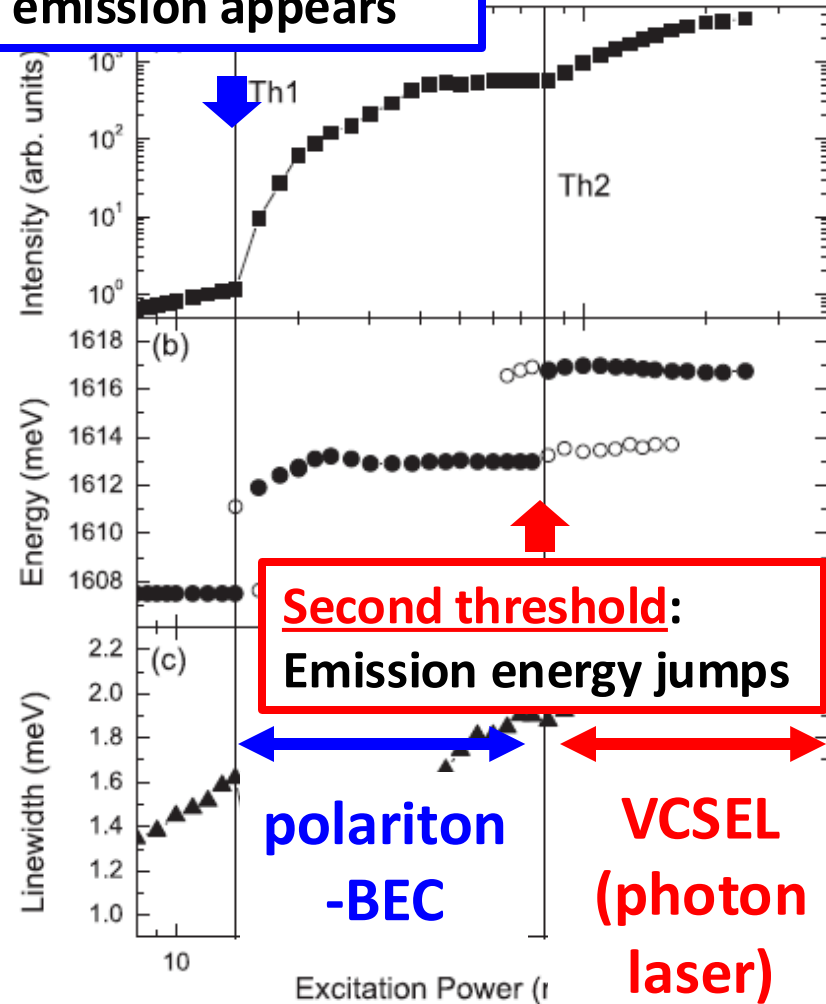
Upper polariton condensate in WS_2 monolayer cavity



Chen et al. Nano Letters 23 (20), 9538-9546

Two-threshold behavior

First threshold:
Condensate emission appears



Second threshold:
Emission energy jumps

“ **Second threshold**
= strong-to-weak coupling transition ”

Normal = $U(1)$ -symmetric

Polariton-BEC = $U(1)$ -broken

Photon laser = $U(1)$ -broken

... no new broken symmetries!
Why a transition?

Minimal model* - two-component damped driven Gross-Pitaevskii equation with noise

$$i\partial_t\Psi_\alpha(\mathbf{r},t) = \sum_{\beta=l,g}[A_{GP}]_{\alpha\beta}(\nabla^2)\Psi_\beta(\mathbf{r},t) + \eta_\alpha(\mathbf{r},t),$$

Photon field with dissipation

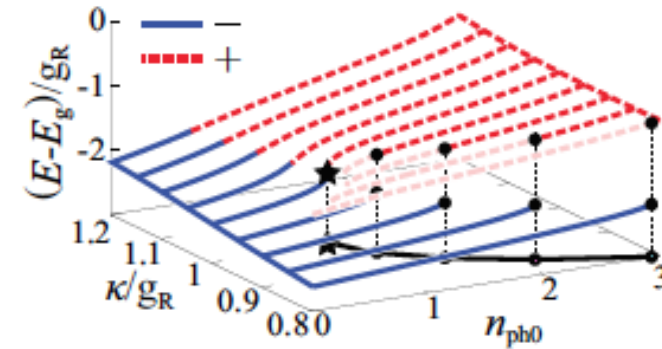
$$A_{GP}(\nabla^2) = \begin{pmatrix} \omega_l - i\kappa - K_l\nabla^2 & g \\ g & \omega_g + iP - \tilde{K}_g\nabla^2 + \tilde{U}_g|\Psi_g|^2 \end{pmatrix},$$

Exciton field with pumping and nonlinearity

Nontrivial stationary solutions have to be oscillating at a fixed frequency: either E_- or E_+

There is a single point in parameter space where both solutions exist – a so-called exceptional point

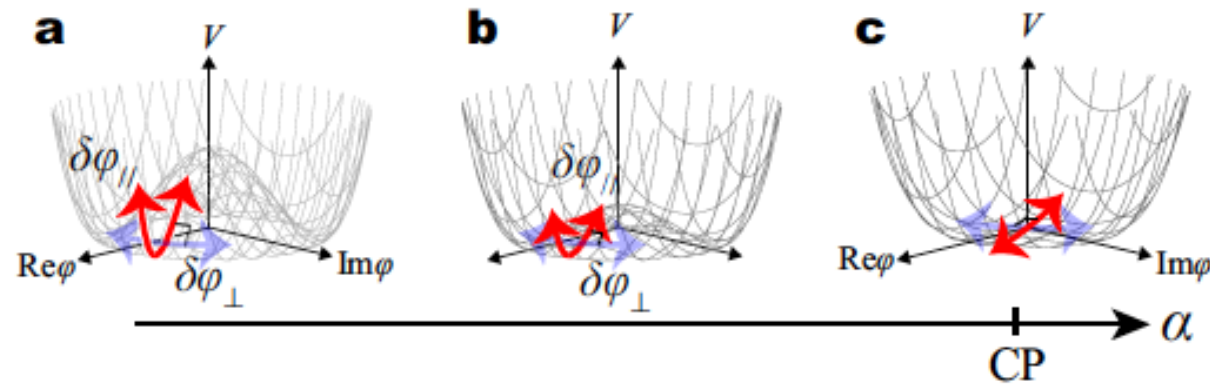
Non-Hermitian matrices do not generally have real eigenvalues;
in our case the balance of dissipation and decay self-tunes to a stationary state



* M. Wouters and I. Carusotto, PRL 99, 140402 (2007).

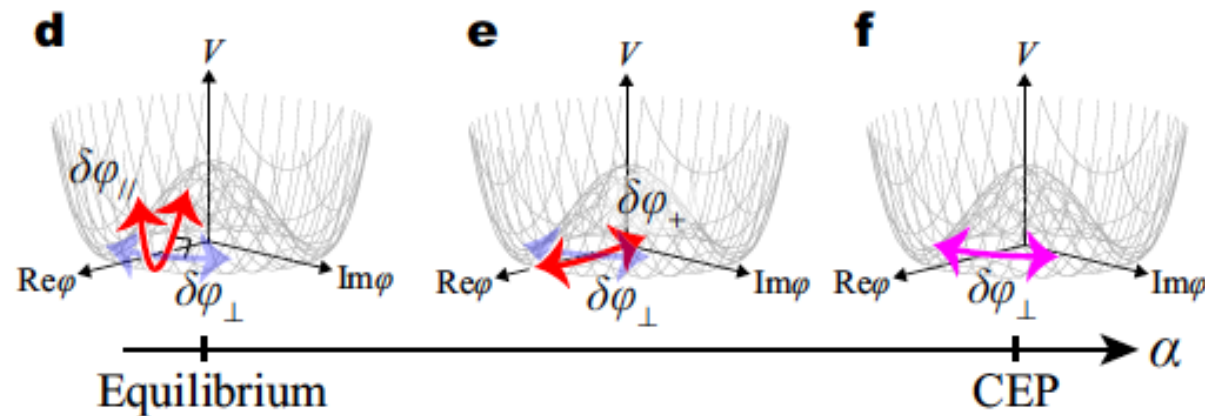
Near a critical exceptional point, the two modes coalesce: all fluctuations project onto a massless mode

Conventional critical point (CP)



Conventionally at a critical point a massive mode becomes gapless
But is always orthogonal to transverse (Goldstone) mode

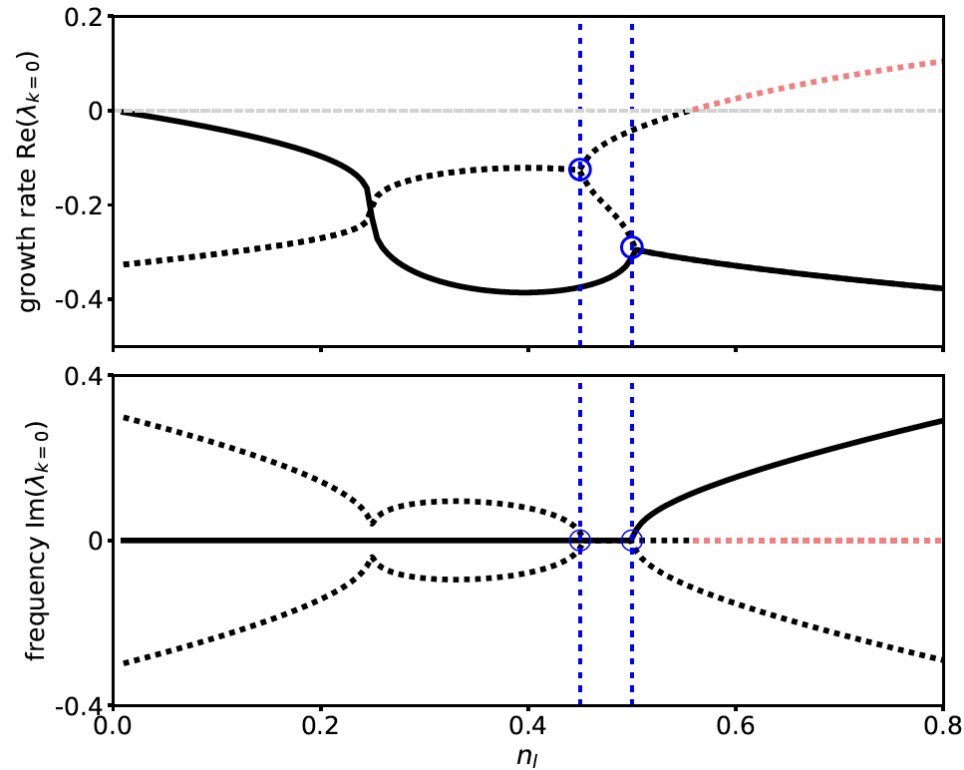
Critical exceptional point (CEP)



At a critical exceptional point the two modes coalesce and are indistinguishable
Much stronger fluctuations: no long-range order below 4 dimensions (?)
Upper critical dimension 8 dimensions (?)

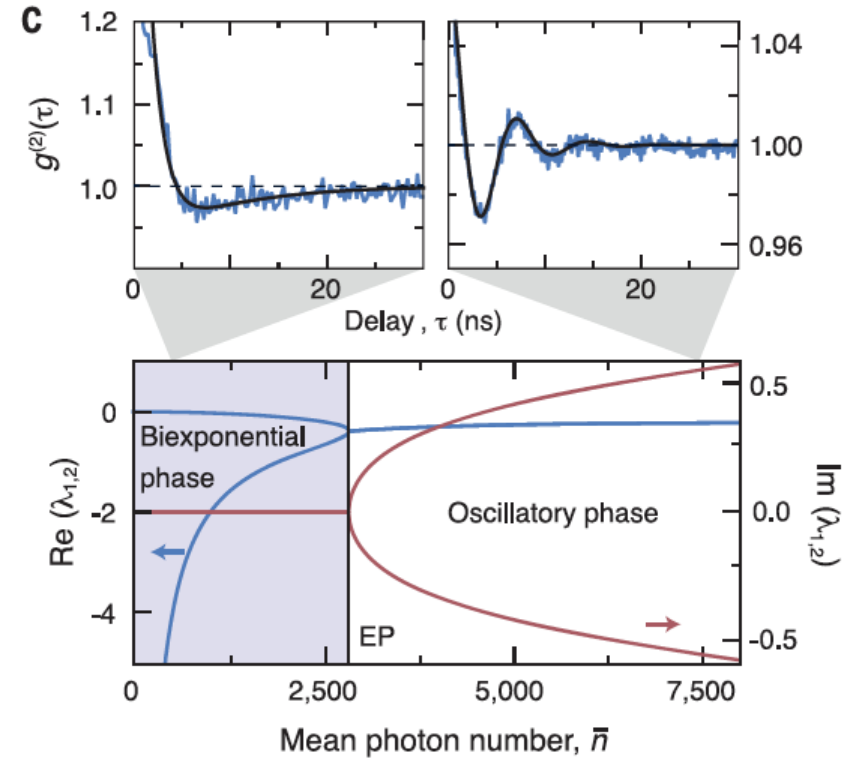
Exceptional points can emerge in the spectrum of linear modes perturbing the condensed state

Simulations from GPE



Huang and PBL unpublished

Intensity correlation g_2 measures fluctuation spectrum



Öztürk *et al.*, *Science* **372**, 88–91 (2021)

Toward a general theory

Non-reciprocal phase transitions

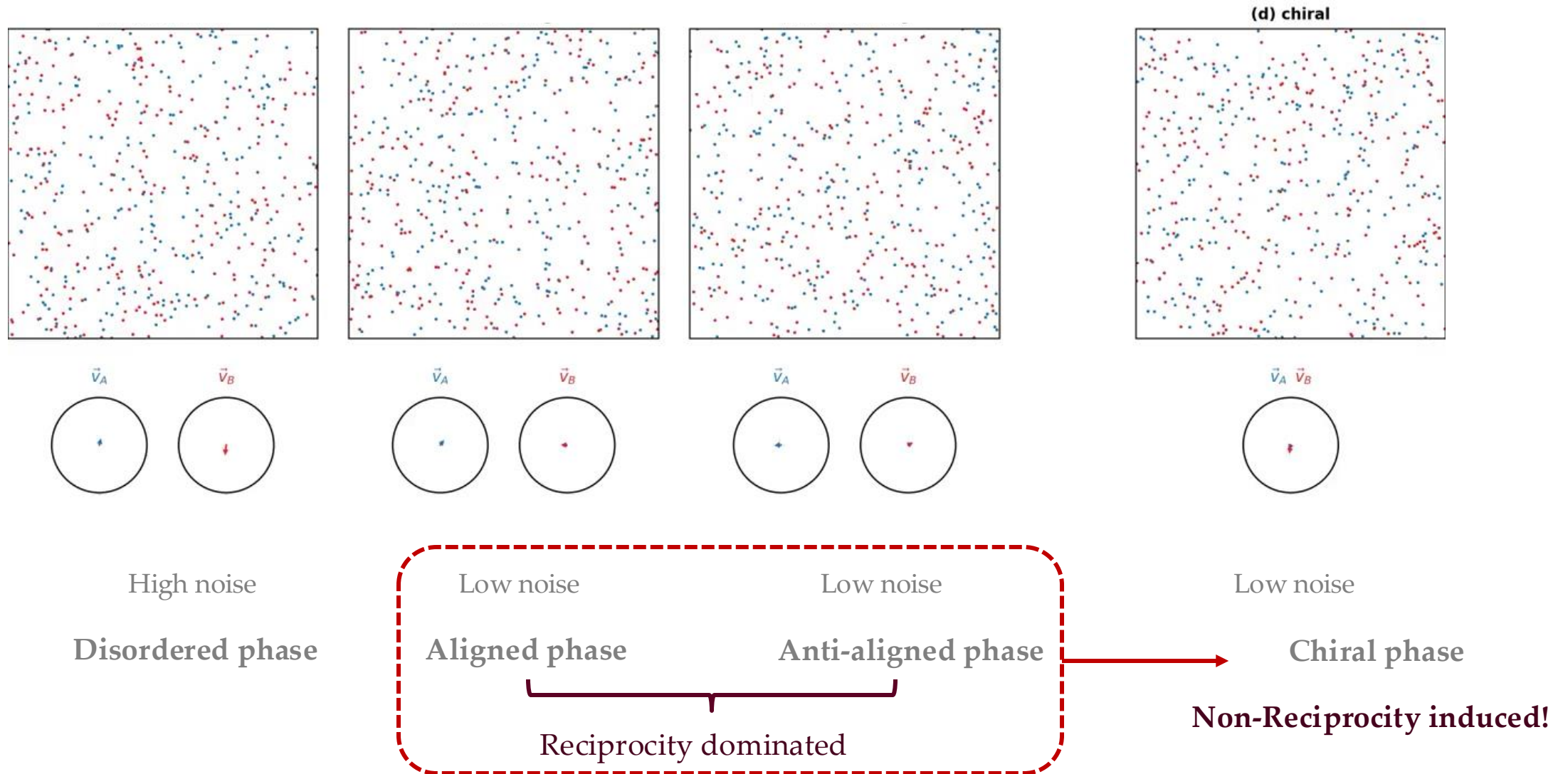


Michel Fruchart *, Ryo Hanai *,
Peter B. Littlewood, Vincenzo Vitelli

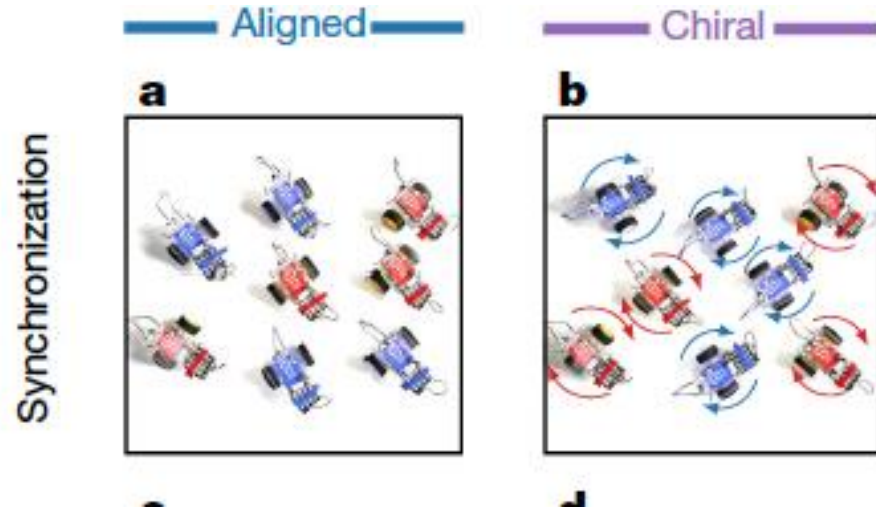
Nature (2021), arXiv:2003.13176

* These authors contributed equally to this work

Non-reciprocal Phase Transition : hawks and doves



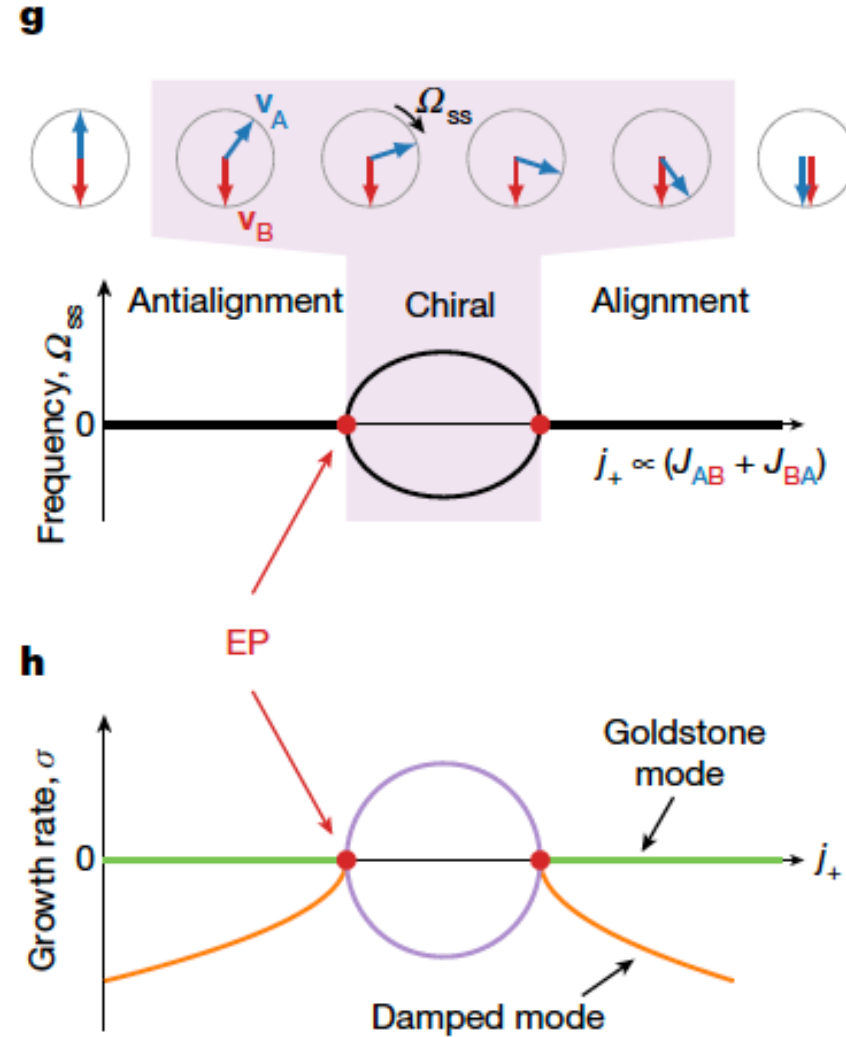
Transition from aligned/chiral/antialigned occurs when linear response in ordered phase reaches an exceptional point



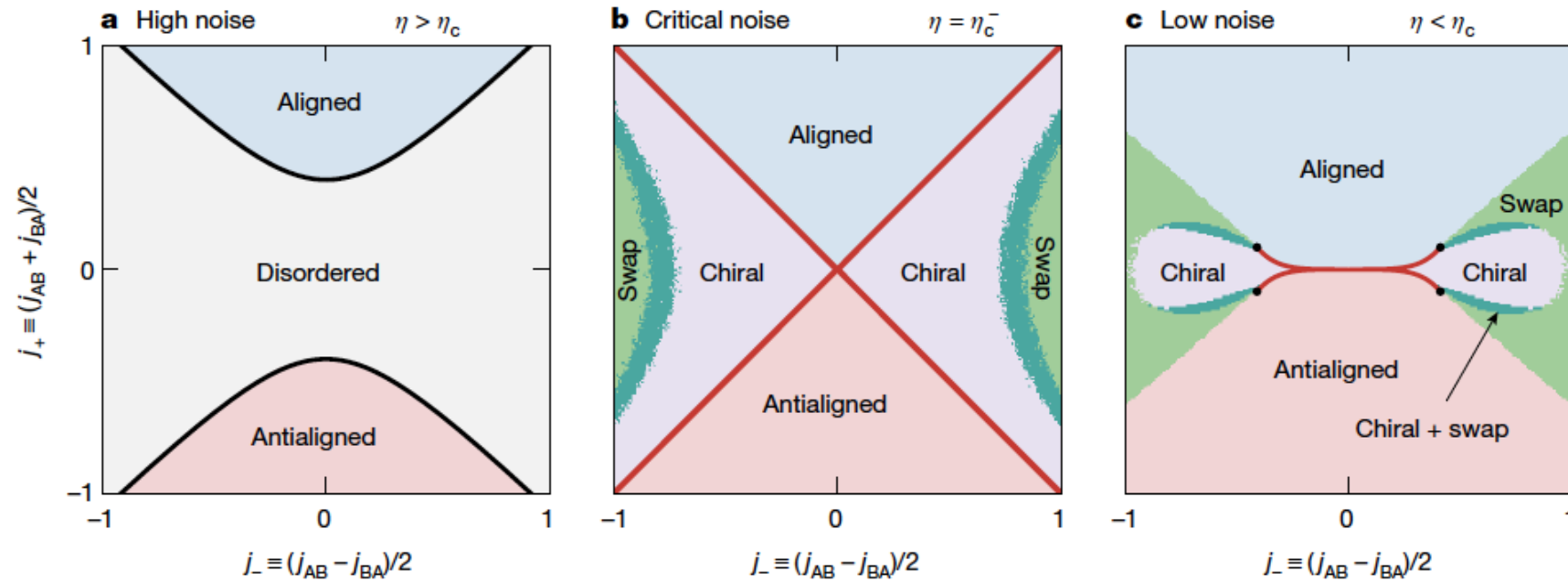
A Hamiltonian system is hermitian/symmetric
Eigenvalues are real, and eigenvectors are orthogonal

A non-Hermitian/non-symmetric system:
Eigenvalues can be complex, and eigenvectors are not orthogonal

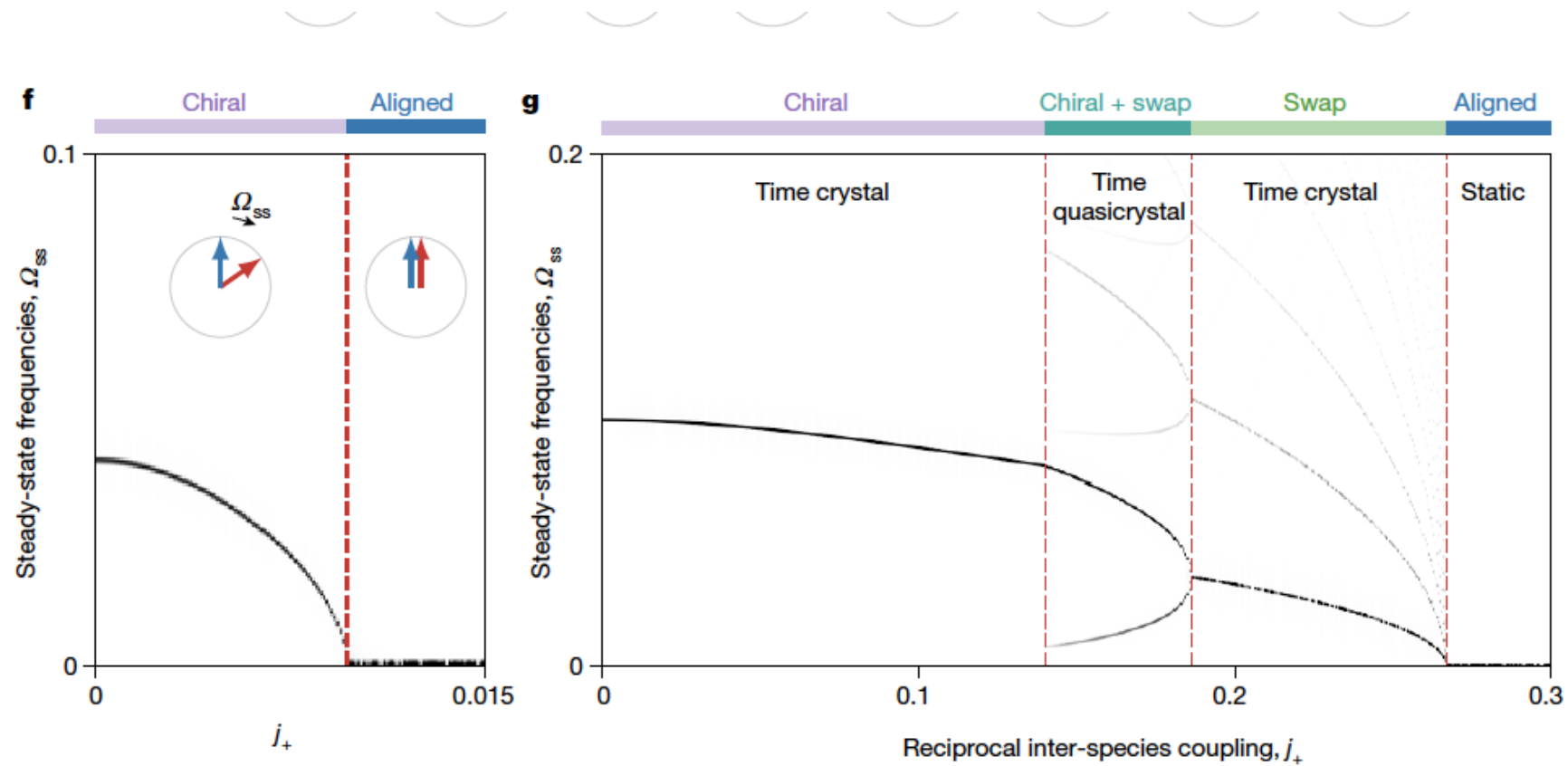
At an exceptional point, two eigenvalues are degenerate, and corresponding eigenvectors merge. System is non-invertible



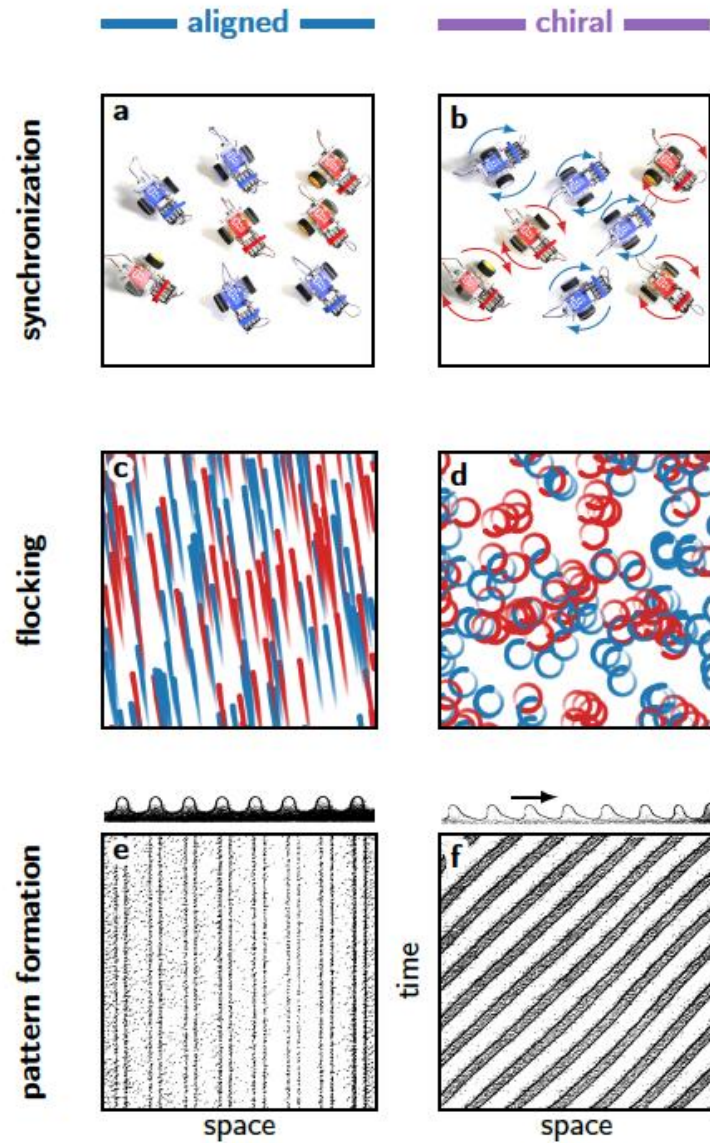
Two component Vicsek model for flocking



Generically there are other periodic and quasiperiodic states



Models with continuous symmetry : $U(1)$, $O(2)$,

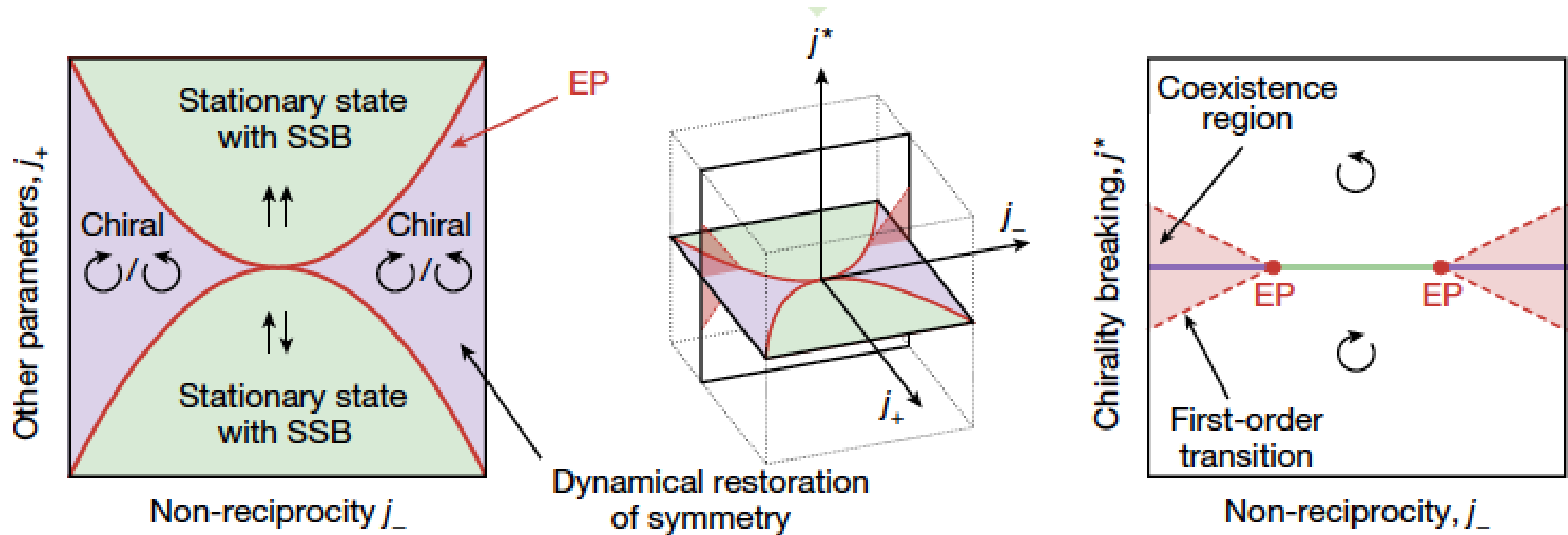


contrarian Kuramoto models

multi-component flocking

Swift-Hohenberg

General phase diagram of 2 - component non-reciprocal condensates



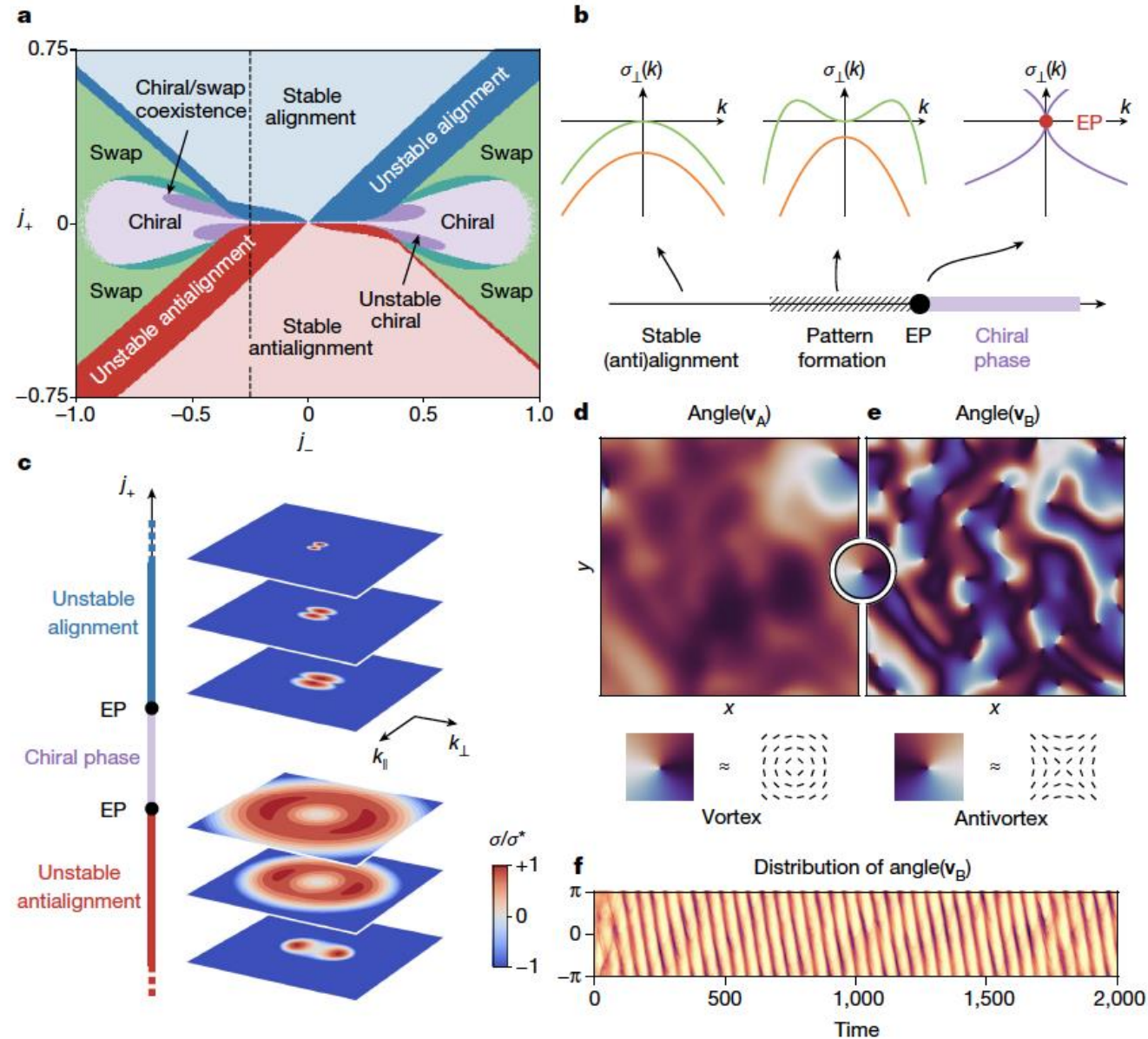
“PT” symmetric model
 Spontaneous chiral symmetry breaking
 EP are lines (co-dimension 1)

Flocking model (Vicsek)
 Interfacial travelling wave
 Wilson-Cowan model of neurons

“PT” non-symmetric model
 Chiral symmetry explicitly broken
 EP's are points (co-dimension 2)

Polariton condensate
 Kuramoto model

Beyond mean field - pattern-forming instabilities

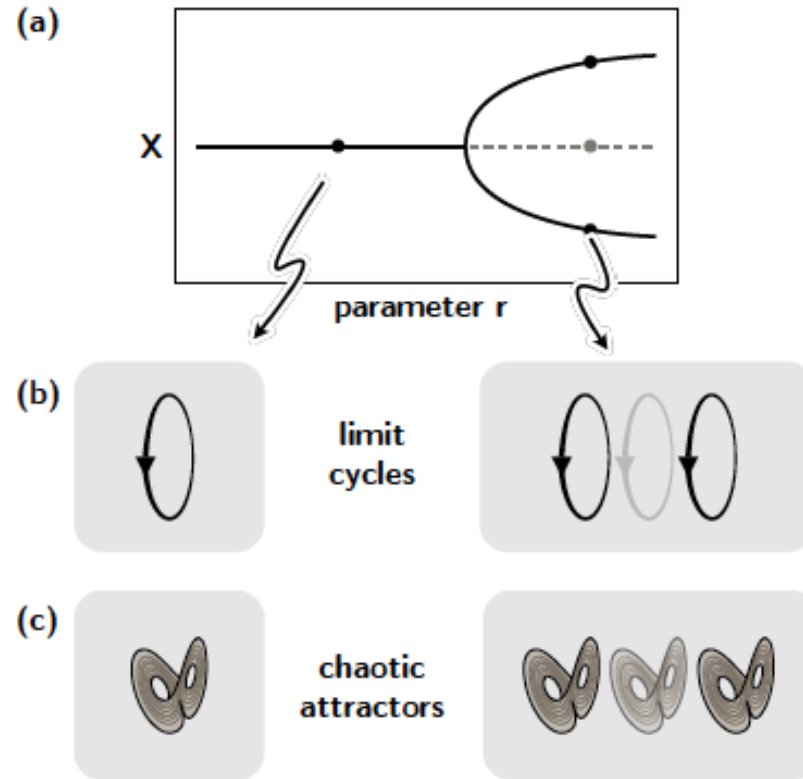
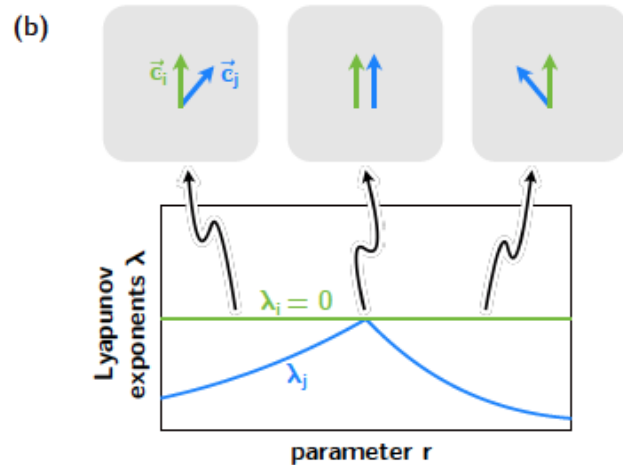
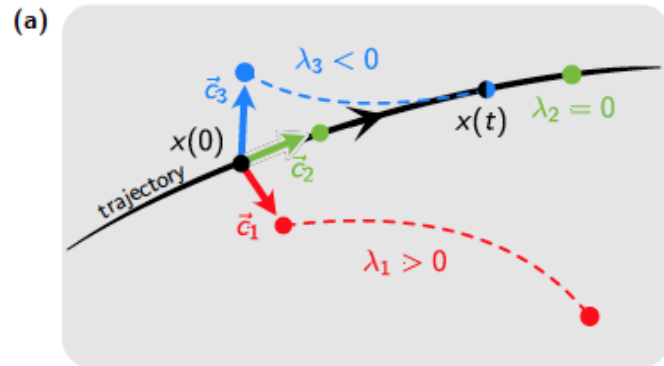


Exceptional points can describe merging of limit cycles

$$\delta\dot{X} = J\delta X.$$

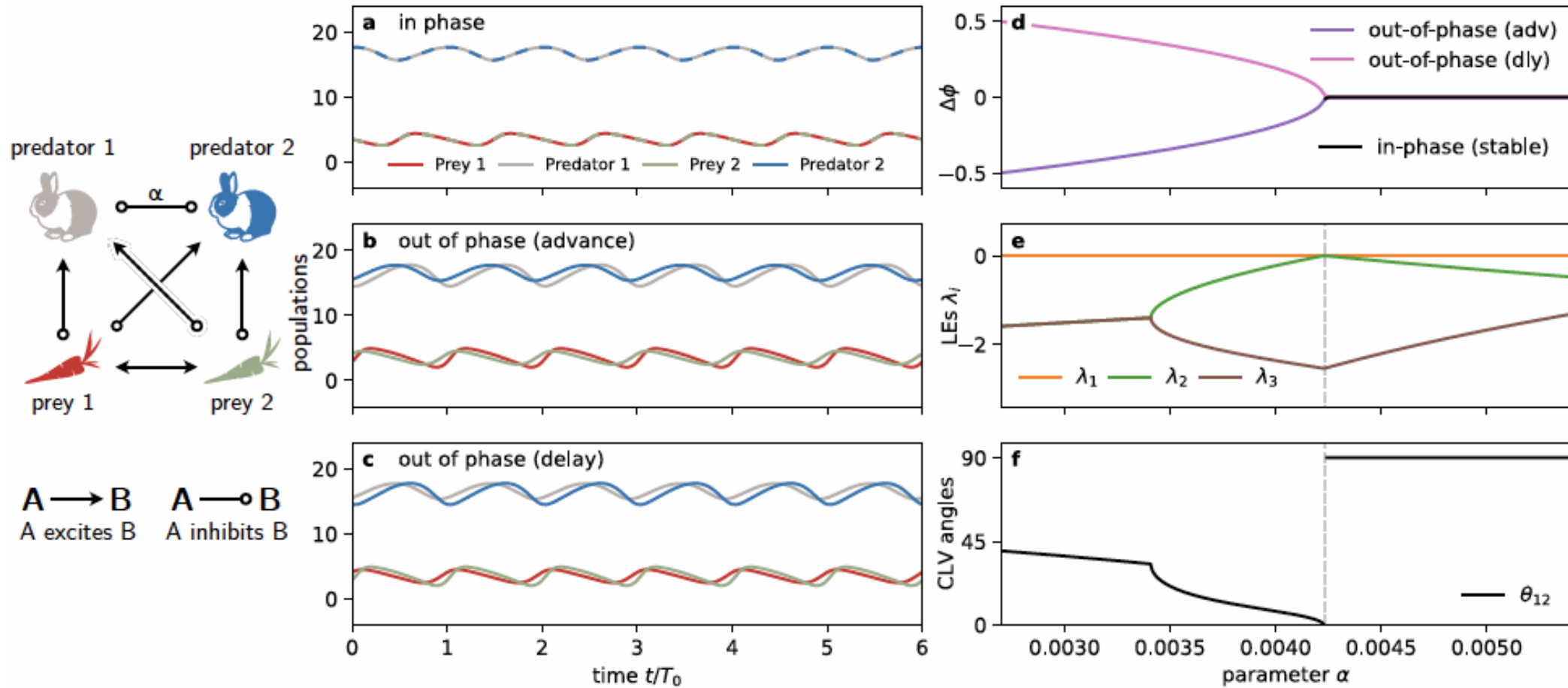
perturbations along the eigenvector c_i of J with eigenvalue $\lambda_i + i\omega$ grows or decays as $e^{\lambda_i t}$.

Merger of Lyapunov eigenvectors



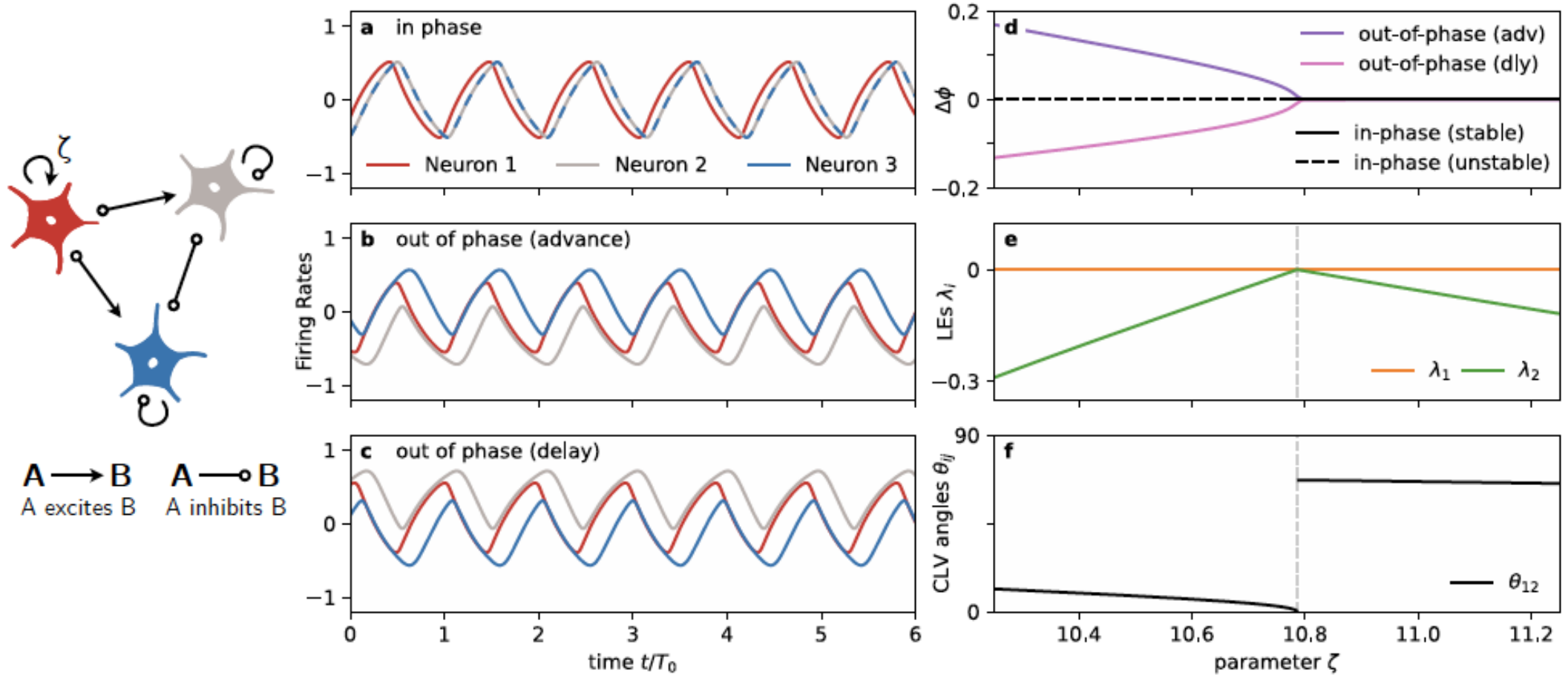
C Weis, et al., arXiv:2207.11667, 2022

Rosenzweig-McArthur predator-prey model



C Weis, et al., arXiv:2207.11667, 2022

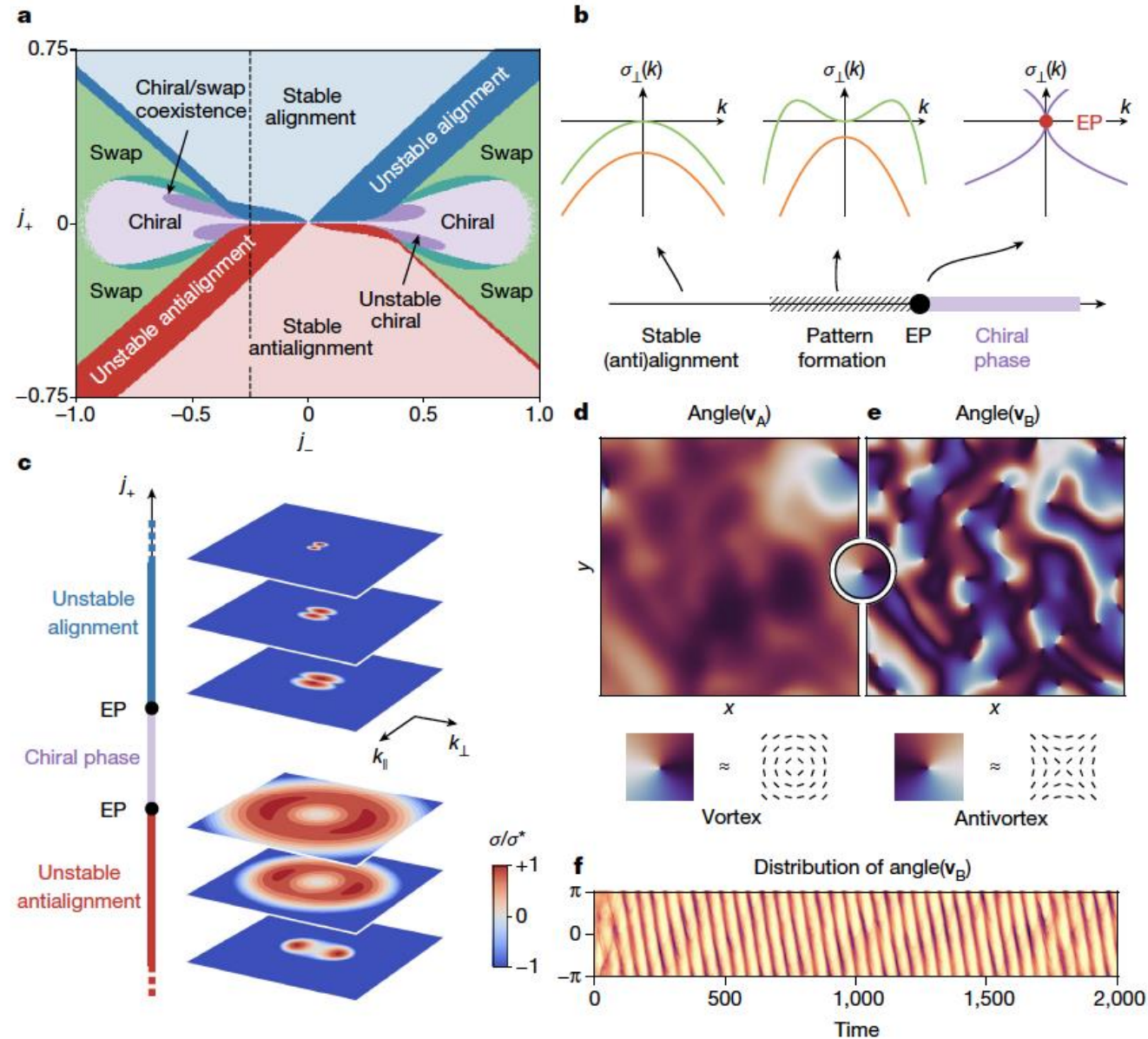
Wilson-Cowan model of excitatory/inhibitory neurons



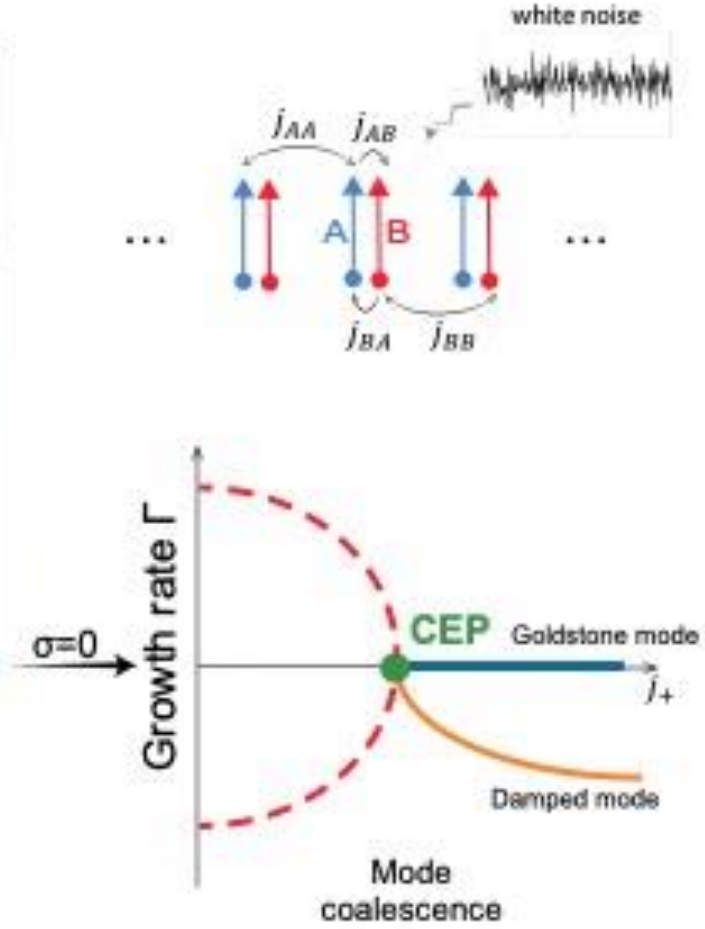
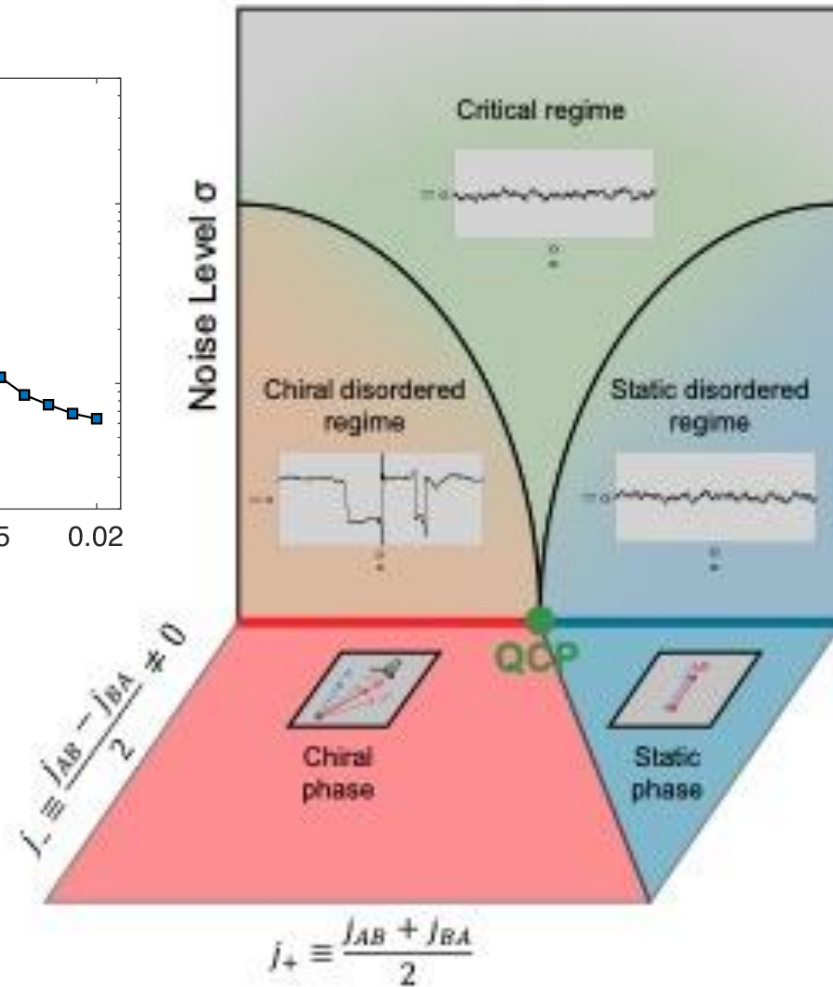
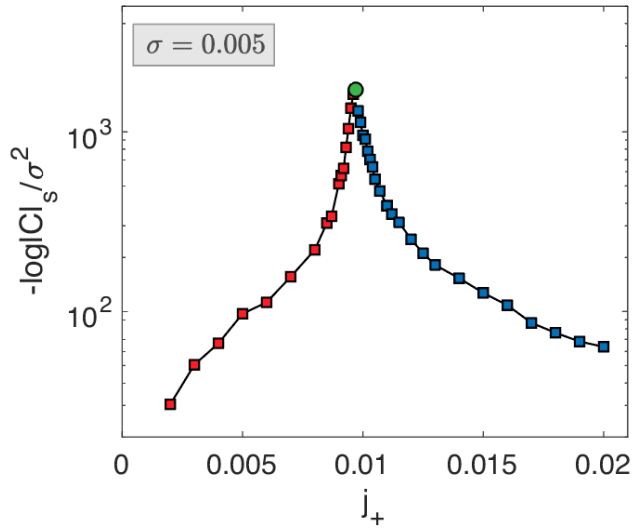
C Weis, et al., arXiv:2207.11667, 2022

Beyond mean field: dynamics, roughening, and pattern formation

Beyond mean field - pattern-forming instabilities



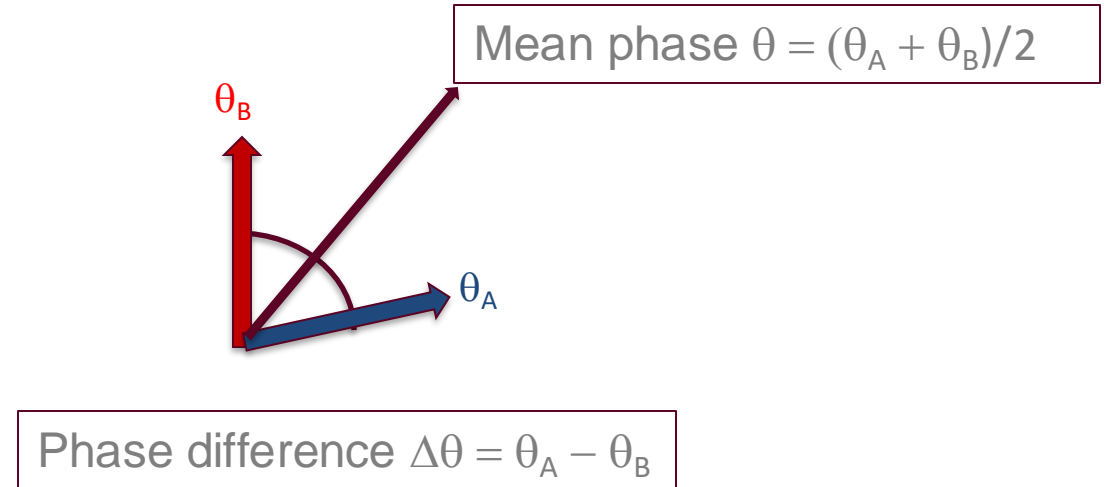
Adding noise : Analogy to a quantum critical point? One-dimensional non-reciprocal XY model



1D non-reciprocal XY model with noise: roughening transition

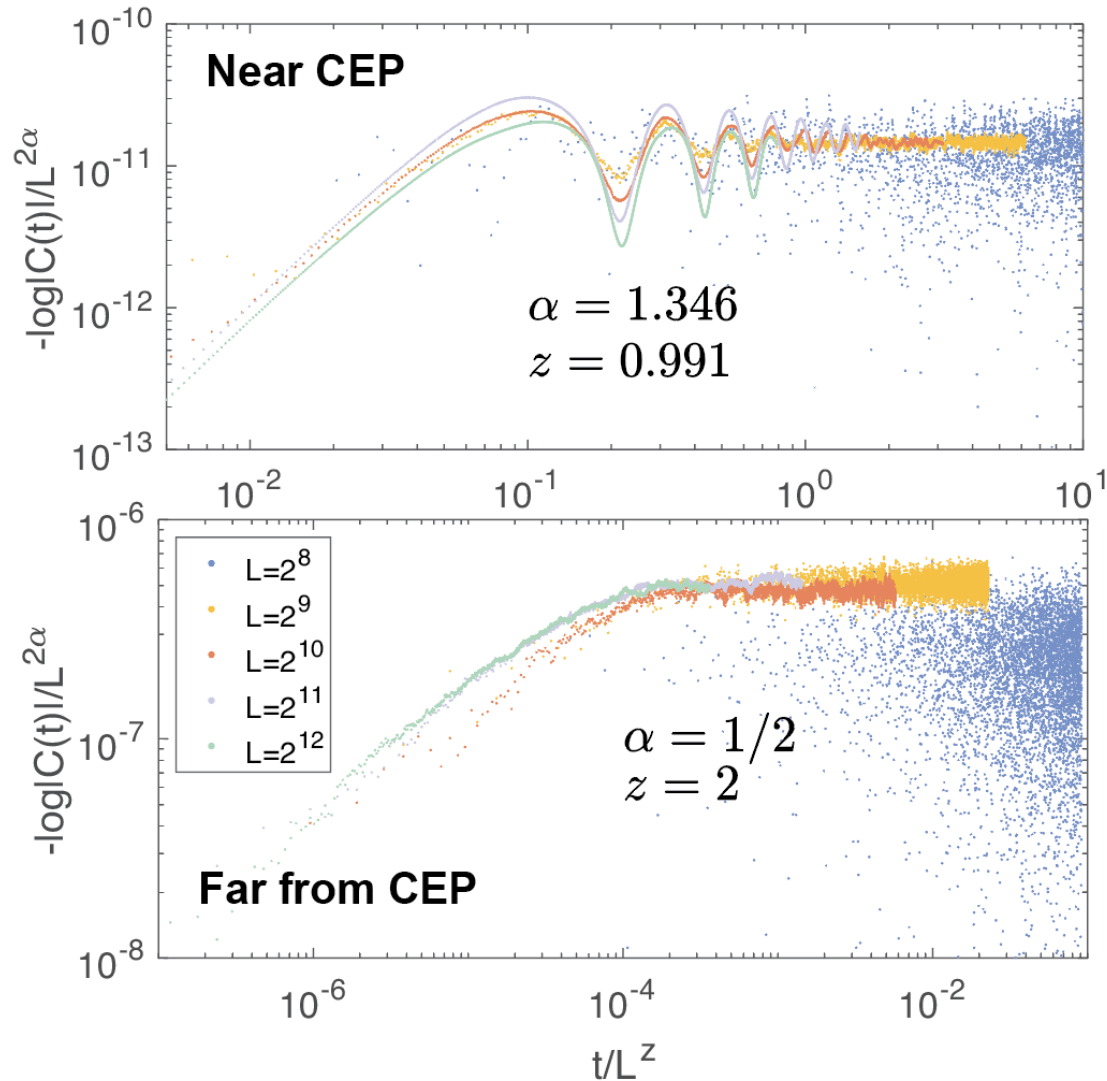
Model described by non-reciprocal spins
U(1) symmetry for average phase
 Z_2 (Ising like) for phase difference
The effect of noise should destroy the
ordered state(s) and create a roughening
transition.

[Note: Kardar-Parisi-Zhang term is
forbidden by symmetry]



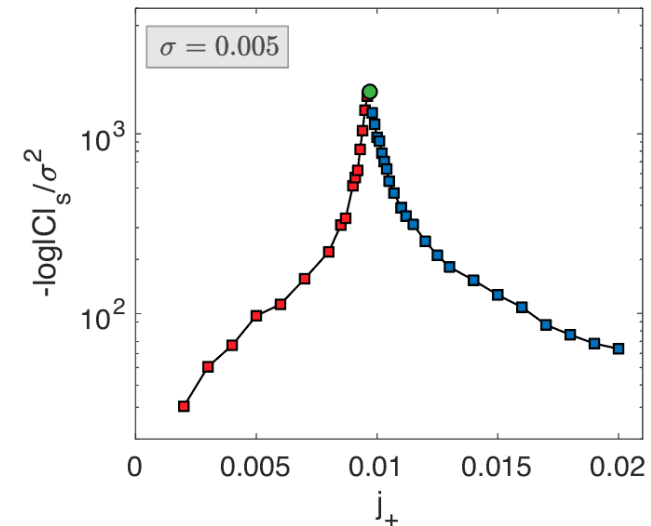
$$C(t, t + \Delta t) = \frac{\langle \overline{\Psi^*(t + \Delta t, x) \Psi(t, x)} \rangle}{\langle |\Psi(t)|^2 \rangle} \sim e^{-(\Delta t/\tau)^{2\beta}},$$

Finite size scaling near CEP vs. Far from CEP



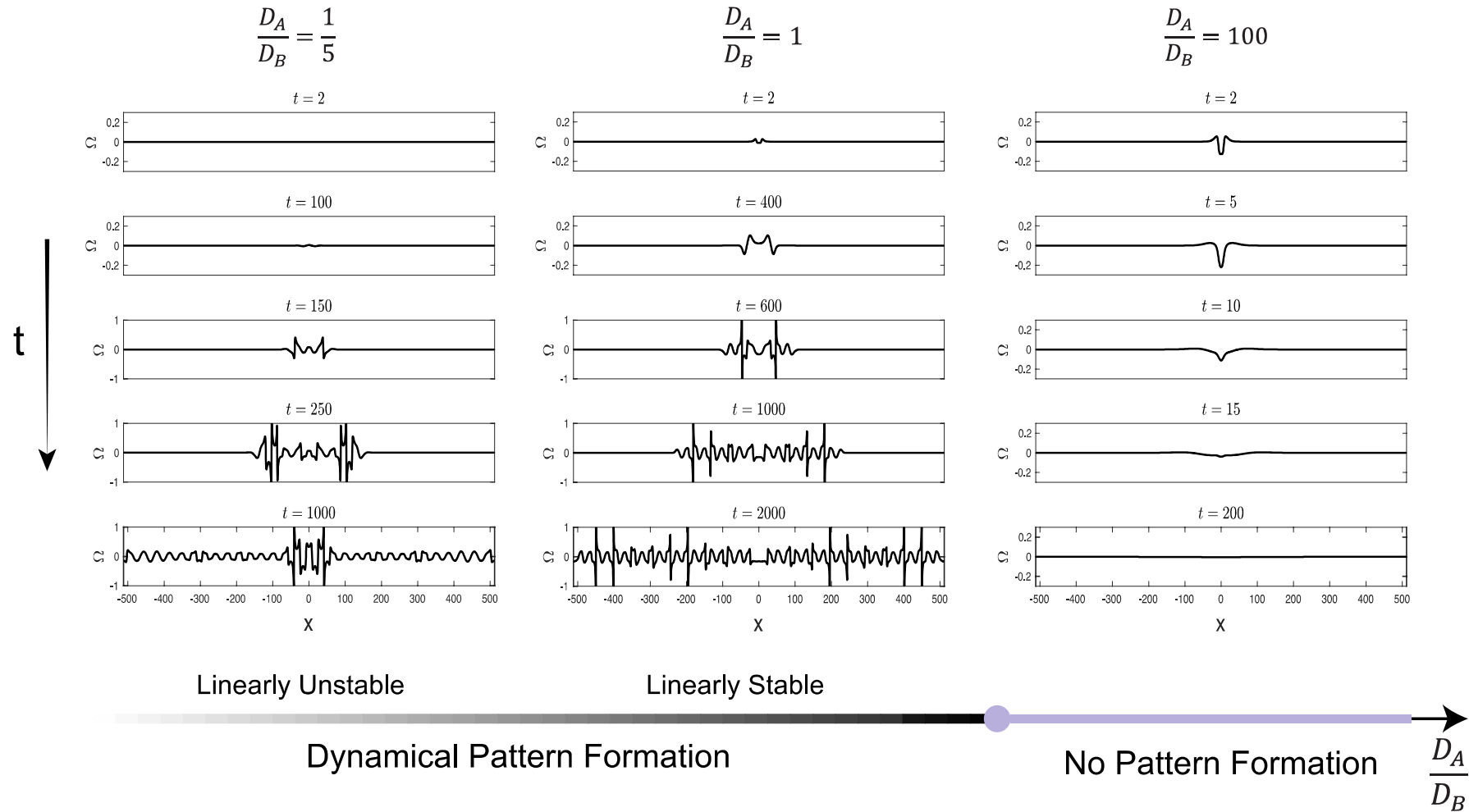
$$C(t, t + \Delta t) = \frac{\langle \overline{\Psi^*(t + \Delta t, x) \Psi(t, x)} \rangle}{\langle |\Psi(t)|^2 \rangle} \sim e^{-(\Delta t/\tau)^{2\beta}},$$

Enhanced fluctuations near CEP
Underdamped sound mode

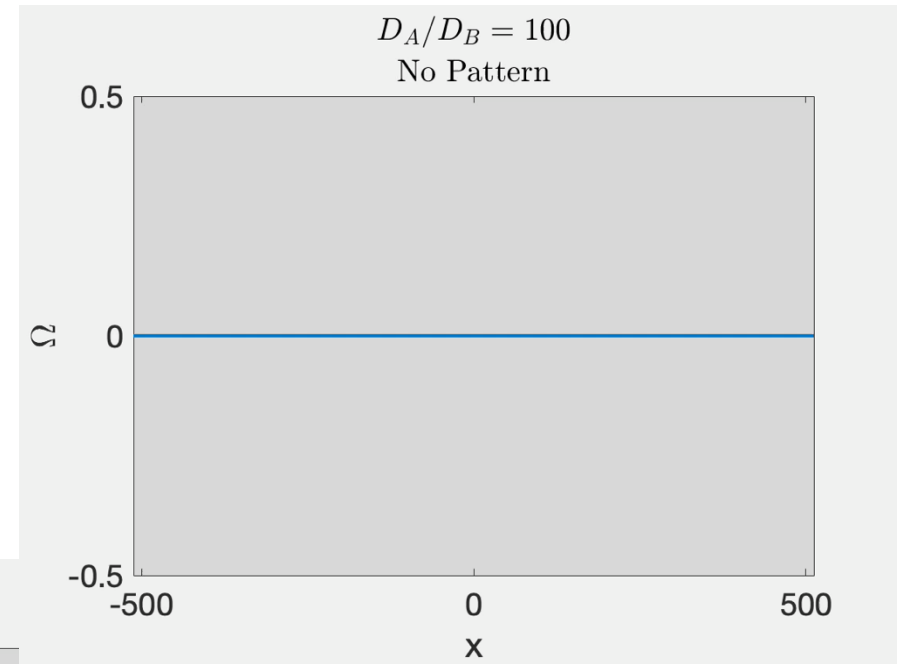
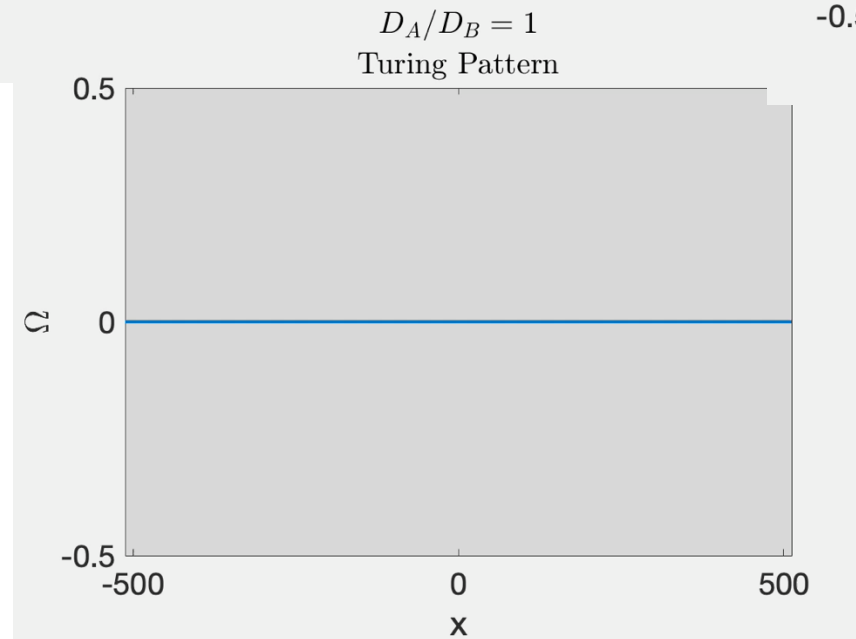
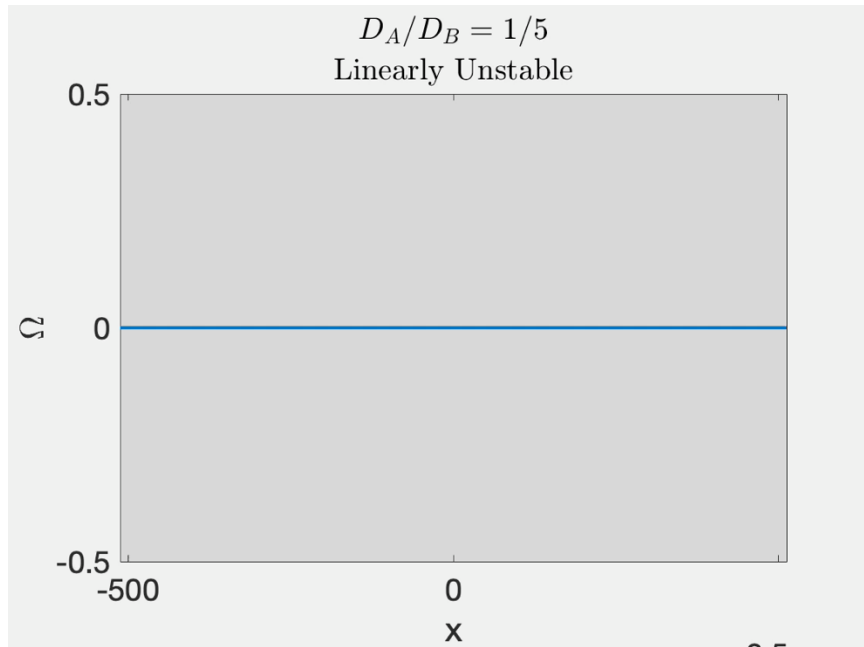


Crossover to Edwards-Wilkinson
scaling away from CEP

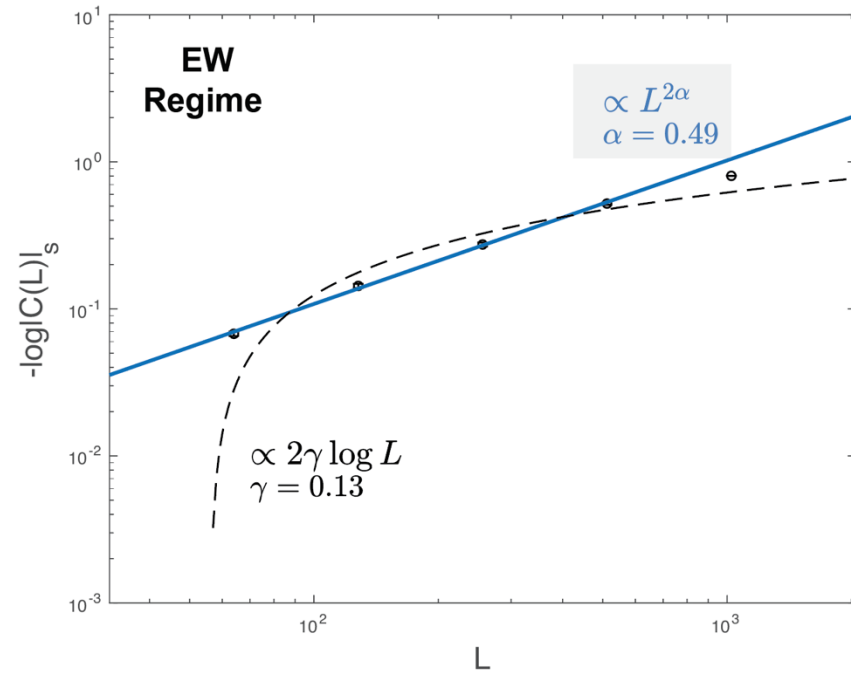
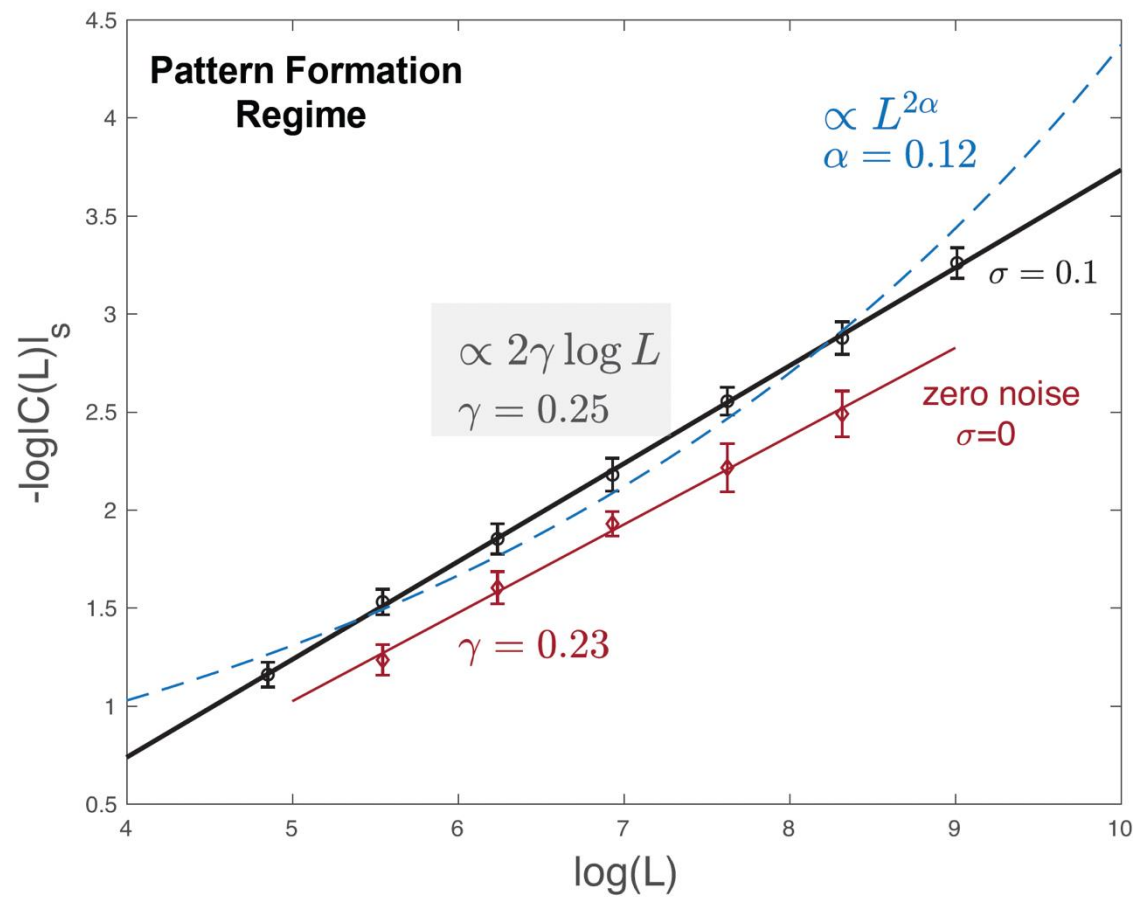
Diffusion-Driven Dynamical Pattern Formation - Turing-Hopf bifurcation



Diffusion-Driven Dynamical Pattern Formation - Turing-Hopf bifurcation



Deterministic Quasi-Long Range Order



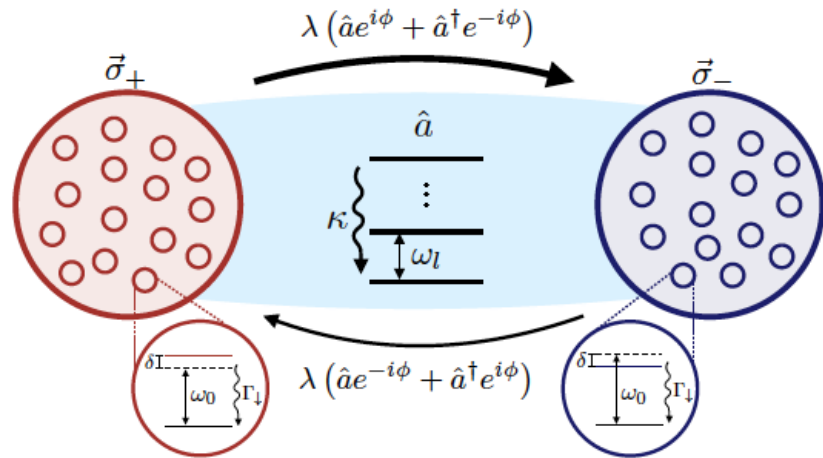
Can we construct “non-reciprocal quantum matter” ?

Some proposals

The non-reciprocal Dicke model

Ezequiel I. Rodríguez Chiacchio,¹ Andreas Nunnenkamp,^{2,*} and Matteo Brunelli^{3,†}

Physical Review Letters 131 (11), 113602

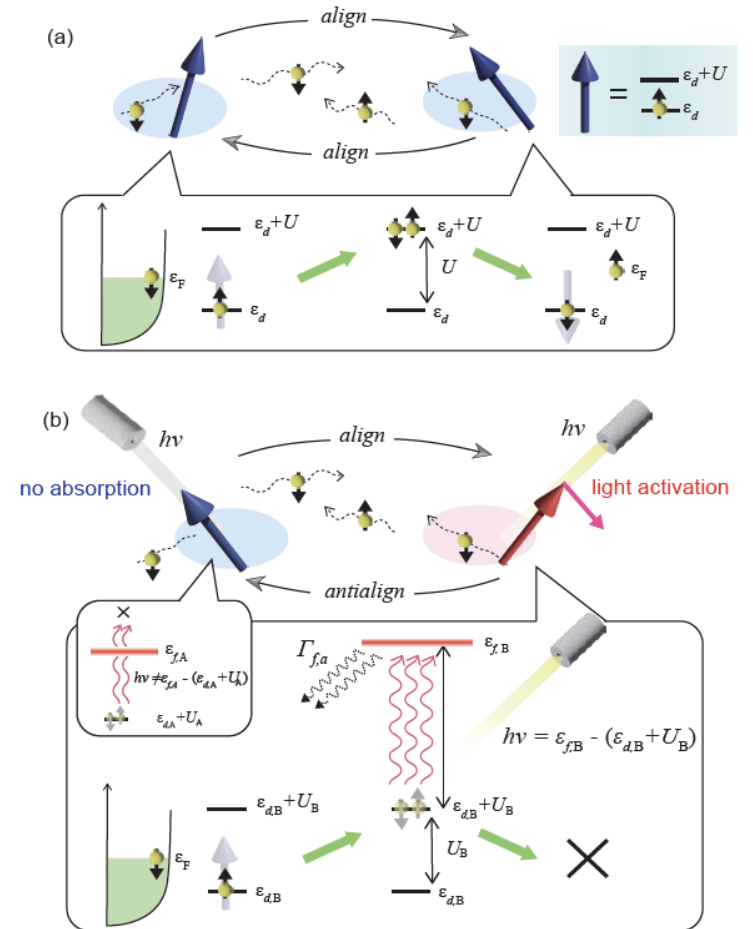


$$\hat{H} = \hat{H}_0 + \frac{\lambda}{2\sqrt{N}} \sum_{j=1}^N \sum_{m=\pm} (e^{-im\phi} \hat{a} + e^{im\phi} \hat{a}^\dagger) \hat{\sigma}_{j,m}^x$$

Photoinduced non-reciprocal magnetism

Ryo Hanai,^{1,*} Daiki Ootsuki,^{2,†} and Rina Tazai^{1,‡}

arXiv:2406.05957



$$\dot{S}_a = \sum_{b(\neq a)} J_{ab} S_a \times S_b - \alpha_a S_a \times \dot{S}_a - \gamma_a n S_a - \sum_{b(\neq a)} \Omega_{ab} S_b,$$

Conclusion

There are classes of dynamical many-body phase transition between different “stationary” states of non-Hermitian (or non-reciprocal) systems that are marked by Critical Exceptional Points where fluctuations merge with a Goldstone mode and become strongly enhanced

Nearby there are unusual states – “time crystals”

CEP’s in extended systems are the dynamical system analog of a critical end point in a thermodynamic system

“Universality classes” (if they exist) are different from thermodynamic phase transitions. No consensus yet on systems beyond mean field

Easy to make in biology ... possible in microscopic driven quantum systems?

The Team

Ryo Hanai



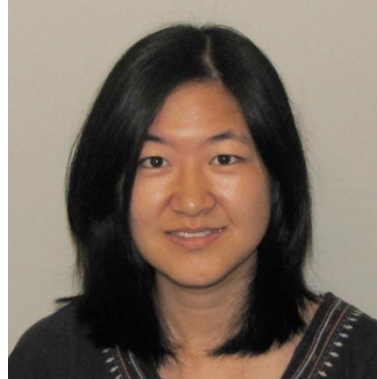
Michel Fruchart



Alex Edelman



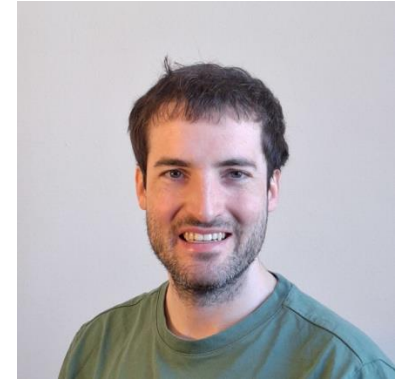
Shuoguang Liu



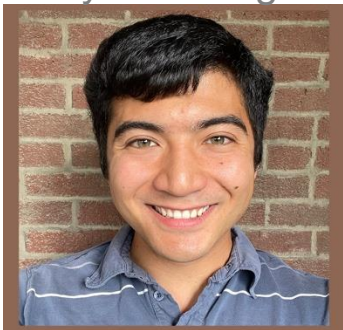
Vincenzo Vitelli



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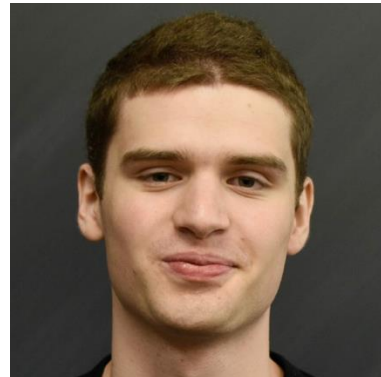
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