# Nonequilibrium Transport and Thermalization in Two-Dimensional Bad Conductors

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**NHMFL**
**(NSF and the State of Florida)**

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**Samples: IBM T. J. Watson Research Center, USA**

# Outline

**I. Introduction**

- **Far from equilibrium phenomena**
- **Quantum materials; interplay of disorder and Coulomb interaction**

  **⇒ Nonequilibrium dynamics**

  a) Glassiness

  b) Many-body localization (MBL)

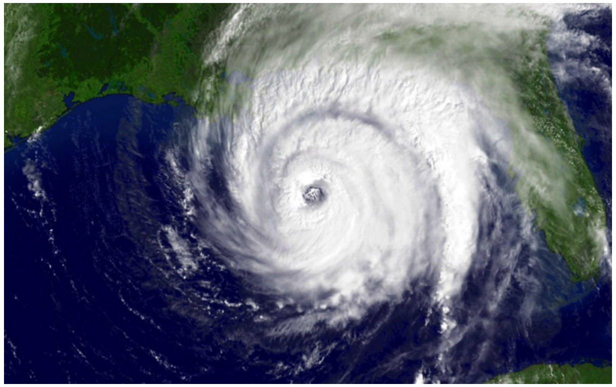
**II. Nonequilibrium dynamics in electronic systems**

- **Dynamics in a** strongly disordered 2D **electron system (2DES) in Si using time-resolved conductivity measurements**
- Long-range Coulomb **interaction: Glassy dynamics**
- Short-range Coulomb **interaction: MBL (prethermal) dynamics**

**III. Summary and outlook**

# Far-from-Equilibrium Physics

Galaxies



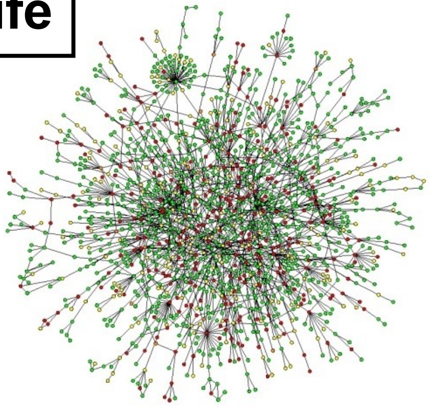
Hurricanes

Avalanches

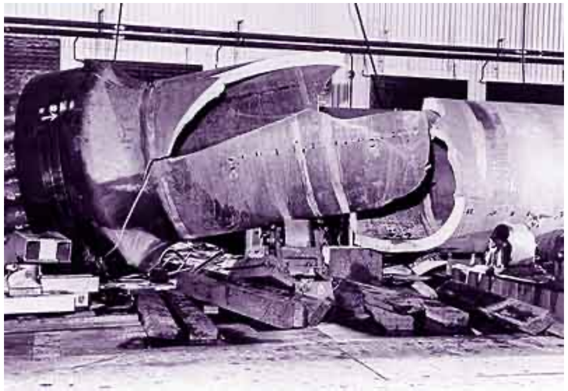
Growth of snowflakes

Schools of fish

**Life**

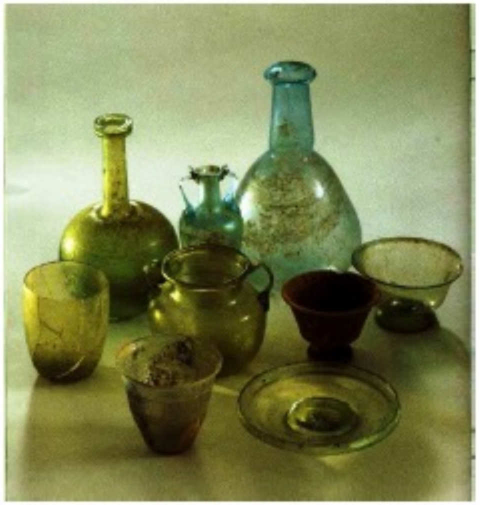


Map of interacting yeast proteins



Materials fatigue and fracture

# Far-from-Equilibrium Physics



Glasses

Plastics

Foams

Granular matter

Gels

Powders

Pills

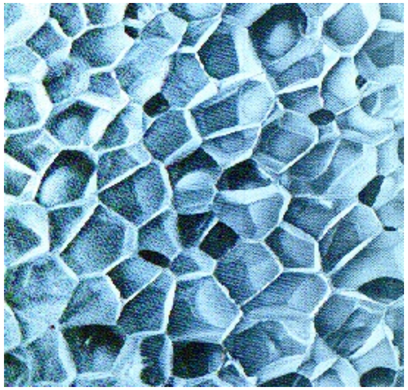
# Far-from-Equilibrium Behavior is Ubiquitous

## How do systems reach the far-from-equilibrium regime?

**Driven systems**
(*e.g.* by solar energy, electric fields, mechanical stresses)

**Systems trapped far from equilibrium**
(*e.g.* glasses, powders, foams, polymers)

Thermal energy supplied by the surroundings too small to allow them to equilibrate on experimental timescales

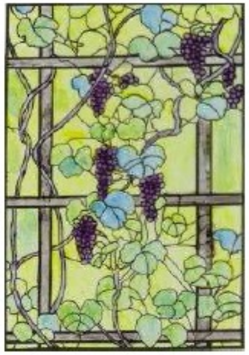


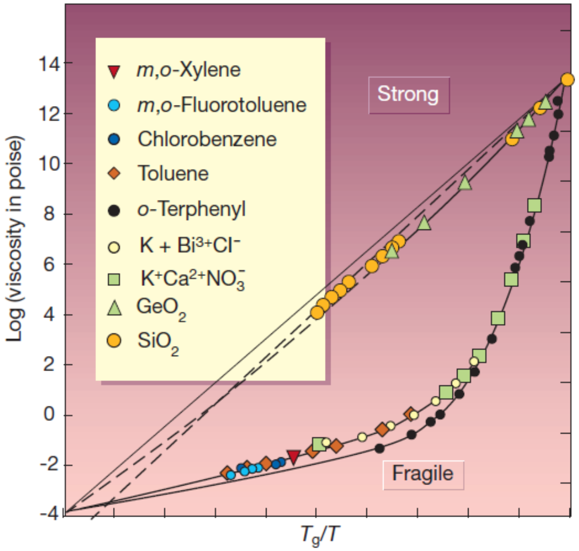
**Memory foam**

**They retain a memory of the preparation conditions**

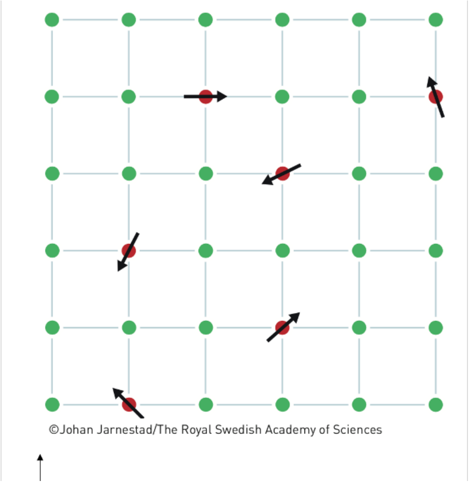
# Glasses

- **Structural glasses** (supercooled liquids)





- **Spin glasses** (e.g. Cu:Mn, Au:Fe)



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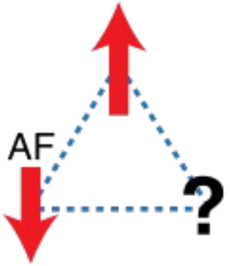
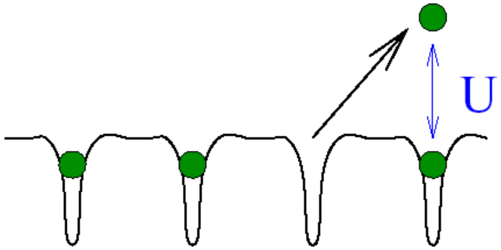
**Frustration**

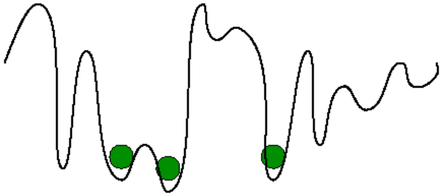


# Frustration in a low-density electron system with Coulomb interaction and disorder

- **Coulomb repulsion: keep electrons apart (uniform density)**

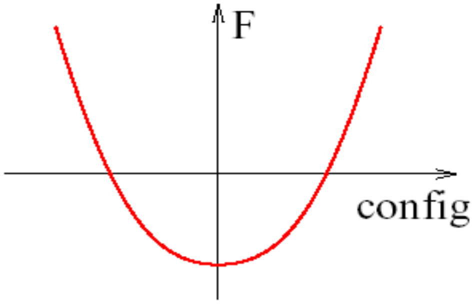


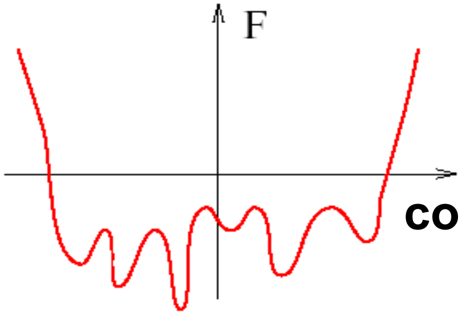
- **Random potential: nonuniform density**



**⇒ Frustration!**
**⇒ Emergence of (exponentially) many metastable states with similar (free) energy**

**Fluid**





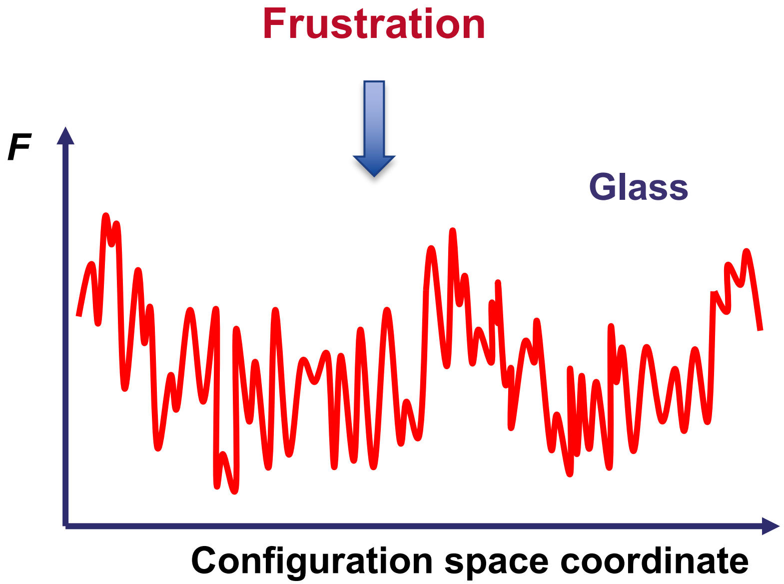
**Glass**

**configuration space**

**Coulomb glass**

**Experimental signature: slow, out-of-equilibrium dynamics**

# Glasses - unifying concept: “Rugged” (free) energy landscape



Valleys – metastable states

- **Breaking of ergodicity**
- **Robust with respect to coupling to an external bath**

**Giorgio Parisi**
**2021 Nobel Prize in Physics**

“His fundamental discoveries … not only influenced physics, but also mathematics, biology, neuroscience and machine learning, because all these fields include problems that are directly related to frustration.”
[Nobel Prize website]

# Thermalization in isolated quantum many-body systems

- When an **isolated** quantum system is prepared far **out of equilibrium**:

Does it **reach thermal equilibrium** at long times?

## Thermalizing

- Ergodic; system acts as a heat bath for its subsystems (exchange of energy and particles)
- Equilibrium statistical mechanics
- Quantum information lost with time

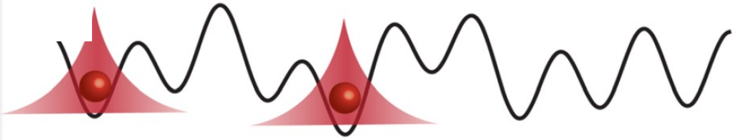
## Nonthermalizing

- Some memory of initial state retained at long times
- Integrable systems
- Many-body-localized systems
- Quantum many-body scar states
- …

[Reviews: Polkovnikov *et al*., Rev. Mod. Phys. 83, 863 (2011); Nandkishore & Huse, Annu. Rev. Condens. Matter Phys. 6, 15 (2015); Mori *et al*., J. Phys. B: At. Mol. Opt. Phys. 51, 112001 (2018); Abanin *et al*., Rev. Mod. Phys. 91, 021001 (2019); Gopalakrishnan & Parameswaran, Phys. Rep. 862, 1 (2020); … Sierant *et al*., arXiv:2403.07111 (2024)]

# Disorder + interactions: Can localization survive?

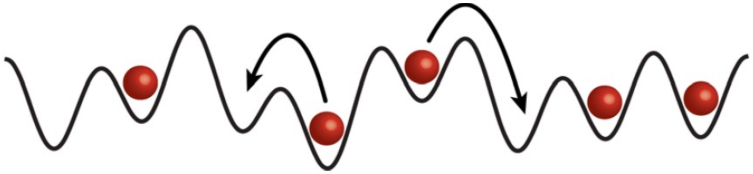
Anderson localization



Single particle

Conductivity $\sigma = 0$

**Many-body localization (MBL)**

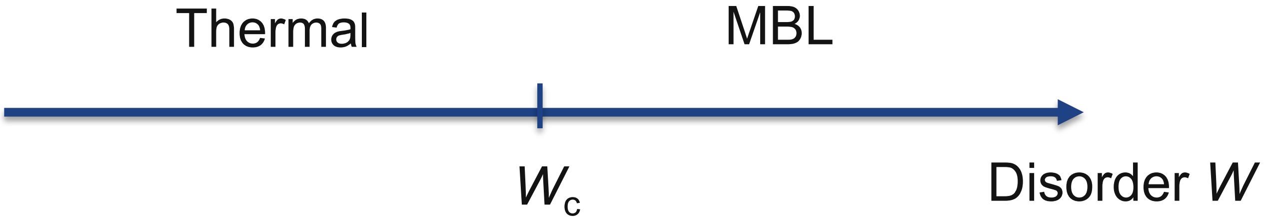


Conductivity $\sigma = 0$ at $0 < T < T_{\mathrm{cr}}$

[Figs. from Abanin *et al.*, Rev. Mod. Phys. **91**, 021001 (2019)]

- No particle transport and slow spreading of quantum information

**Dynamical phase diagram**


Thermal

MBL

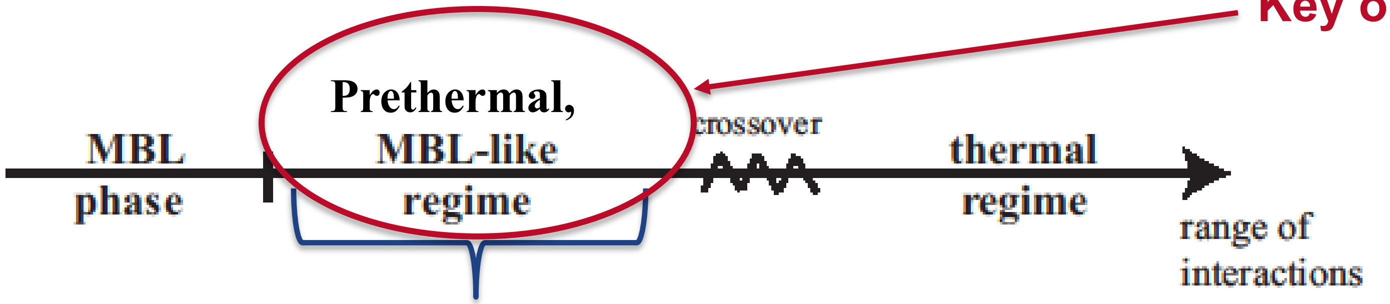
$W_c$

Disorder $W$

# Effect of the range of interactions on MBL? Role of dimensionality?

- MBL: strong disorder + short-range interactions
  ( + highly nonequilibrium conditions)
- MBL with longer-range interactions?
  Longer range interactions generally favor thermalization

**Key open question**



**Thermalizes as time *t* ⟶∞**

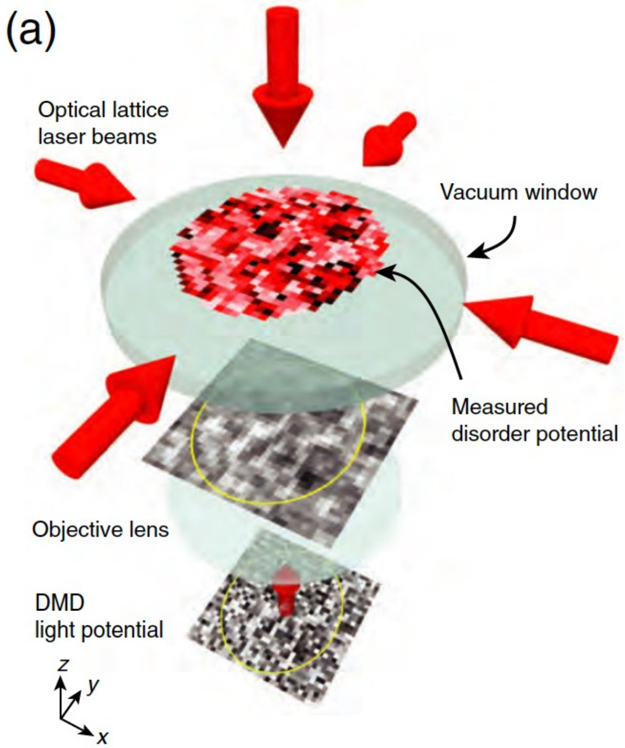
**and system size *L* ⟶∞**

[Gornyi *et al*., Phys. Rev. Lett. 95, 206603 (2005); Basko *et al*., Ann. Phys. 321, 1126 (2006); Nandkishore & Sondhi, Phys. Rev. X 7, 041021 (2017); Tikhonov & Mirlin, Phys. Rev. B 97, 214205 (2018); Gopalakrishnan & Huse, Phys. Rev. B 99, 134305 (2019); Sajna & Polkovnikov, Phys. Rev. A 102, 033338 (2020)…]

- MBL in dimensions D > 1? (In D=1?)
- MBL in mixed dimensionality systems?

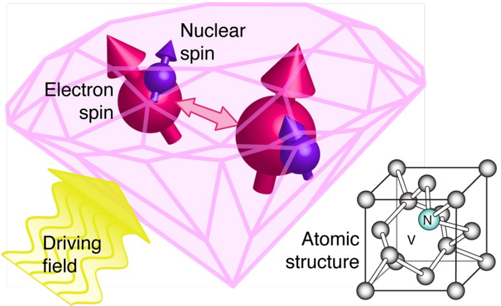
# MBL in synthetic many-body systems

- Ultracold atoms in optical lattices



[Choi *et al.*, Science 352, 1547 (2016)]

- Trapped ions
- Superconducting qubits
- Spins of NV centers in diamond

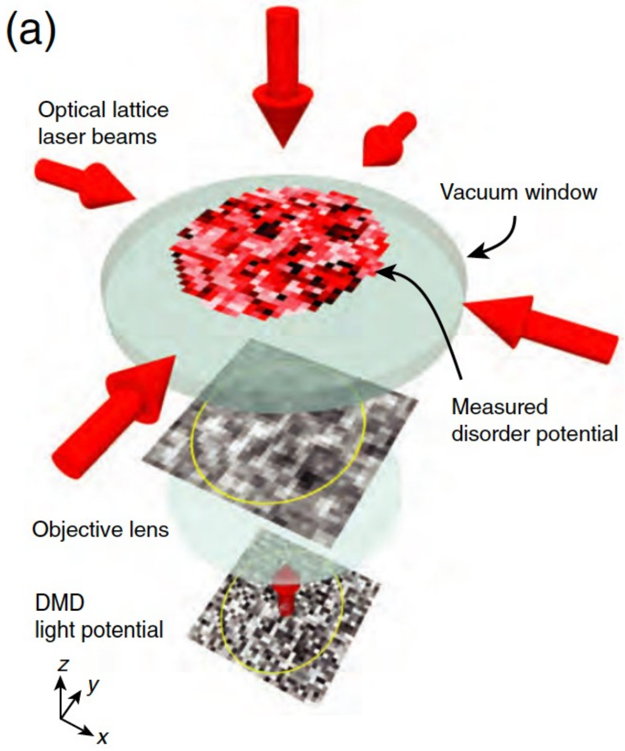


[Fig. from https://physics.aps.org/articles/v4/78]

# MBL in a disordered 2D bosonic optical lattice

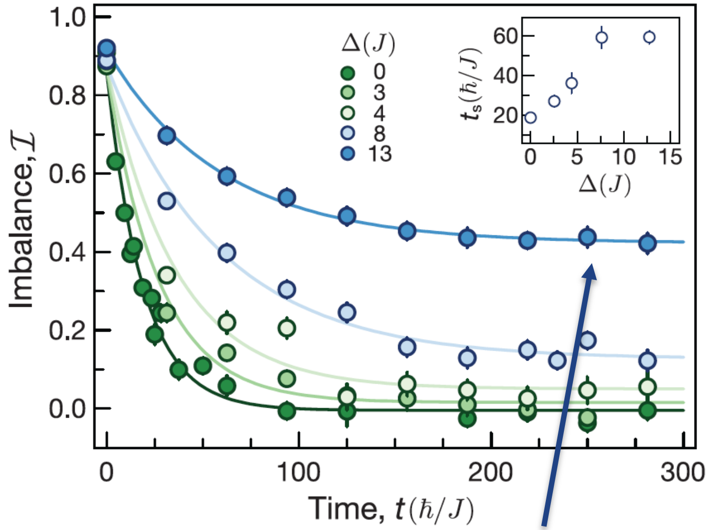
- Ultracold atoms in optical lattices

- On-site interaction
- Ground state in the absence of disorder: Mott insulator
- Track time evolution of the initial out-of-equilibrium state (density step)



[Choi *et al.*, Science 352, 1547 (2016)]

Density asymmetry (imbalance) vs time for different disorder strength **Δ**



Imbalance persists at long times for high enough disorder ⇒ **MBL**

Transition to MBL when all characteristic energy scales are comparable

# MBL in an open quantum system: Effect of thermal coupling to the environment

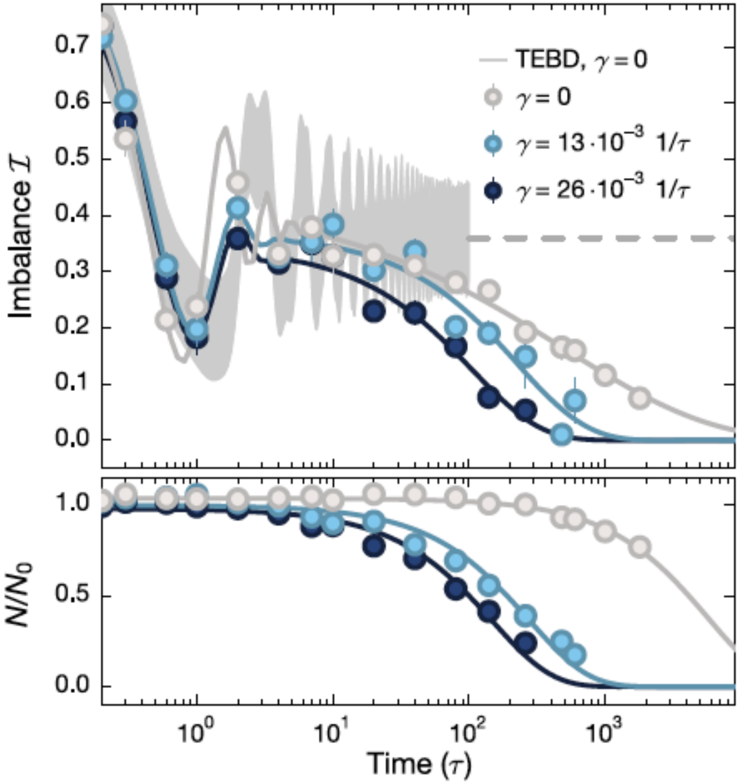
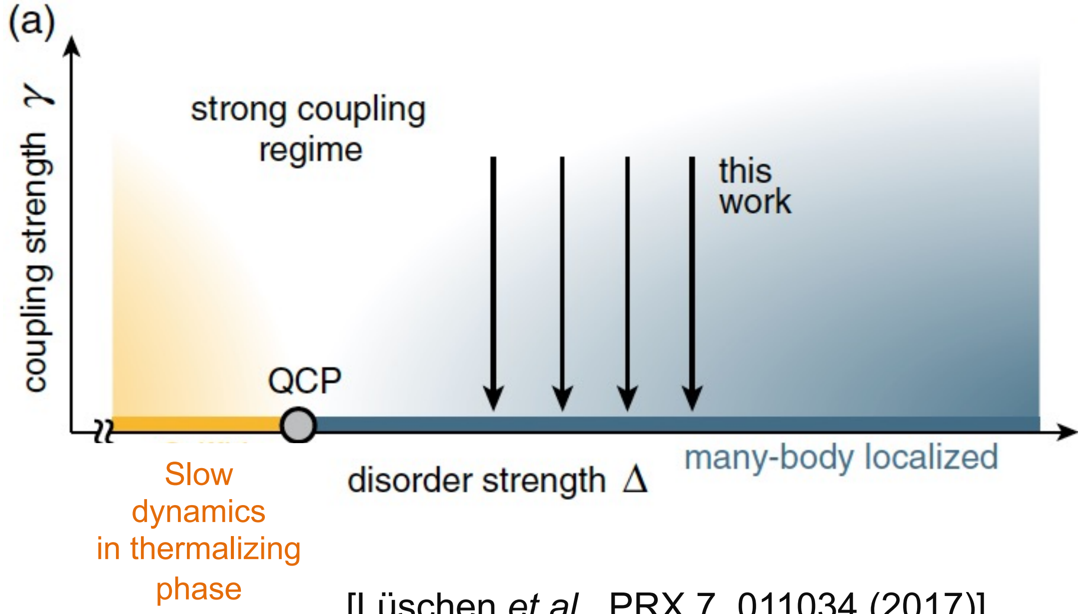


FIG. 2. Time evolution of a charge-density wave in the presence of photon scattering. Upper panel: An initially prepared one-dimensional charge-density wave evolves in the presence of quasiperiodic disorder of strength $\Delta = 4J$ at $U = 2J$ under the influence of varying scattering rates $\gamma$. Higher scattering rates result in shorter lifetimes of the imbalance. The finite imbalance lifetime at $\gamma = 0$ is due to residual couplings between different 1D tubes [14] and off-resonant scattering of lattice photons,

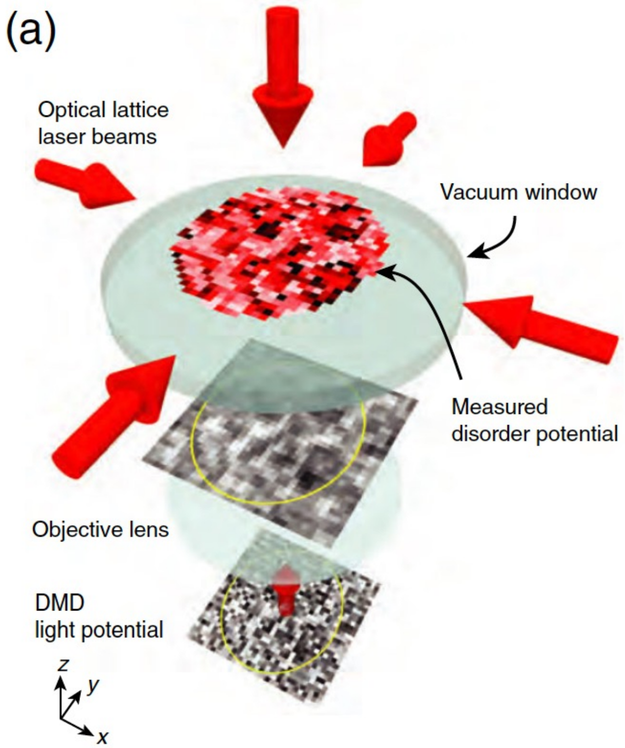


[Lüschen *et al.*, PRX 7, 011034 (2017)]

- Thermalization time inversely proportional to the coupling strength of the bath
- **Sensitivity to coupling to a thermal bath: key experimental signature of MBL, in contrast to glassy systems**

# MBL in synthetic many-body systems

- Ultracold atoms in optical lattices



[Choi *et al.*, Science 352, 1547 (2016)]

- Trapped ions
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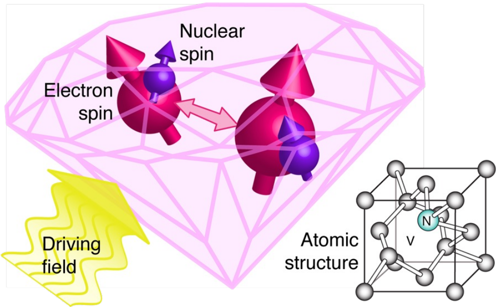


[Fig. from https://physics.aps.org/articles/v4/78]

- Systems remain isolated only up to some time (time scales: ~ $10^{-3}$ s)
  ⇒ Thermalization
- Finite-size effects

# MBL in real, solid-state materials?

## Finite-size effects

- Electronic systems: larger system sizes

  ⇒ Much closer to the thermodynamic limit

- Power-law interactions ($\sim 1/r^{\alpha}$) of special interest;

  **$D < \alpha < 2D$ case has been under debate**

## Finite coupling to a bath

- Electronic systems: generally strong coupling to phonons ⇒ MBL lost

  **Sensitivity to coupling to a thermal bath: key experimental signature of MBL, in contrast to glassy systems**

- Some coupling to an external bath is unavoidable in any system ⇒

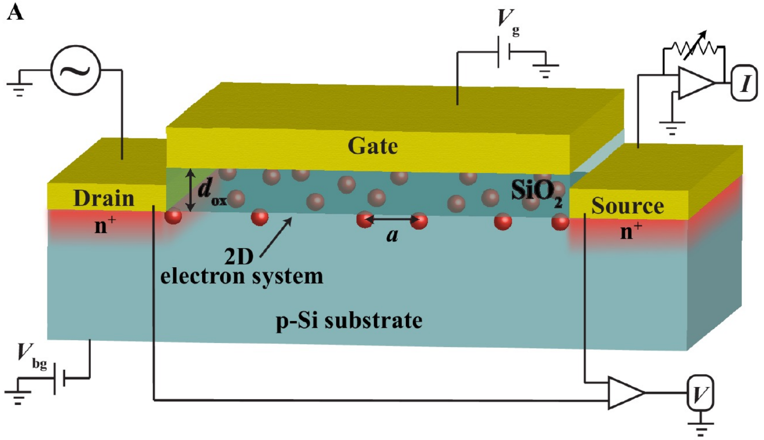
  Comparison to theory can be done only in a prethermal regime

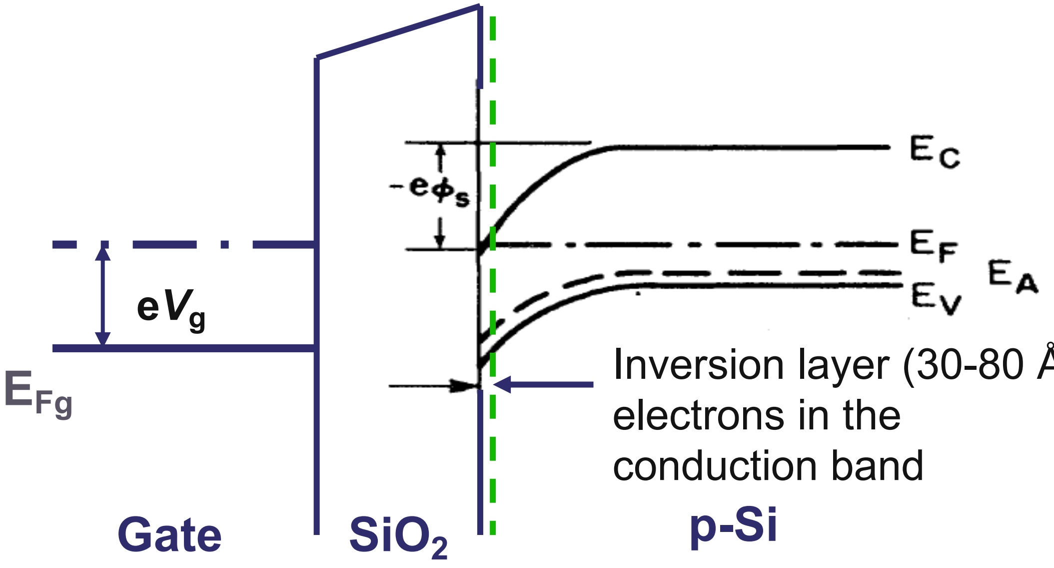
# 2D electron system in Si metal-oxide-semiconductor field-effect transistors

Si MOSFET:

- **Basic building block of modern electronics:** well-developed, mature technology (device aspects well-understood, good contacts even at low *T*…)
- Discovery of 2D behavior of electrons in 1966!
  [Fowler, Fang, Howard, Stiles, PRL 16, 901 (1966)]

**MOSFET** (metal-oxide-semiconductor field-effect transistor) – **capacitor!**





Inversion layer (30-80 Å) - electrons in the conduction band

**Electric field effect: conductivity $\sigma(V_g)$**

Vary total density $n_s$ using $V_g$

$n_s = C_{ox}(V_g - V_T)/e$

# 2D electron system in Si MOSFETs: Excellent model system to study charge dynamics

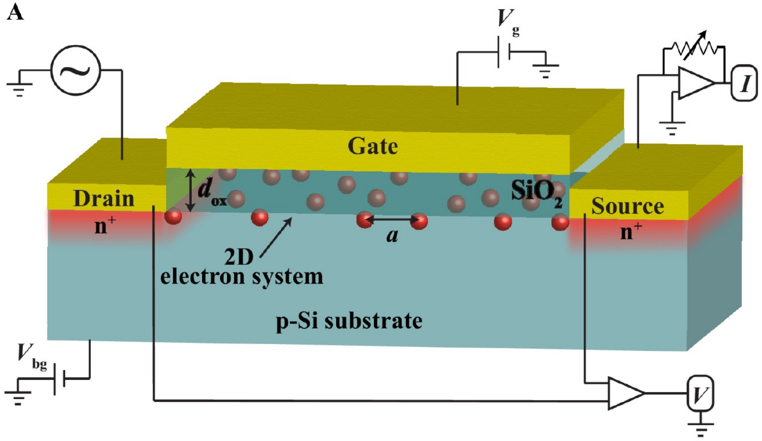
Si MOSFET:

- Electron **density $n_s$ varied easily** up to three orders of magnitude by changing gate voltage $V_g$

⇒ a) Can be prepared far out of equilibrium

b) Can study nonequilibrium dynamics across the quantum metal-insulator transition (MIT)

**MOSFET** (metal-oxide-semiconductor field-effect transistor) – **capacitor**!



- **Total density** in a 2DES changes quickly, within the device time constant $\tau = RC$ (~5 ns at most, for our devices); instantaneously within our expt. resolution

Perform **quantum quench**

# 2D electron system in Si MOSFETs: An excellent candidate for observing MBL

Si MOSFET:

- Electron **density $n_s$ varied easily** up to three orders of magnitude by changing gate voltage $V_g$

⇒ a) Can be prepared far out of equilibrium

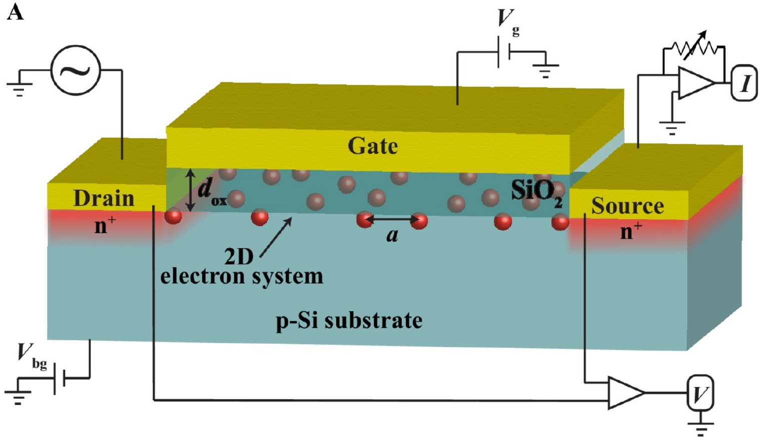
b) Can study nonequilibrium dynamics across the quantum metal-insulator transition (MIT)

**- Weak electron-phonon coupling at low *T***
($T \lesssim 1.6$ K for our samples in the regime of interest)

⇒ Heat transfer by **electron diffusion through contacts** and leads

Coupling further reduced by placing samples and leads in **vacuum**

**MOSFET** (metal-oxide-semiconductor field-effect transistor) – **capacitor**!

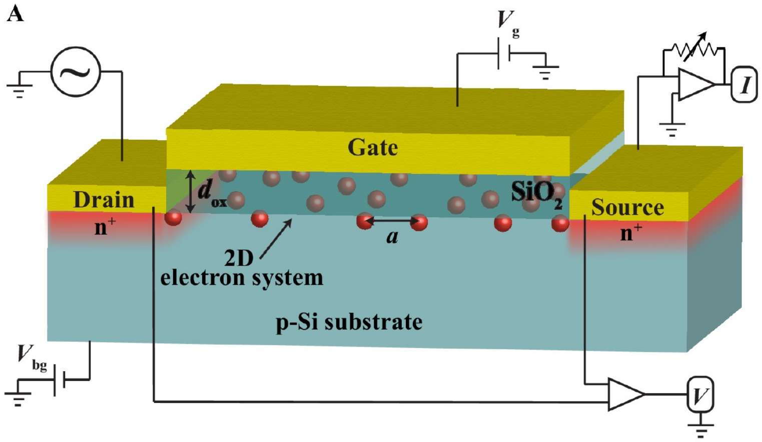


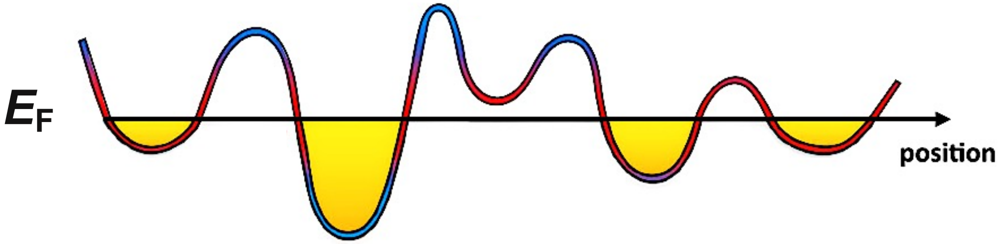
Si: $T_{\text{Debye}} = 645$ K

[Zieve *et al.*, Phys. Rev. B 57, 2443 (1998); Altshuler *et al.*, Physica E 9, 209 (2001)]

# Disorder and interactions in a 2DES in Si MOSFETs

- **Disorder:** Smooth random potential due to **Na$^+$ ions in SiO$_2$** (frozen below ~150-200 K) – spatially separated from the 2DES



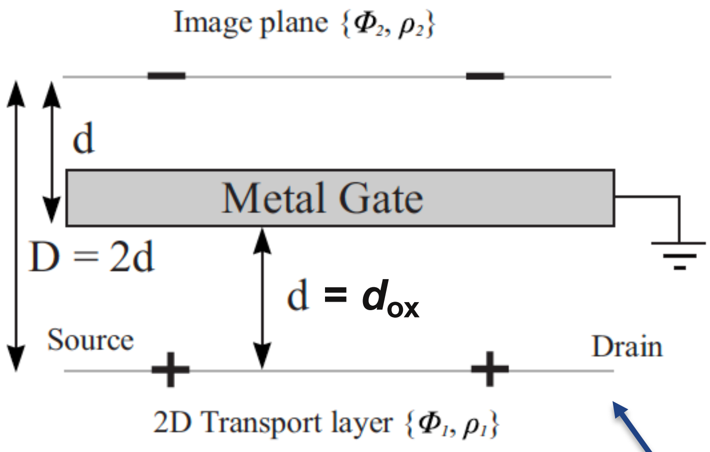


- **Coulomb interaction ∝ 1/*r***

# 2D electron system in Si MOSFETs: Varying the range of the Coulomb interaction

Si MOSFET:

- **Screening the Coulomb interaction** within the 2DES by reducing the oxide thickness $d_{ox}$ (distance from the gate)



[Widom & Tao, PRB 38, 10787 (1988); Ho et al., PRB 80, 155412 (2009); Skinner & Shklovskii, PRB 82, 155111 (2010); Skinner & Fogler, Phys. Rev. B 82, 201306(R) (2010); Fregoso & Sá de Melo, PRB 87, 125109 (2013)]

**Coulomb interaction in the presence of the gate**

$$\sim\left|1/r - \underbrace{1/(r^2 + 4d_{ox}^2)^{1/2}}_{\text{Coulomb interaction of an electron with an image charge of another electron in a 2DES}}\right|$$

At large distances $r \gg 2d_{ox}$, **interaction ~$1/r^3$**

Realized at low enough densities, such that the mean electron separation

$$2a = 2(\pi n_s)^{-1/2} \gg 2d_{ox}$$

# 2D electron system (2DES) in Si: Our samples

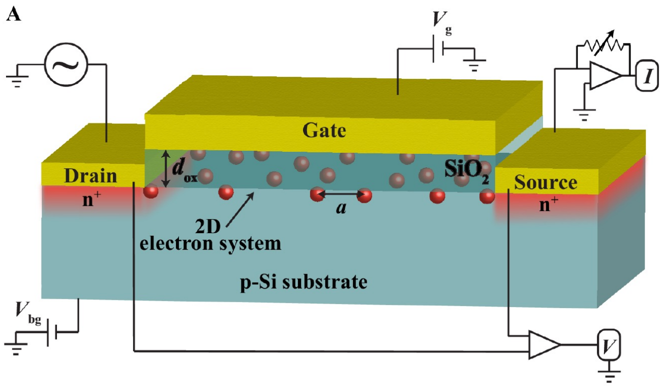
- We focus on the case of **strong disorder** (smooth random potential due to $Na^+$ ions in the oxide; frozen below ~150-200 K); $\mu_{\text{peak}} \approx 0.05 - 0.06$ $m^2/Vs$

**Long-range Coulomb interaction**

- **Thick oxide:**
  - **$d_{ox}$ = 50 nm**
  - $5.3 \lesssim d_{ox}/a \leq 8.0$
- Coulomb interaction:

$$\sim 1/r$$

Mean carrier separation: $2a = (\pi n_s)^{-1/2}$



The two sets of devices are otherwise identical

**Screened Coulomb interaction ("short-range")**

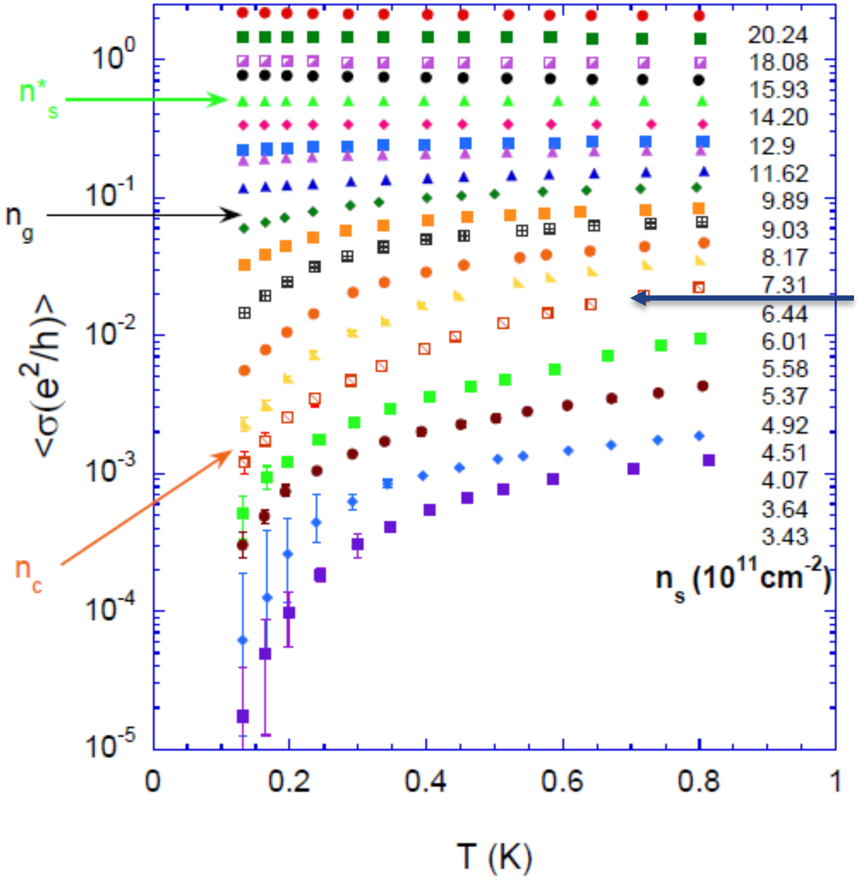
- **Thin oxide:**
  - **$d_{ox}$ = 6.9 nm**
  - $0.7 \lesssim d_{ox}/a \lesssim 1.5$
- For $\frac{d_{ox}}{a} \ll 1$, screened Coulomb interaction:

$$\sim 1/r^3$$

Samples: IBM T. J. Watson Research Center, USA

# Equilibrium transport in a strongly disordered 2DES: Temperature dependence of the conductivity

## Long-range Coulomb interaction



[S. Bogdanovich & D. Popović, Phys. Rev. Lett. 88, 236401 (2002)]

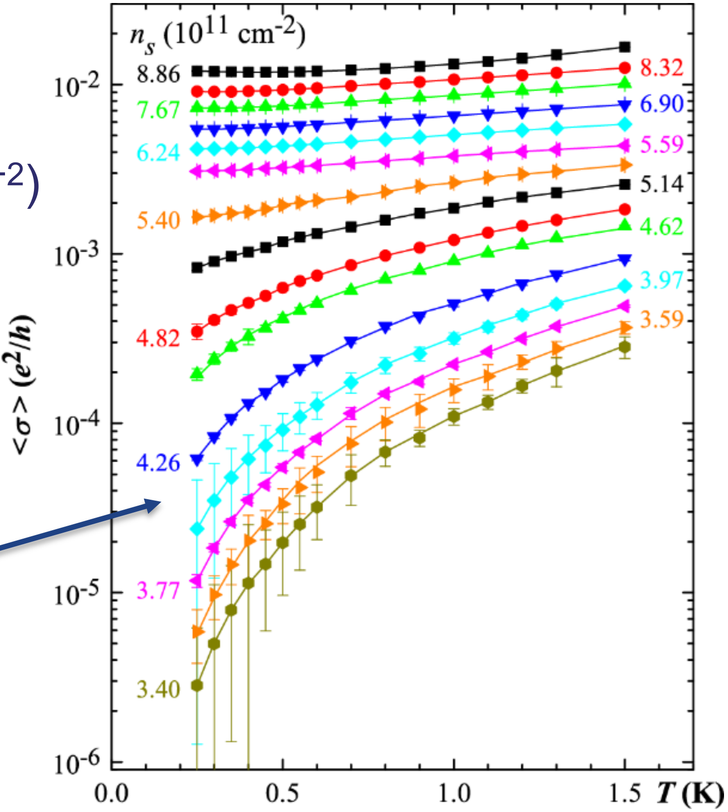
2D MIT

$n_c\ (10^{11}\ cm^{-2})$

(5.0 ± 0.3)

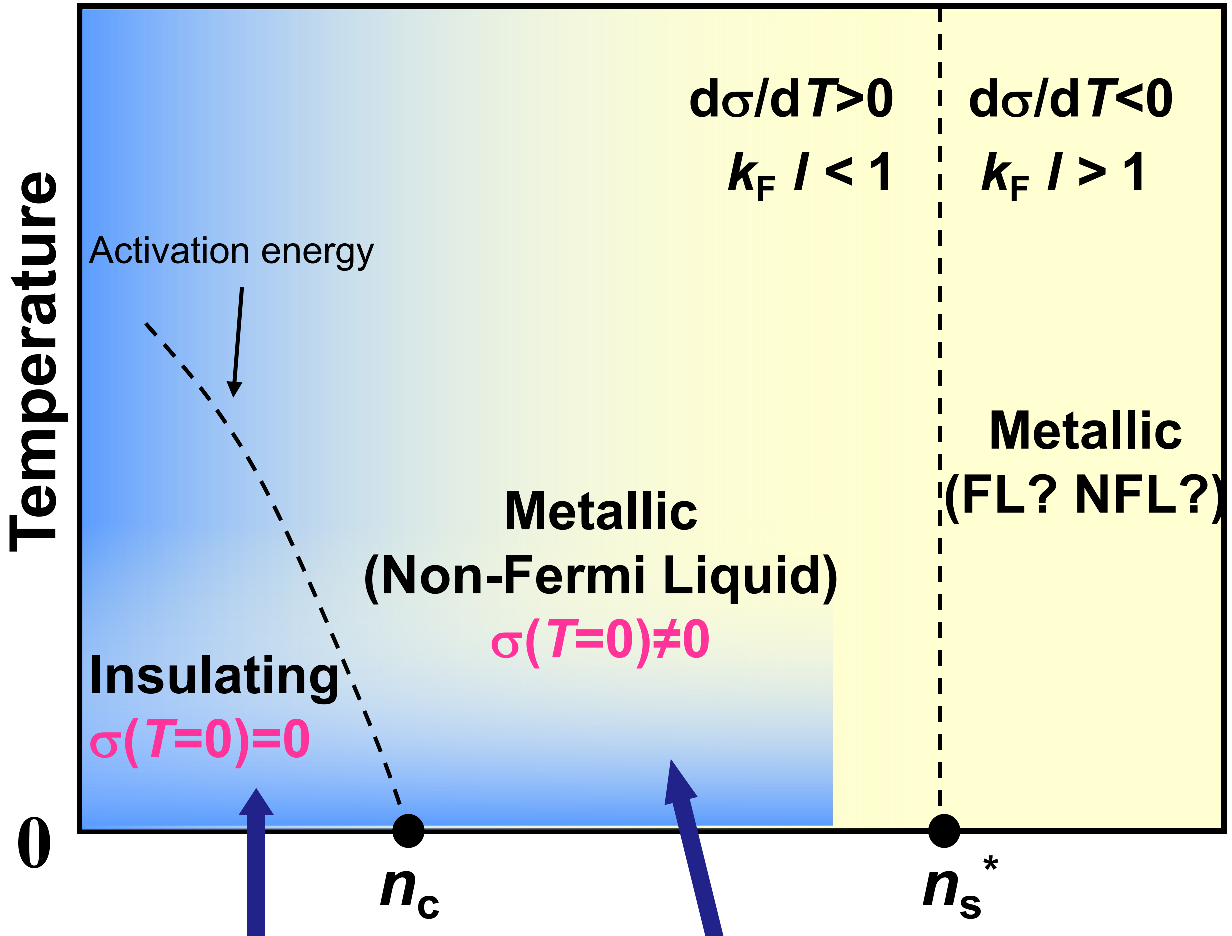
(4.2 ± 0.2)

## Screened ("short-range") Coulomb interaction



[P. V. Lin & D. Popović, Phys. Rev. Lett. 114, 166401 (2015)]

# Phase diagram of a strongly disordered 2DES in Si: Equilibrium transport



Exponential localization

"Bad" metal: $\sigma(n_s,T)=\sigma(n_s,T=0)+b(n_s)T^{3/2}$

**Reviews** in book chapters:
D. P. in
a) "Conductor-Insulator Quantum Phase Transitions", edited by Dobrosavljević, Trivedi, Valles (Oxford Univ. Press, 2012)
b) "Strongly Correlated Electrons in Two Dimensions", edited by Kravchenko (Pan Stanford Publishing, 2017)

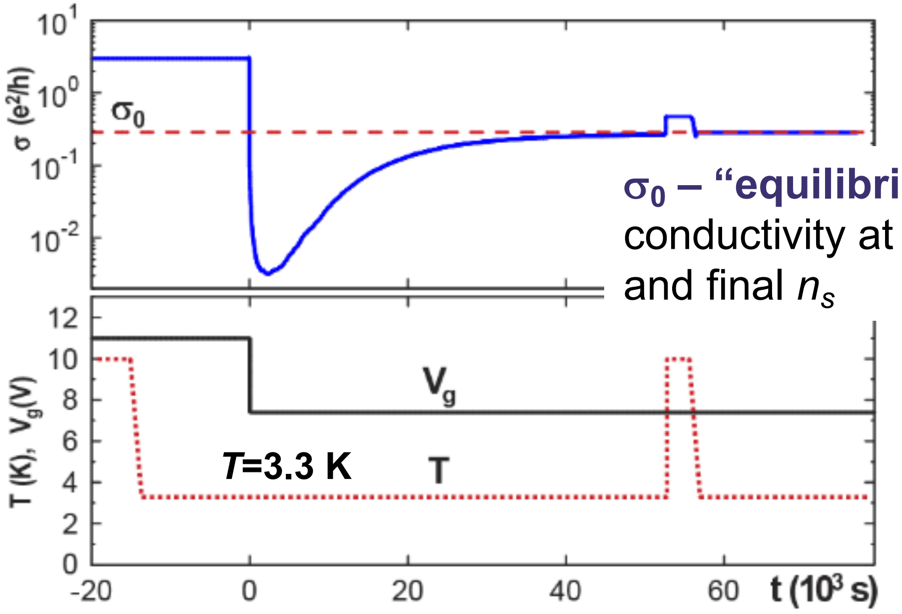
False color

# Quench dynamics in a disordered 2DES: Relaxations of $\sigma$ after a large, rapid change of $n_s$

## Experimental protocol

- Sample is warmed up to ~7-20 K to "reset" the sample
- At ~7-20 K, the gate voltage is set to some $V_g^i$ where $k_F l \geq 1$
- The sample is **cooled to the measurement *T*** and allowed to equilibrate ( $\Leftrightarrow$ $\sigma$ saturates)
- Then *V*g, i.e. ***n*$_s$ is quickly (<3 s) changed to a lower, final value**
- $\sigma$ is measured continuously throughout the process

**Long-range case (~1/*r* )**



**$\sigma_0$ – "equilibrium"** conductivity at *T* and final $n_s$

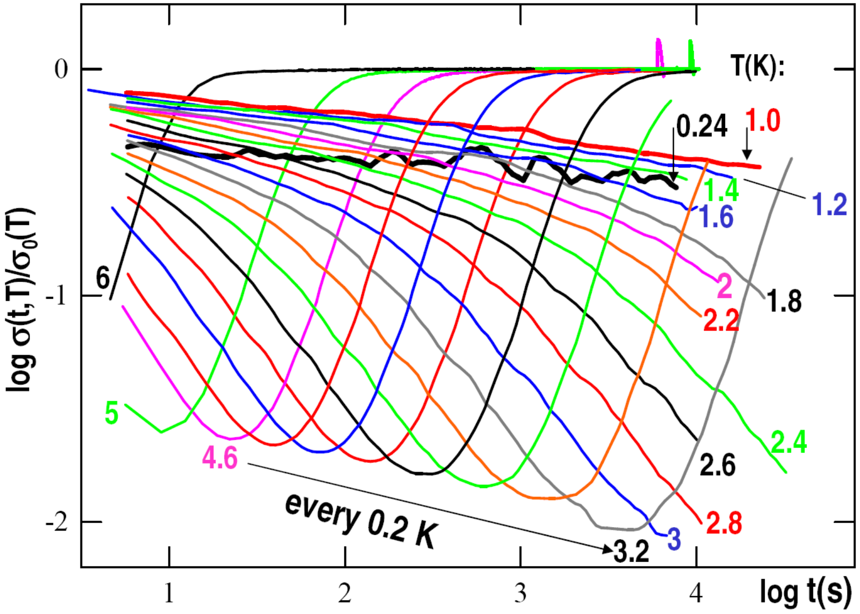
**Time**

Initial $n_s(10^{11}\text{cm}^{-2})$ =20.26 ; $k_F l$ ~1

Final $n_s(10^{11}\text{cm}^{-2})$ = 4.74 $\geq n_c$

- Large perturbation: **Δ** $E_F$ ~ $E_F$
  ($k_B T \ll E_F$)

# Quench dynamics in a high-disorder 2DES: Relaxations of $\sigma$ after a large, rapid change of $n_s$



- Overshooting of equilibrium
- $\sigma$ moves away from $\sigma_0$ at intermediate times
- Slow, **nonexponential** relaxation of σ at intermediate times (before the minimum)

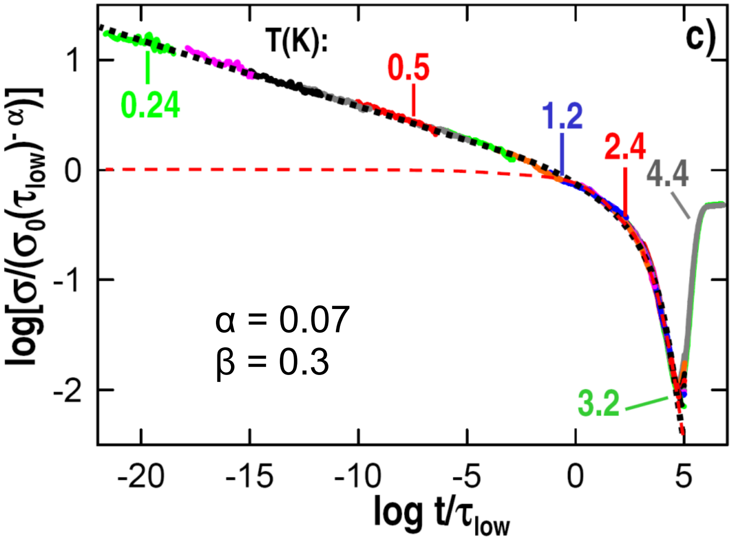
- Dynamical scaling at **intermediate times:**

$$\sigma(t,T)/\sigma_0 \propto t^{-\alpha(n)} \exp\{-[t/\tau(n_s,T)]^{\beta(n)}\}$$

$$\alpha(n_s) < 0.4, 0.2 < \beta(n_s) < 0.45$$

Broad distribution of relaxation times

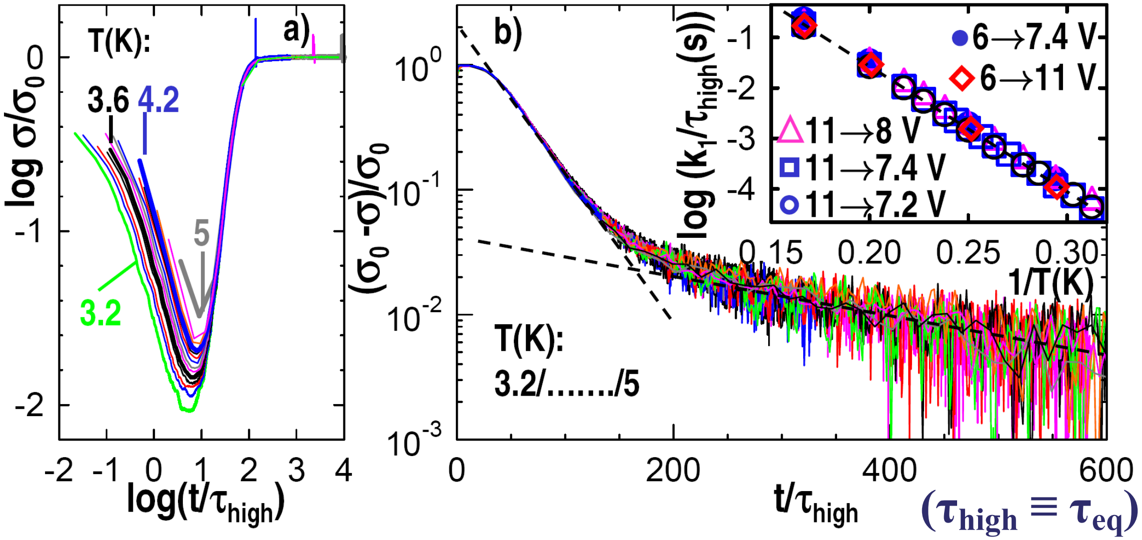
$$\boldsymbol{\sigma/\sigma_0 \propto t^{-\alpha} \quad \text{as } T \to 0}$$



**Similar to spin glasses above $T_g$**

# Quench dynamics in a high-disorder 2DES: Relaxations of $\sigma$ after a large, rapid change of $n_s$

## Long times (approach to "equilibrium"):



- Different *T* data collapse for times **after** the minimum
- The system reaches "equilibrium" after a long enough time *t*
- Approach to "equilibrium" **exponential in time**

- Thermalization at **long times**:

  **$\tau_\sigma \propto \exp(E_A/T)$**

- As **$T \rightarrow 0$, $\tau_\sigma \rightarrow \infty$**

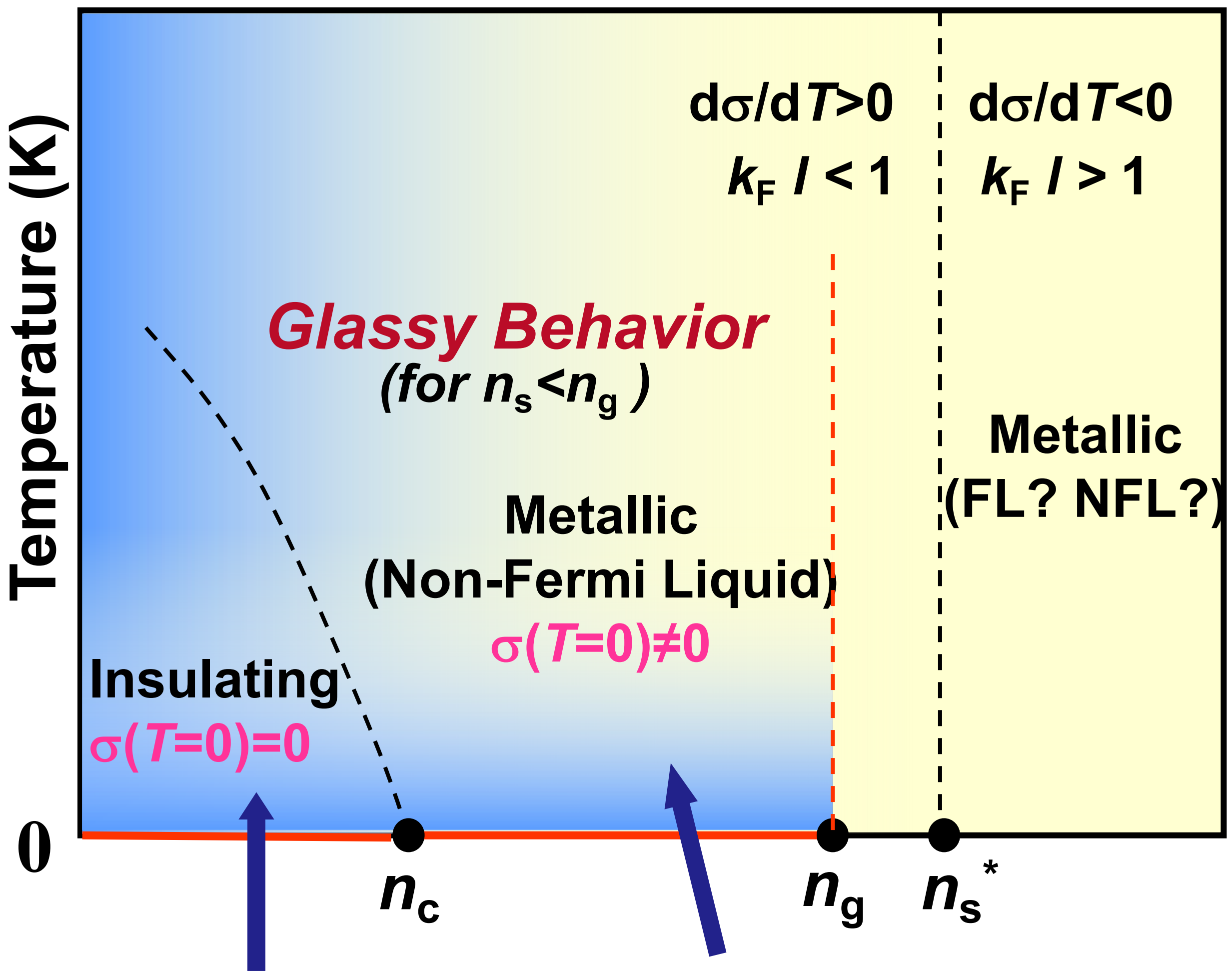
  (glass transition at $T_g$=0)

$E_A \approx 57$ K

[Jaroszyński & Popović, Phys. Rev. Lett. 96, 037403 (2006); also Jaroszyński & Popović, Phys. Rev. Lett. 99, 046405 (2007); Jaroszyński & Popović, Phys. Rev. Lett. 99, 216401 (2007)]

# Phase diagram of a strongly disordered 2DES in Si: Dynamics with long-range Coulomb interaction

Long-range Coulomb (~1/$r$) + disorder ⇒ **Frustration**



**$T$=0 glass transition for $n_s < n_g$**

$n_g \approx 7.5\times10^{11}\text{cm}^{-2}$
$n_c \approx 5.0\times10^{11}\text{cm}^{-2}$

**2D MIT: Melting of the Coulomb glass**

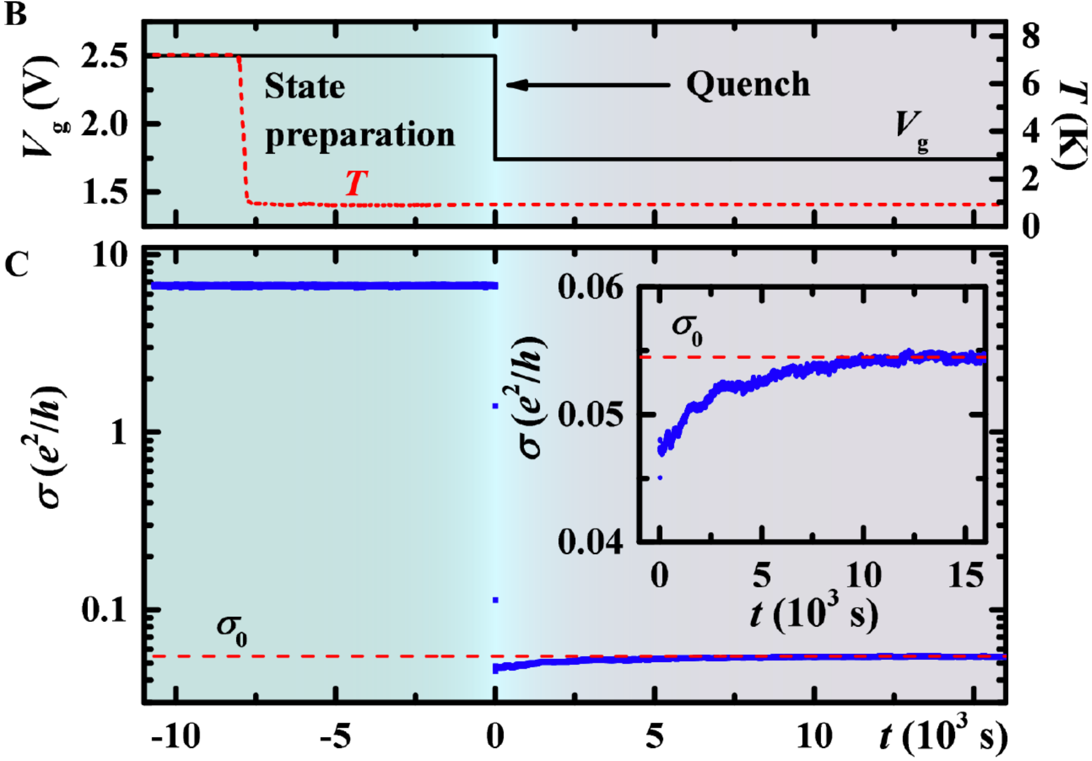
**Reviews** in book chapters:
D. P. in
a) "Conductor-Insulator Quantum Phase Transitions",
edited by Dobrosavljević, Trivedi, Valles
(Oxford Univ. Press, 2012)
b) "Strongly Correlated Electrons in Two Dimensions", edited by Kravchenko
(Pan Stanford Publishing, 2017)

Exponential localization;
Glassy insulator

"Bad" metal: $\sigma(n_s,T)=\sigma(n_s,T=0)+b(n_s)T^{3/2}$
Intermediate, glassy phase

# Quench dynamics in strongly disordered 2DESs

**Short-range Coulomb interaction: ~1/$r^3$**



Initial $n_s$($10^{11}$cm$^{-2}$) =32.2 ; $k_F l$ >1

Final $n_s$($10^{11}$cm$^{-2}$) = 8.44 $\geq n_c$ ; $T$=0.92 K

$n_c \approx 4.2 \times 10^{11}$cm$^{-2}$

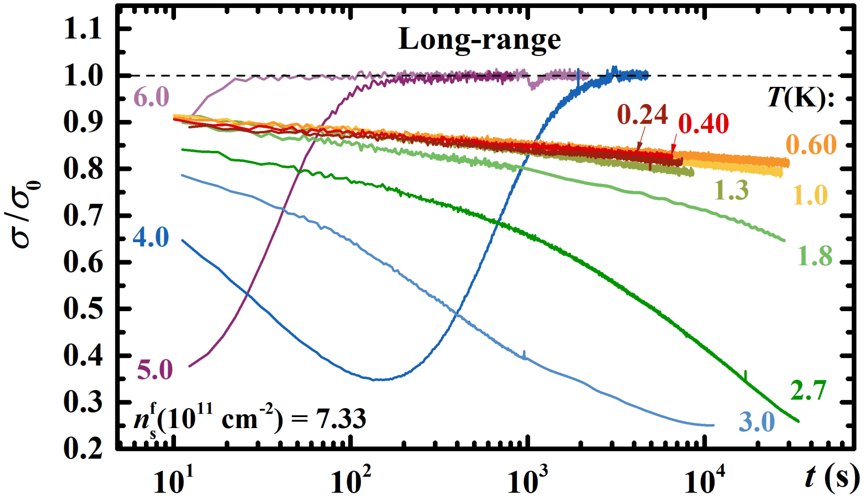
**Same experimental protocol for both sets of samples; samples and leads in vacuum**

- Sample is warmed up to ~7-20 K to "reset" the sample
- At ~7-20 K, the gate voltage is set to some $V_g^i$ where $k_F l \geq 1$
- The sample is **cooled to the measurement *T*** and allowed to equilibrate
- Then $V$g, i.e. ***n*$_s$ is quickly (<3 s) changed to a lower, final value**
- $\sigma$ is measured continuously throughout the process

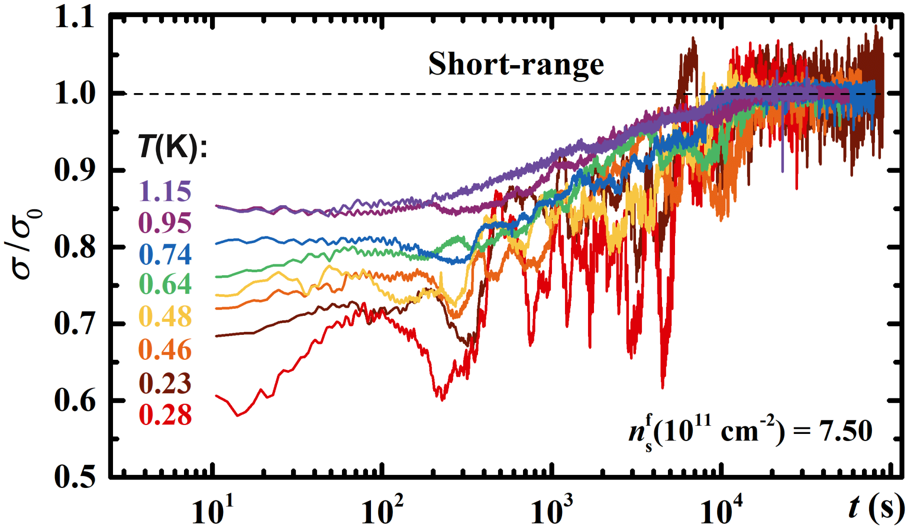
- **Large perturbation**: **Δ** $E_F \sim E_F$

($k_B T \ll E_F$)

# Quench dynamics in strongly disordered 2DESs: Behavior at intermediate times



$n_c\,(10^{11}\ \mathrm{cm}^{-2}) = (5.0 \pm 0.3)$



$n_c\,(10^{11}\ \mathrm{cm}^{-2}) = (4.2 \pm 0.2)$

**Dynamical scaling at intermediate times** (similar to spin glasses above $T_g$):

$$\sigma(t,T)/\sigma_0 \propto t^{\alpha(n)} \exp\{-[t/\tau(n_s,T)]^{\beta(n)}\}$$

$$\alpha(n_s) < 0.4, 0.2 < \beta(n_s) < 0.45$$

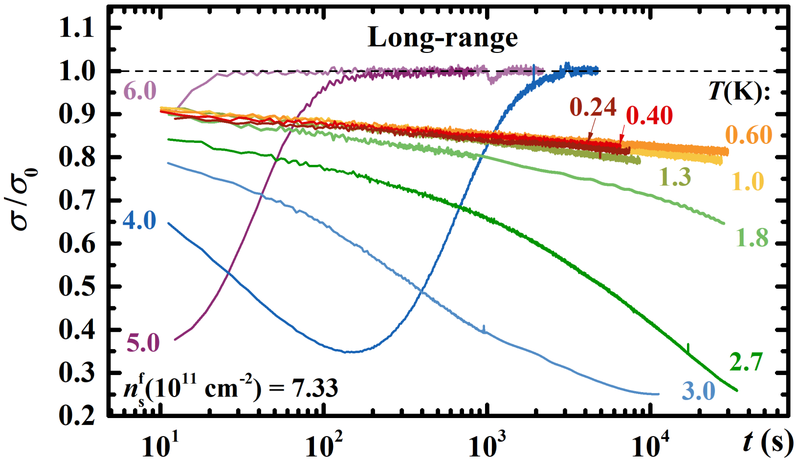
$$\boldsymbol{\sigma/\sigma_0 \propto t^{-\alpha} \quad \text{as } T \to 0}$$

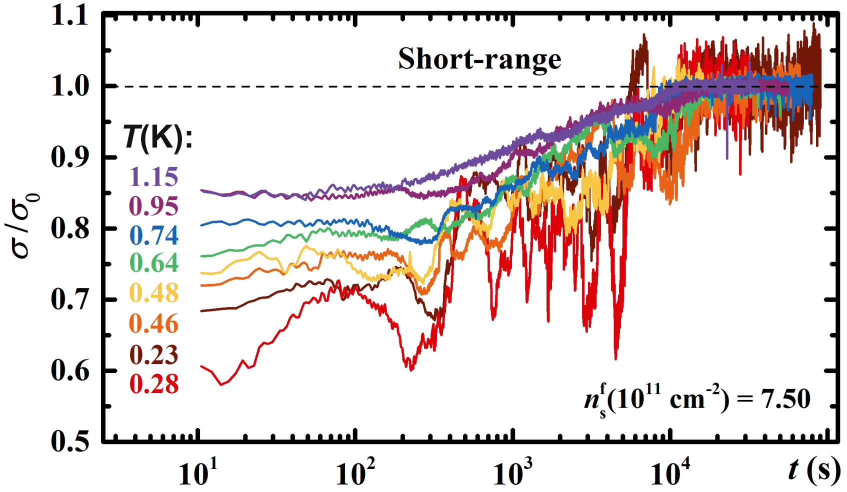
[Jaroszyński & Popović, Phys. Rev. Lett. 96, 037403 (2006); also Jaroszyński & Popović, PRL 99, 046405 (2007) and PRL 99, 216401 (2007)]

- **Negligible relaxation** (no apparent time dependence, as expected for MBL-like dynamics)
- No evidence of glassiness

[L. J. Stanley, P. V. Lin, J. Jaroszyński, D. Popović, Nat. Commun. 14, 7004 (2023)]

# Quench dynamics in strongly disordered 2DESs: Behavior at long times





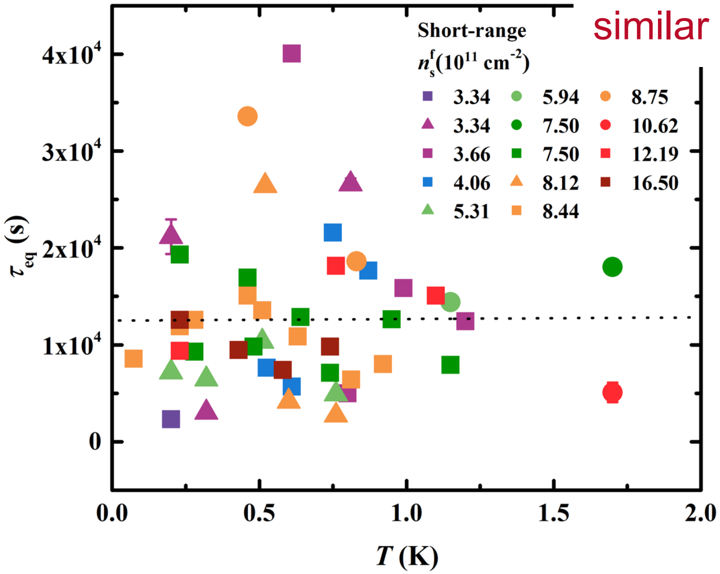
$$\sigma(t) - \sigma_0 \propto \exp(-t/\tau_\sigma)$$

- Thermalization at **long times**:

  $\boldsymbol{\tau_\sigma \propto \exp(E_A/T)}$

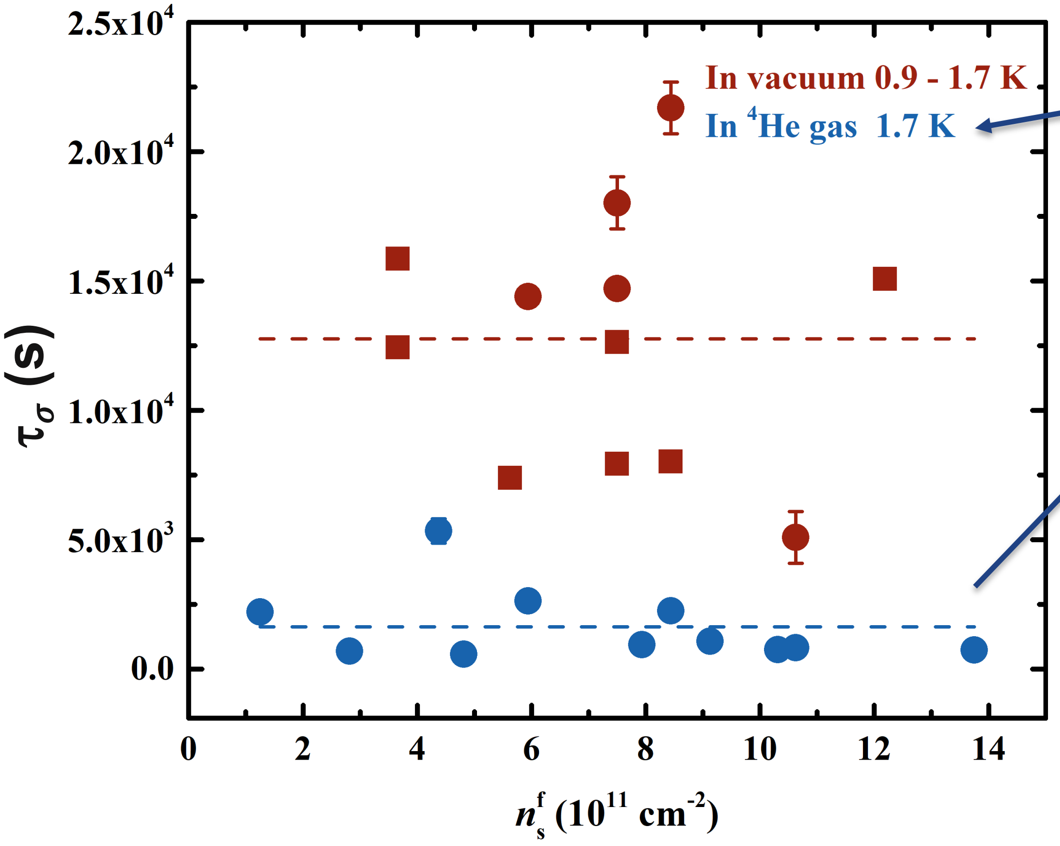
- As $\boldsymbol{T \rightarrow 0,\ \tau_\sigma \rightarrow \infty}$

  (glass transition at $T$=0)



$\boldsymbol{\tau_\sigma \sim 10^4}$ **s,**
similar for all $T$ and $n_s$

# Thermal coupling to the environment sets the time scale for thermalization!



**Short-range Coulomb interaction**

Stronger coupling to the bath when in $^4$He vapor

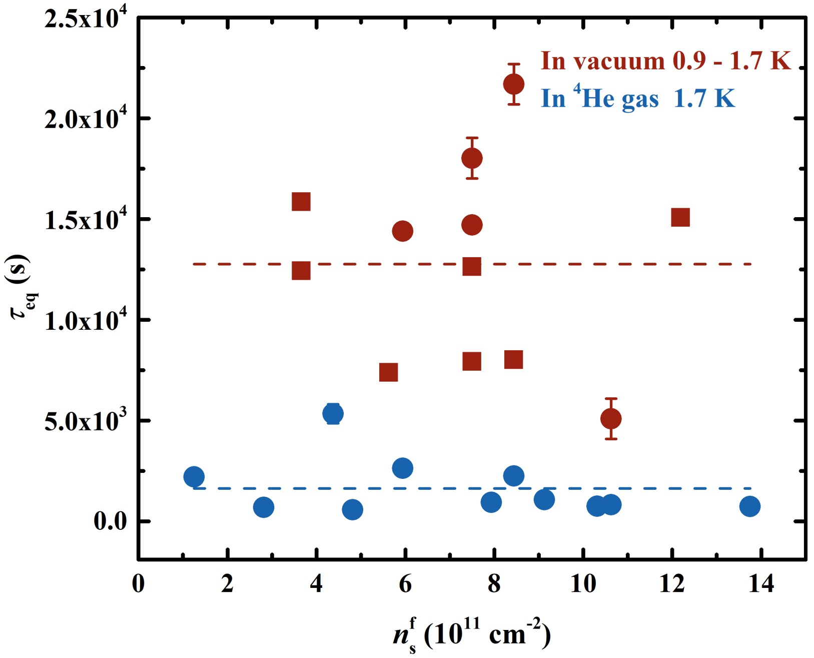
$\tau_\sigma$ reduced by an order of magnitude!

**Prethermal, MBL regime**

[Effect of finite coupling to a bath on MBL: M. H. Fischer, M. Maksymenko, E. Altman, Phys. Rev. Lett. **116**, 160401 (2016); Lüschen et al., PRX **7**, 011034 (2017); also, review by Abanin *et al.*, Rev. Mod. Phys. **91**, 021001 (2019)]

# Quench dynamics in strongly disordered 2DESs: Sensitivity to thermal coupling to environment

## Short-range Coulomb interaction



- ▶ $T < 2$ K: weak electron-phonon coupling

a) Samples in vacuum: $\tau_\sigma \sim 10^4$ s

**weak** thermal coupling to environment

b) Samples in $^4$He vapor: $\tau_\sigma \sim 10^3$ s

**intermediate** thermal coupling to environment

- ▶ $T > 2$ K: cooling via phonons dominant

**Strong** thermal coupling to environment: $\tau_\sigma < 200$ s (at 4.2 K in both vacuum and liquid $^4$He)

## Long-range Coulomb interaction:

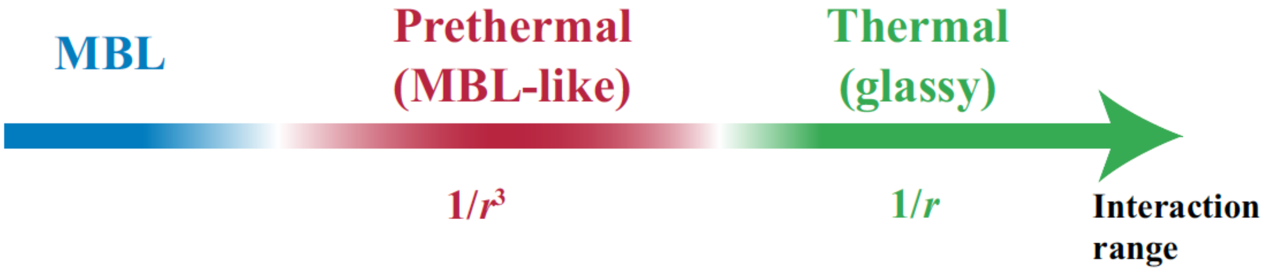
Glassy dynamics unchanged even up to $T \sim 6$ K ⇒ insensitive to thermal coupling to environment

**Strong (exponential?) dependence of thermalization time on the coupling strength: MBL dynamics**

# Quench dynamics and thermalization: Direct observation of MBL-like dynamics

**Effect of interaction range in a 2DES for a fixed disorder strength:**

- **No difference in equilibrium transport properties**
- **Striking difference in nonequilibrium dynamics!** (when $k_F l < 1$; bad conductor)



**MBL**

**Prethermal (MBL-like)**

**Thermal (glassy)**

**$1/r^3$**

**$1/r$**

**Interaction range**

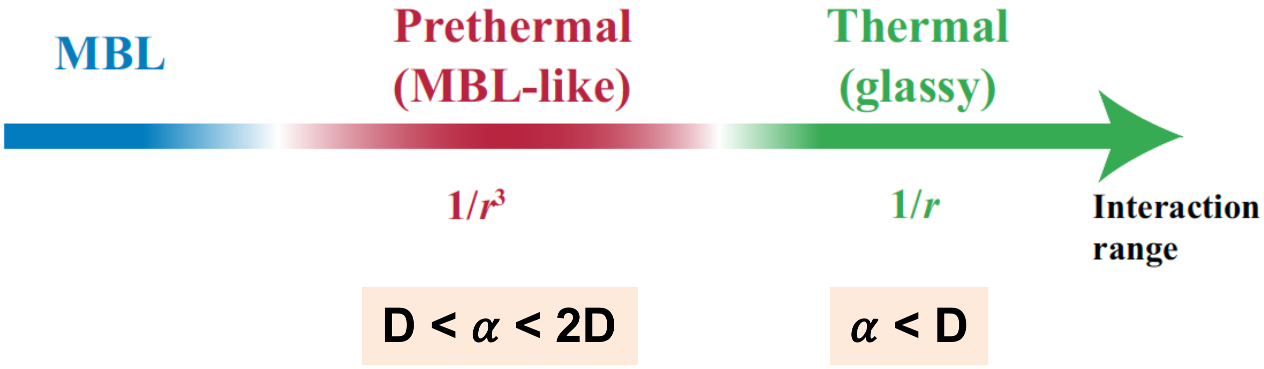
- **Screened Coulomb interaction** leads to a **prethermal, MBL-like** regime at intermediate times
- Time scale: hours!

[L. J. Stanley, P. V. Lin, J. Jaroszyński, D. Popović, Nat. Commun. 14, 7004 (2023)]

# Quench dynamics and thermalization: Direct observation of MBL-like dynamics

**Effect of interaction range in a 2DES for a fixed disorder strength:**

**Power-law interactions: ~ $1/r^{\alpha}$**



MBL

Prethermal (MBL-like)

Thermal (glassy)

D=2

$1/r^3$

$1/r$

Interaction range

D < $\alpha$ < 2D

$\alpha$ < D

[Theory: Yao et al., PRL 113, 243002 (2014); Burin, PRB 92, 104428 (2015); Gutman et al., PRB 93, 245427 (2016)…]

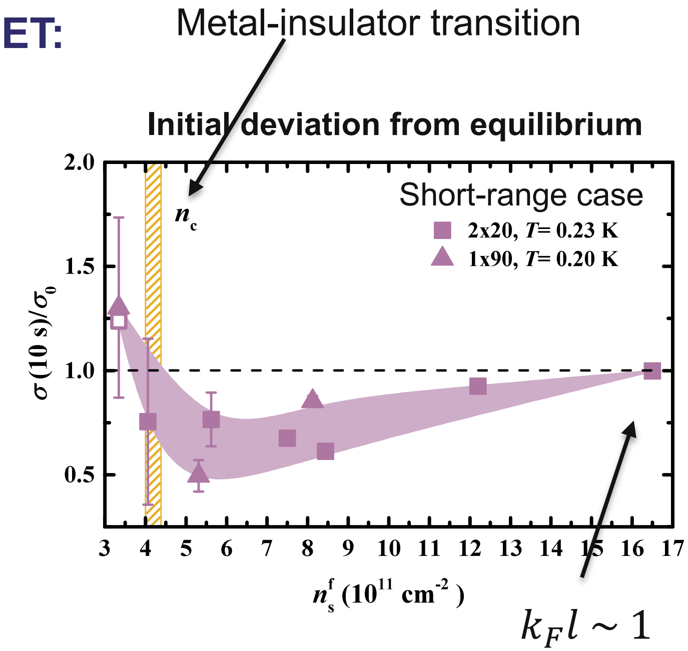
- Short-range (dipolar) case: divergent thermalization time as coupling to bath → 0? True MBL phase??

# Summary and outlook

**Strongly disordered 2D electron system in a Si MOSFET:**

- **Slow dynamics when $k_F l < 1$** (bad conductor)
- Glassy dynamics with long-range Coulomb interaction (~1/*r* )
- **No** glassy dynamics with screened Coulomb interaction (~1/$r^3$); prethermal, MBL regime
- New, **solid-state platform** for studies of thermalization and **MBL regime in large systems**; **time scales: hours!**

- Glassy vs MBL (prethermal) regimes?
  - Wide distribution of timescales to reach local thermal equilibrium?
  - Equilibrium for $t > \tau_\sigma$?
  - Dynamics after cooling with fixed electron density?
  - Waiting time protocols



Metal-insulator transition

Initial deviation from equilibrium

$k_F l \sim 1$