

Hall effect in interacting systems

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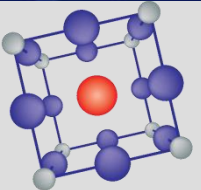
<https://giamarchi.unige.ch/>



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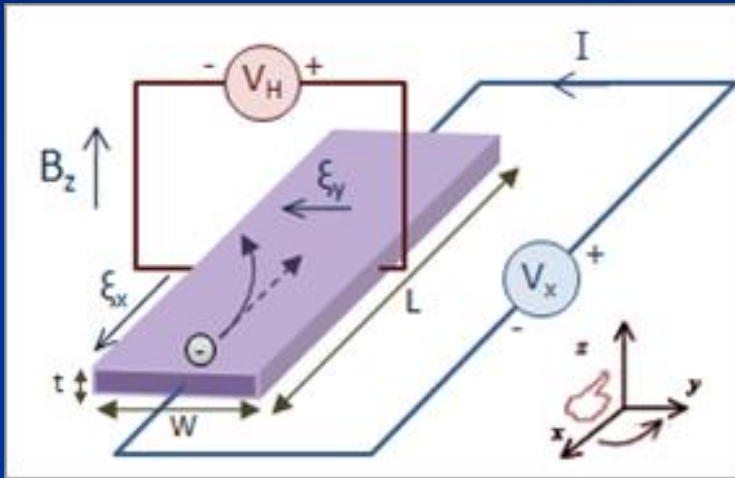


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Hall effect



Non interacting ``simple''

$$R_h = \frac{V_{\perp}}{I_{\parallel} B} \propto \frac{1}{n}$$

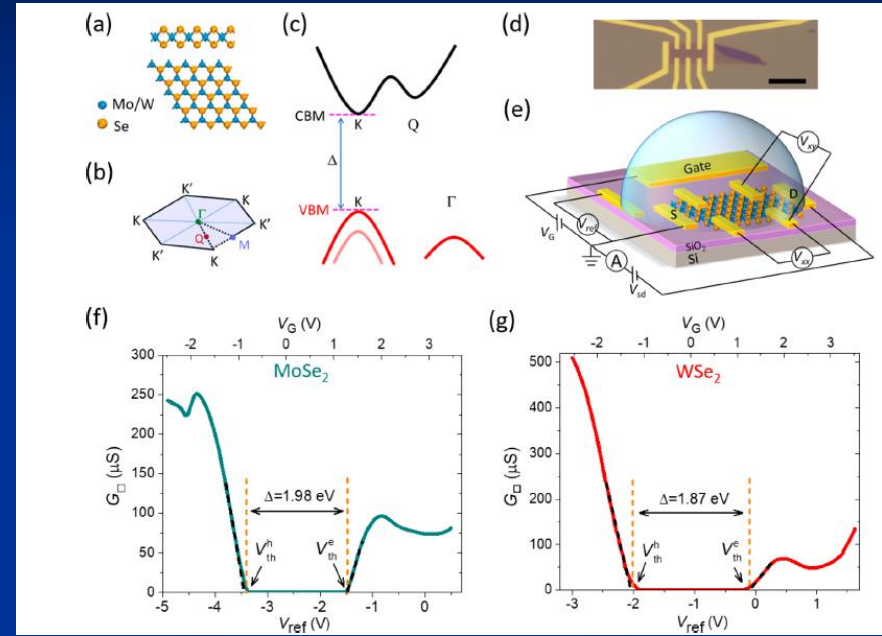
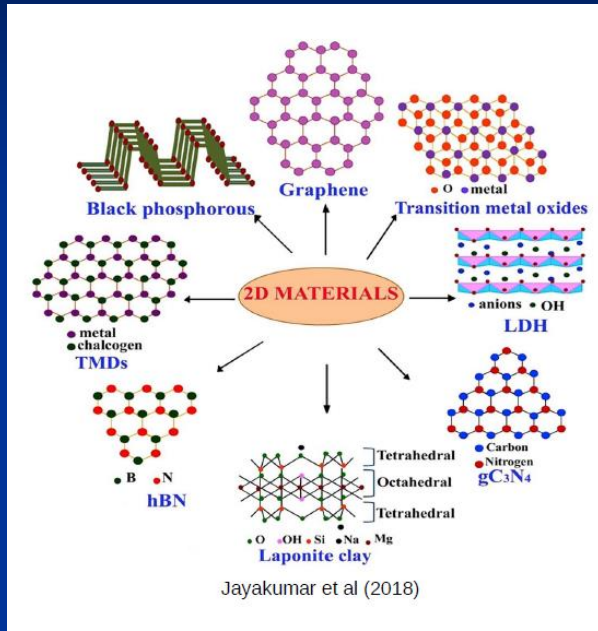
No interactions: curvature of fermi surface
- topological formula (Thouless-Kohmoto)

With **interactions**: open question

Two interesting situations

- Low density systems (electrostatic doping near an empty band)
- Dimensional effects: quasi-1D coupled systems, moiré (??)

Low density systems



- Novel effect:
cross quantum capacitance

$$\frac{1}{C} = \frac{1}{C_G} + \frac{1}{C_Q} + \frac{1}{C_S}$$

H. Zhang, C. Berthod, H. Berger, TG, A. Morpurgo, Nano Letters, 19, 8836 (2019)

C. Berthod, H. Zhang, A. Morpurgo, TG, PRR 3, 043036 (2021)



Hall effect



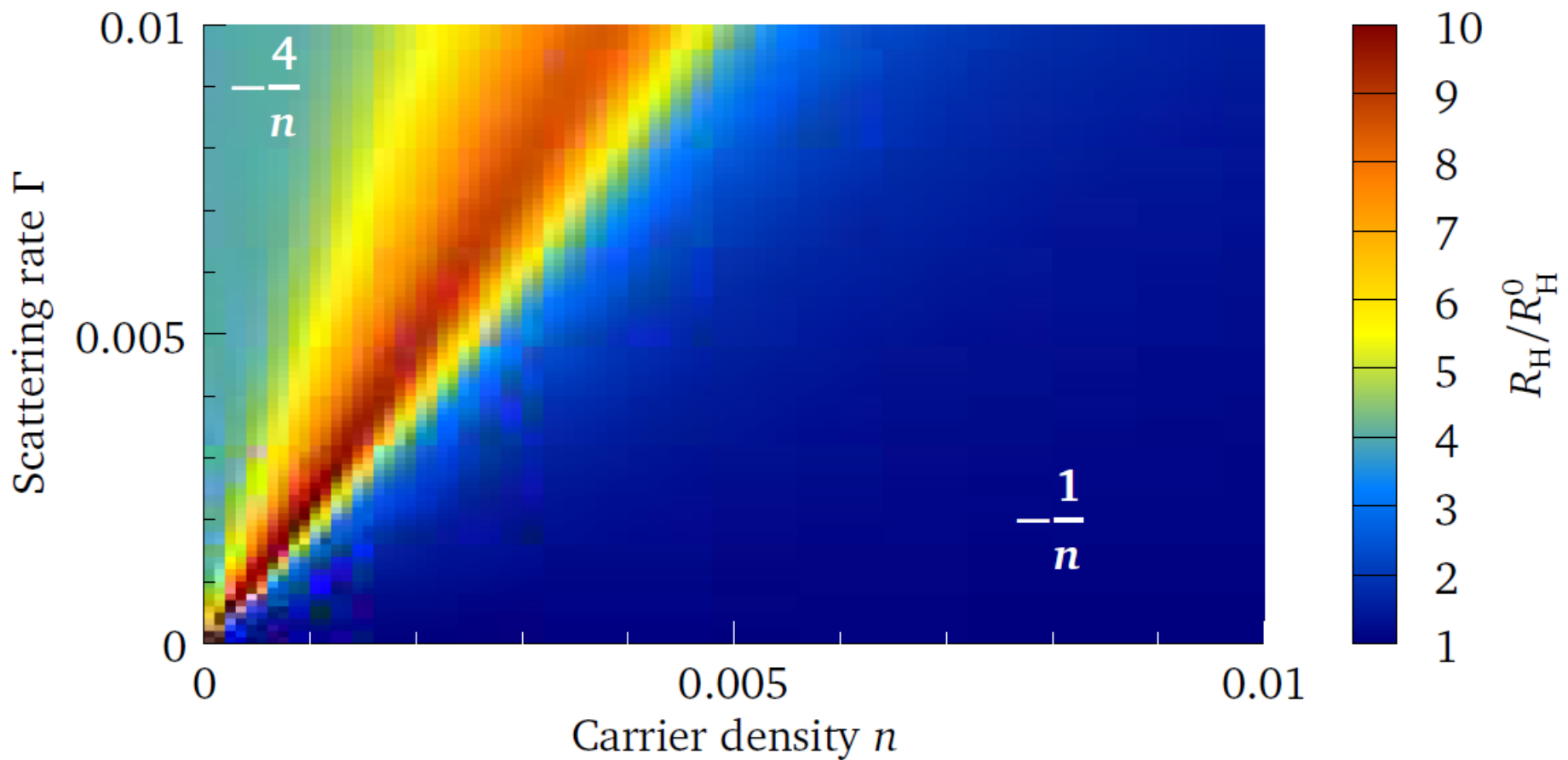
G. Morpurgo, L. Rademaker, C. Berthod, TG, PRR 6 013112 (2024)

$$\sigma_{xx}^{(0)} \propto \int_{-\infty}^{\infty} d\varepsilon [-f'(\varepsilon - \mu)] \int d^2k v_{xx}^2 A^2(\mathbf{E}_k, \varepsilon)$$
$$\sigma_{xy}^{(1)} \propto \int_{-\infty}^{\infty} d\varepsilon [-f'(\varepsilon - \mu)] \int d^2k \left(\frac{v_x v_y}{m_{xy}} - \frac{v_x^2}{m_{yy}} \right) A^3(\mathbf{E}_k, \varepsilon)$$

- We consider a local (k-independent) self energy

Constant self energy $\Sigma = -i\Gamma$

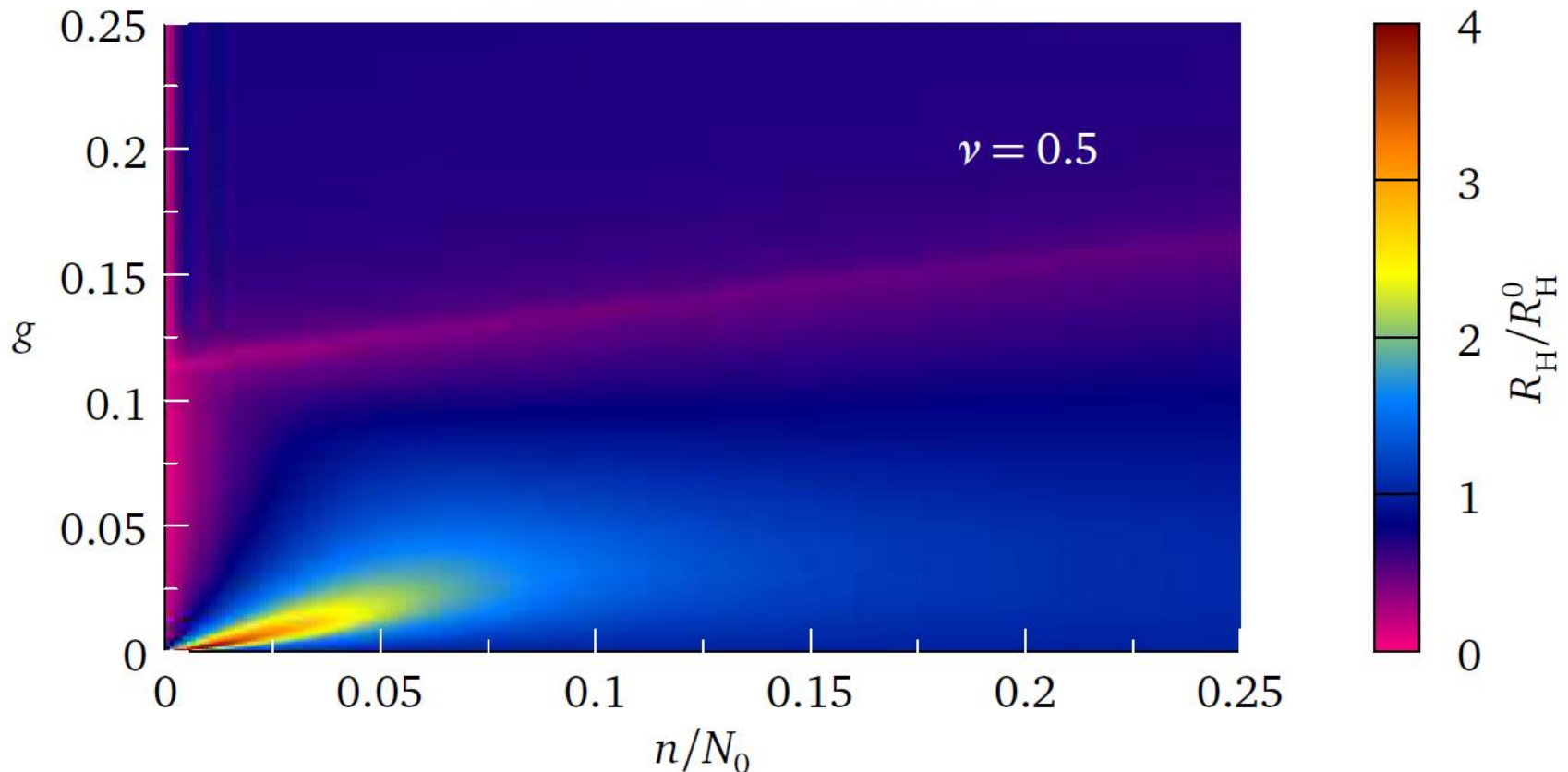
G. Morpurgo, L. Rademaker, C. Berthod, TG, PRR 6 013112 (2024)



Power-law self energy

$$\text{Im}\Sigma(\varepsilon) = -g(\varepsilon - \varepsilon_0)^\nu$$

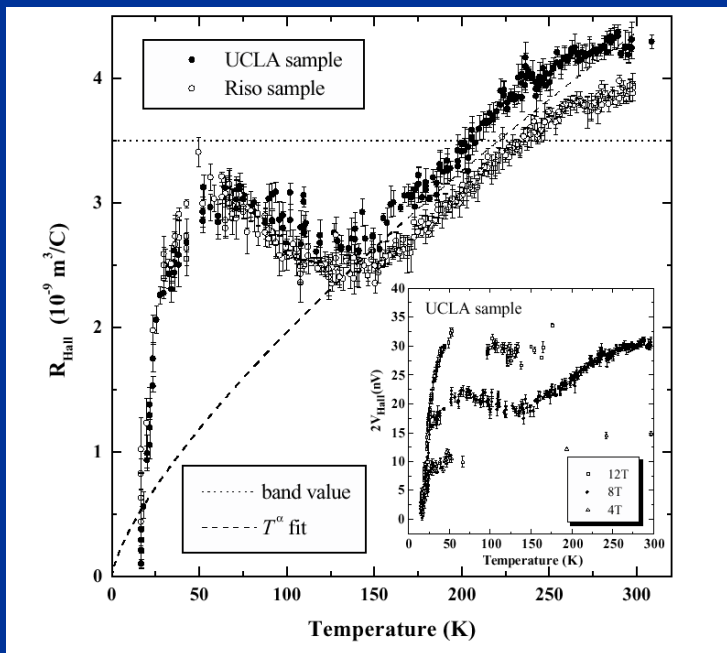
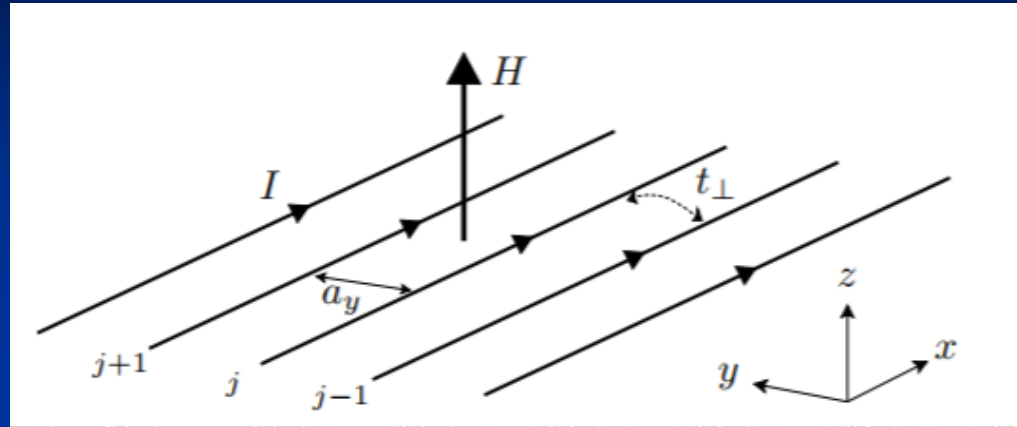
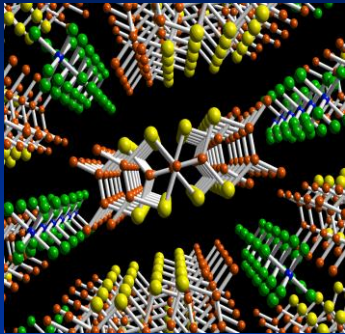
Hall constant



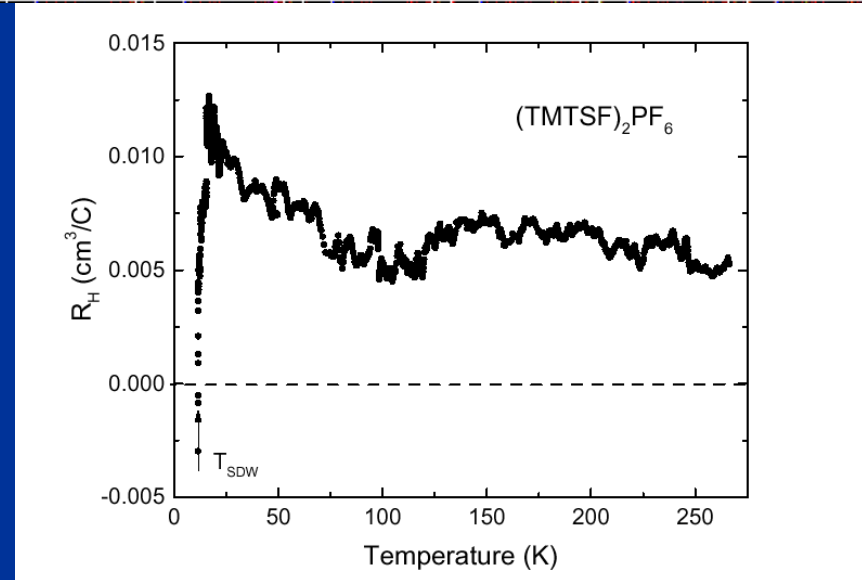
Coupled 1D structures

- Organic conductors/superconductors
(Bechgaard salts)
- Cold atomic systems
- Moiré systems ??

Experiments ?



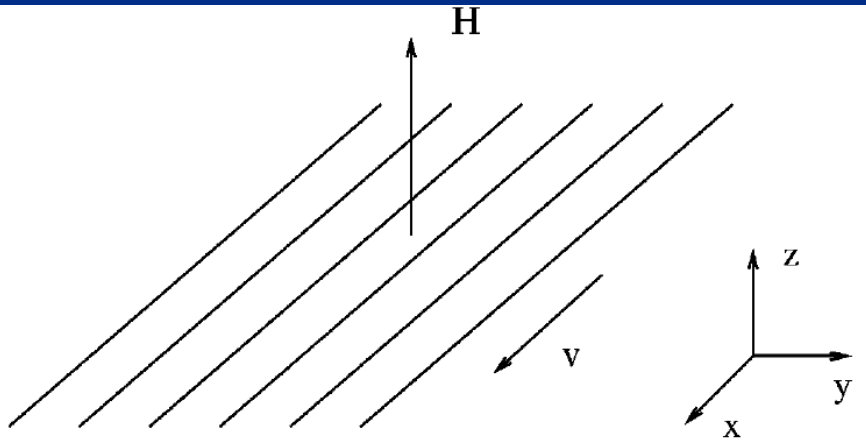
J. Moser et al., PRL 84
2674 (00)



G. Mihaly et al., PRL 84
2670 (00)

Quasi-1D geometry

A. Lopatin, A. Georges, TG PRB 62 (00)



$$H = \int dx \left[\sum_i v_F \hat{\psi}_i^\dagger \tau_3 (-i \partial_x) \hat{\psi}_i - \alpha \sum_i \hat{\psi}_i^\dagger \partial_x^2 \hat{\psi}_i + g \sum_i \hat{\psi}_{i+}^\dagger \hat{\psi}_{i+} \hat{\psi}_{i-}^\dagger \hat{\psi}_{i-} - t_\perp \sum_{\langle i,j \rangle} \hat{\psi}_i^\dagger \hat{\psi}_j e^{-i \frac{e}{c} A_{i,j}} \right]$$

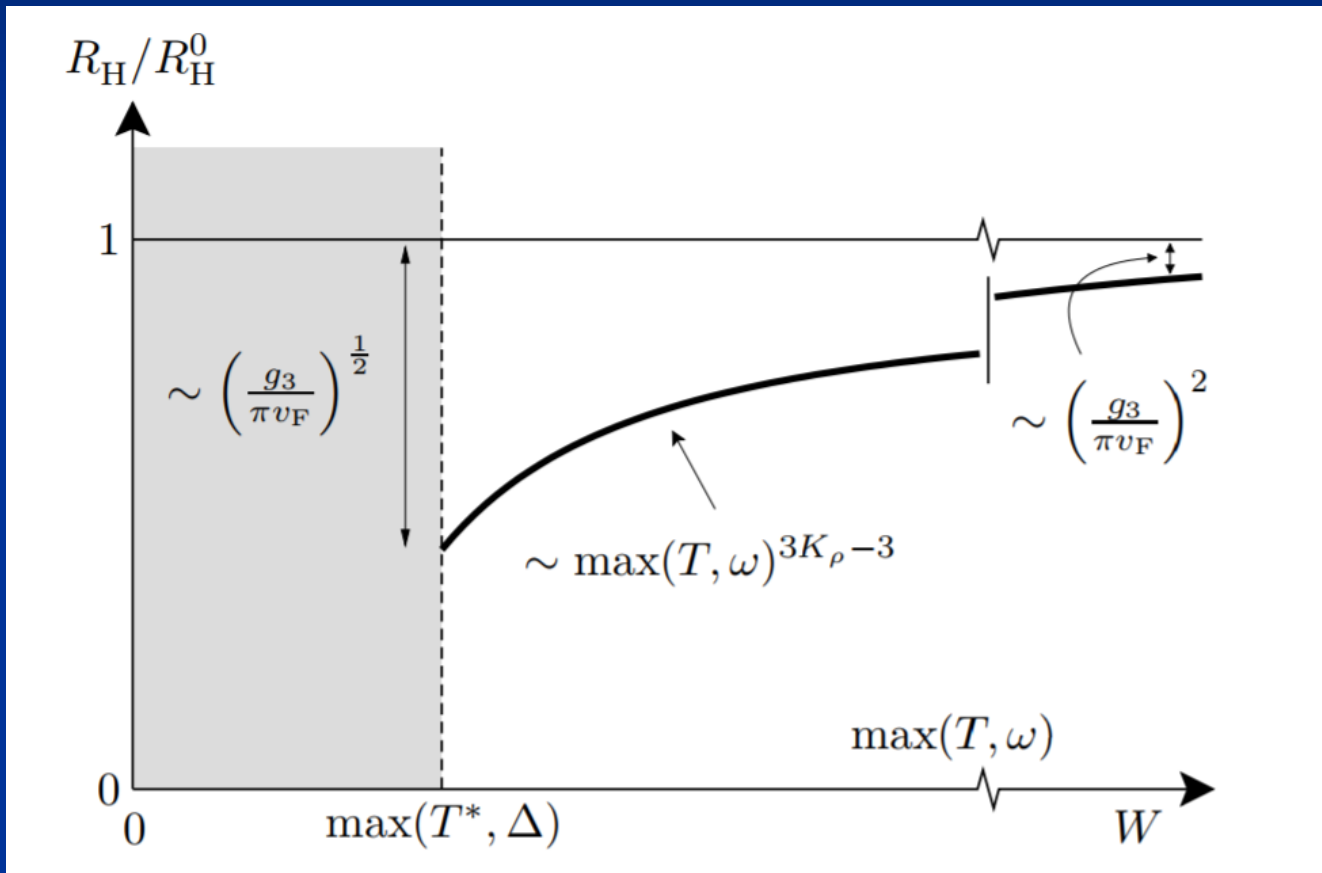
$$\epsilon_\pm = \pm v_F (p \mp p_F) + \alpha (p \mp p_F)^2.$$

$\alpha = 0$: usual TLL description of 1D systems

But: $\alpha \neq 0$: Particle hole symmetry; Hall = 0

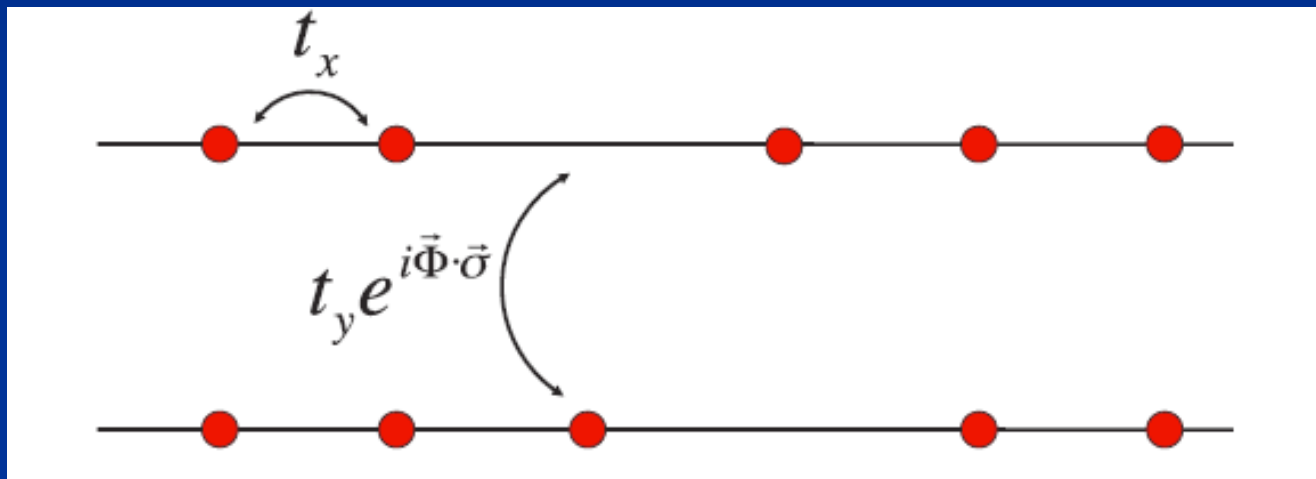


High temperature regime

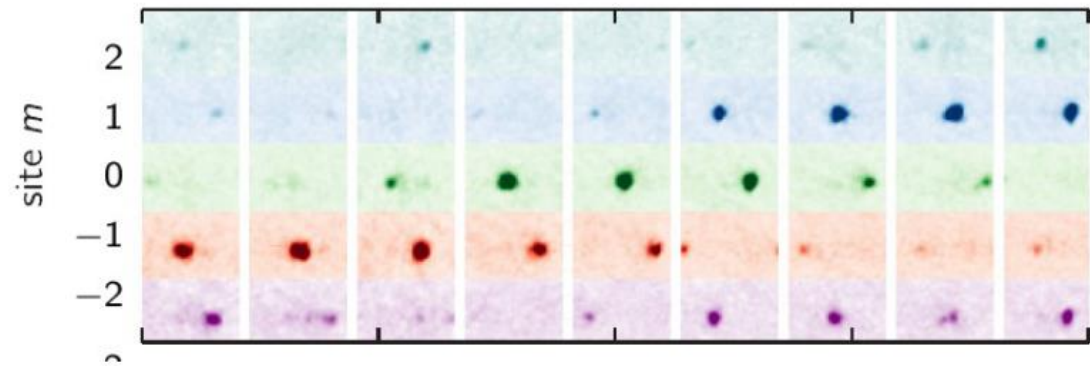
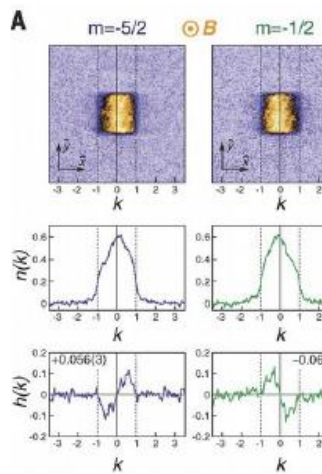
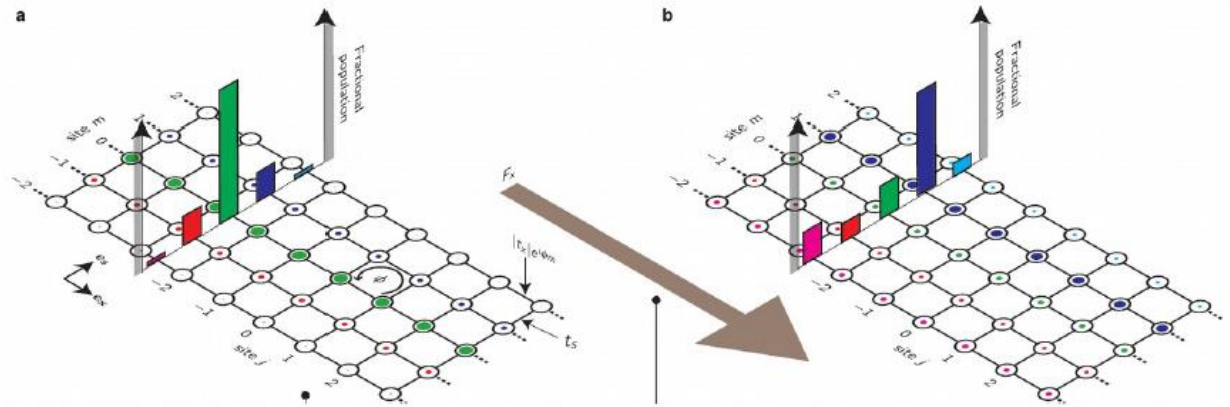
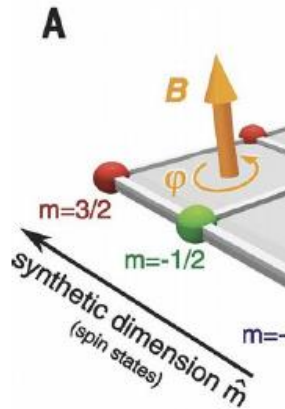


G. Leon, C. Berthod, TG PRB 75, 195123 (2007)

Other geometries: ladders



Synthetic GF / dimensions



Dina Genkina et al 2019 New J. Phys. 21 053021

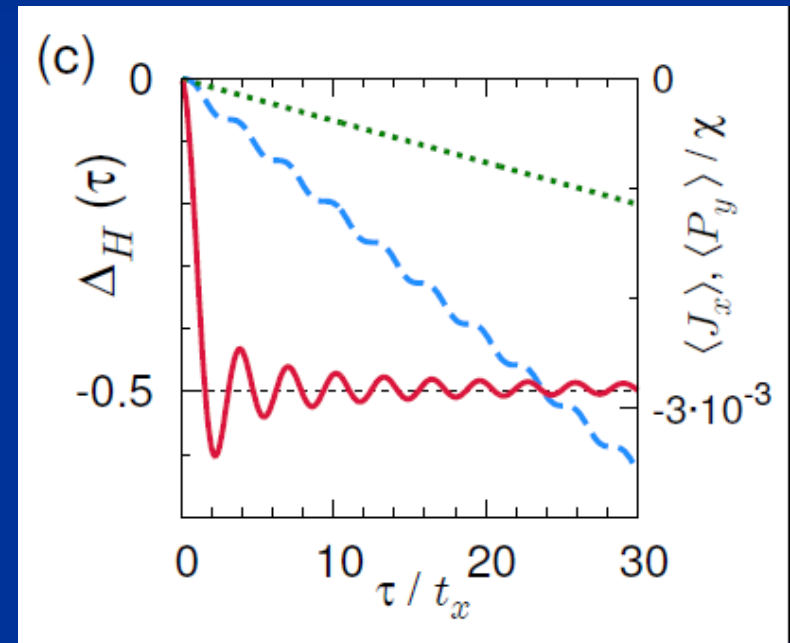
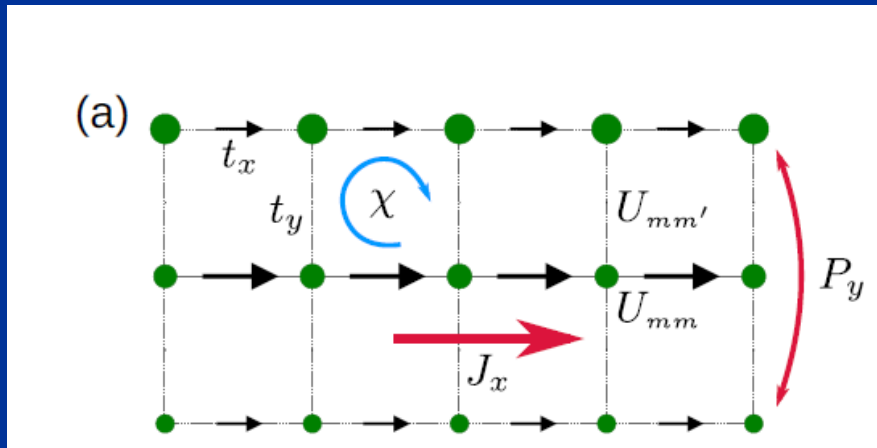
Spielman's group (Maryland) arXiv:1804.06345



Hall effect on ladders



S. Greshner, M. Filippone, TG, PRL 122, 083402 (2019)



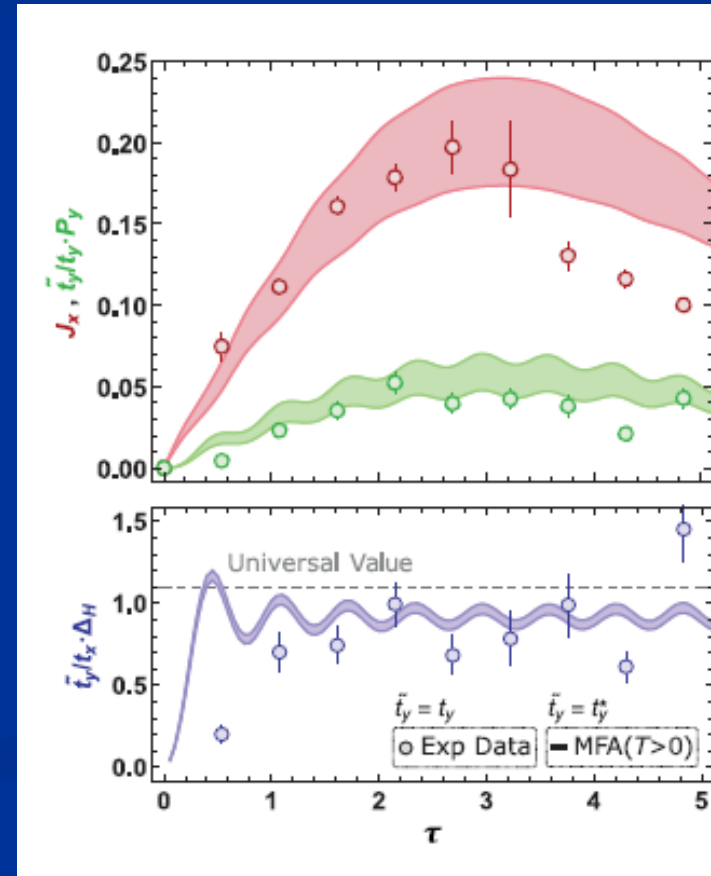
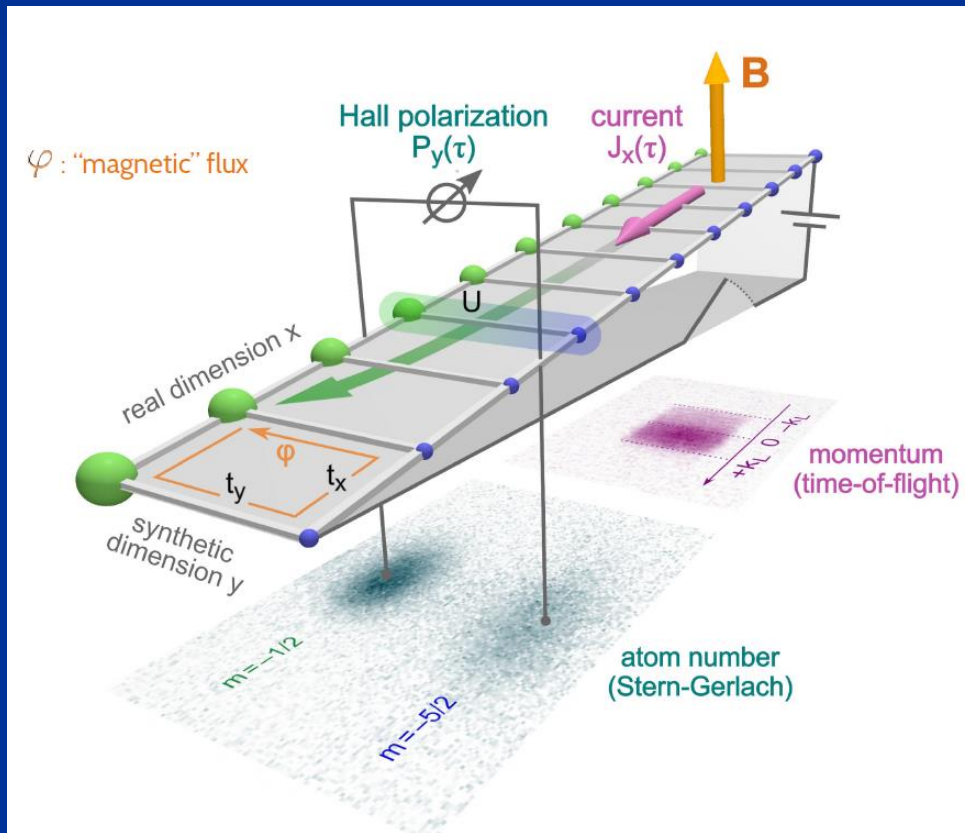
$$\Delta_H = q\mathcal{I}_0(M, t_\perp) \quad \text{and} \quad R_H = -1/n.$$

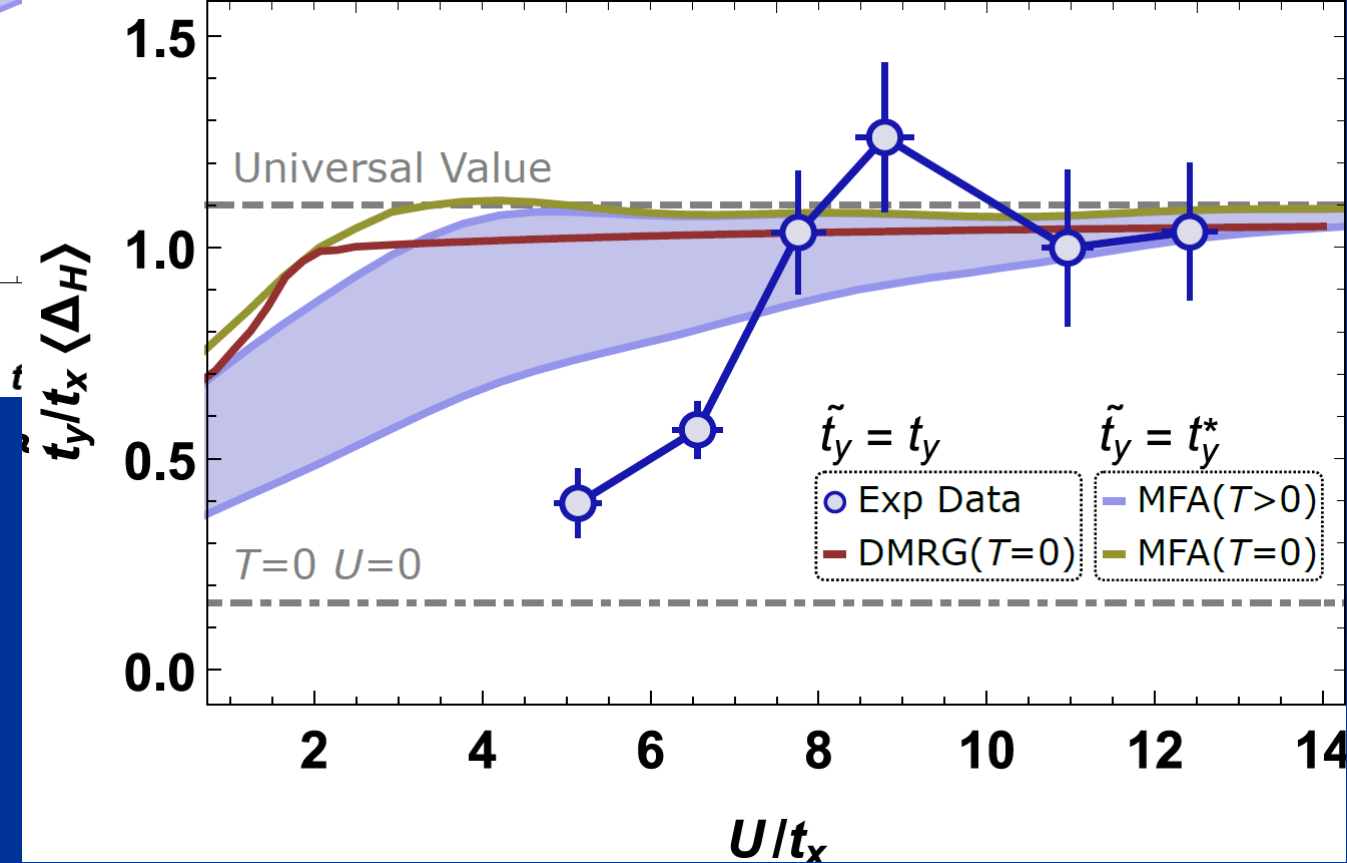
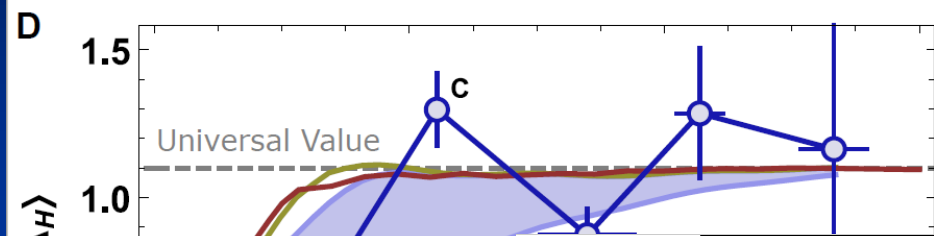
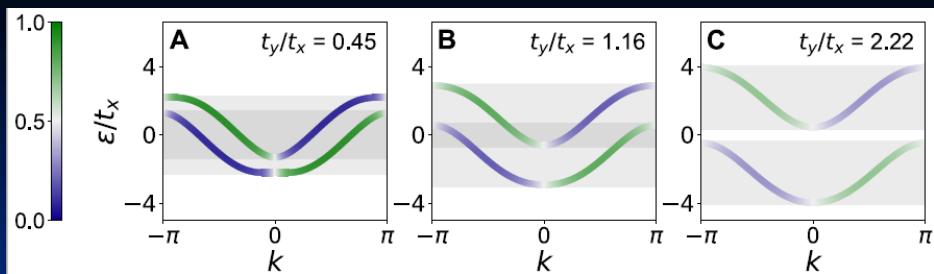
Universal value of Hall resistance



Quantum simulation

T.-W. Zhou, G. Cappellini, D. Tusi, L. Franchi, J. Parravicini, C. Repellin, S. Greschner, M. Inguscio, TG, M. Filippone, J. Catani, L. Fallani, *Science* (2023).

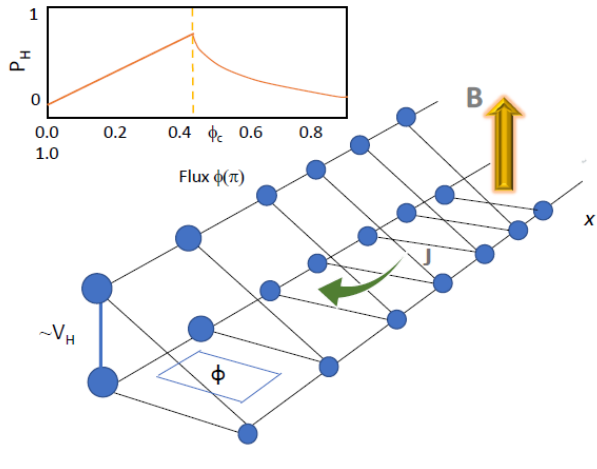




Analytical derivation



Bosonization: small ty



$$\psi(x) = \sqrt{\rho(x)} e^{i\theta(x)}$$

$$\pi\Pi(x) = \nabla\theta(x)$$

$$\rho(x) = -\frac{1}{\pi} \nabla\phi(x)$$

$$\alpha = -\frac{\partial}{\partial\rho_0} \left(\frac{u}{6\pi^2 K} \right),$$

$$\gamma = -\frac{\partial}{\partial\rho_0} \left(\frac{uK}{2\pi^2} \right).$$

$$H = \sum_{p=1,2} H_p + H_{tunn.}$$

$$H_p = \int \frac{dx}{2\pi} \left[uK(\pi\Pi_p)^2 + \frac{u}{K}(\partial_x\phi_p)^2 \right] + \int dx \left[\alpha(\partial_x\phi_p)^3 + \gamma(\pi\Pi_p)^2\partial_x\phi_p \right],$$

$$H_{tunn.} = -\frac{t_{\perp}}{\pi\alpha} \int dx \cos(\theta_1 - \theta_2),$$

Hall voltage

$$V_H = -\frac{\pi^2 \gamma \langle j_\rho \rangle}{e u_\rho K_\rho} B a = -\langle j_\rho \rangle \frac{B a}{2e} \frac{1}{u_\rho K_\rho} \frac{\partial}{\partial \rho_0} (u_\rho K_\rho),$$

■ Hall resistance

$$R_H = \frac{a}{e u_s K_s} \frac{\partial}{\partial \rho_0} (u_s K_s).$$

Charge stiffness !

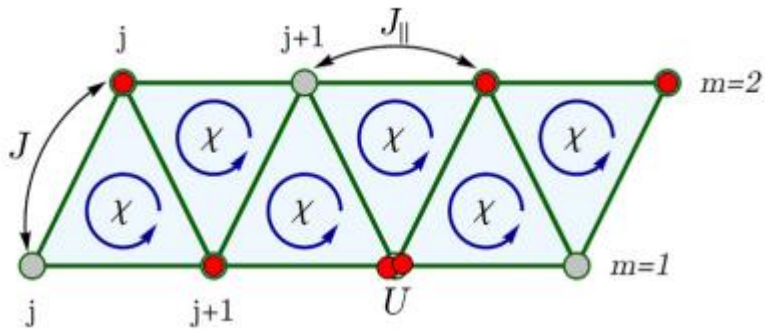
■ Galilean system

$$R_H = \frac{aB}{e\rho_0},$$

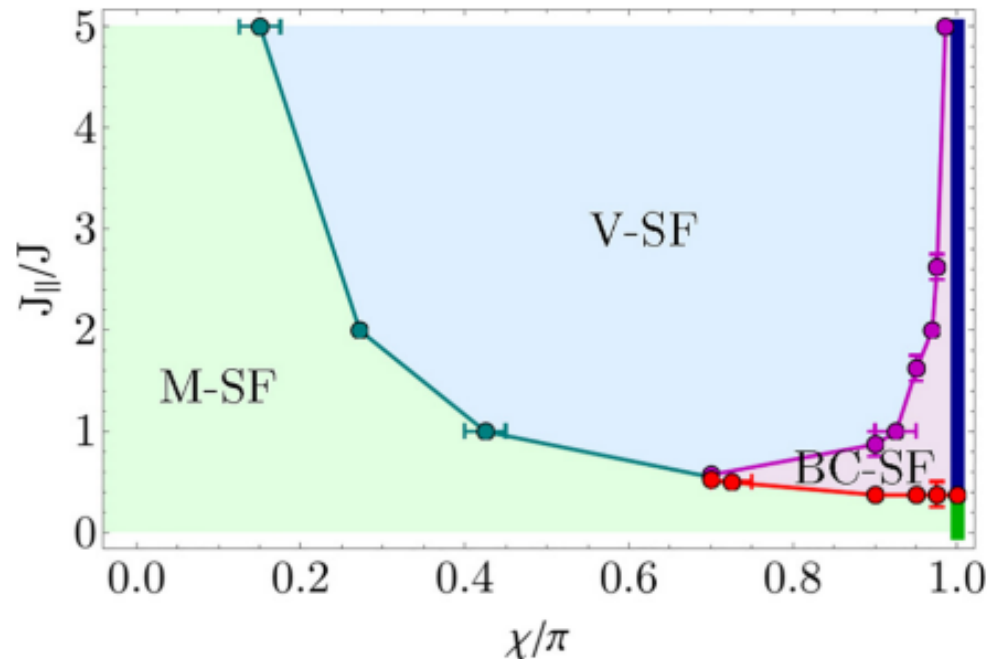
■ Near Mott

$$R_H \sim \frac{a \operatorname{sign}(\rho_0 - \rho_{0,c})}{e |\rho_0 - \rho_{0,c}|},$$

Triangular ladders

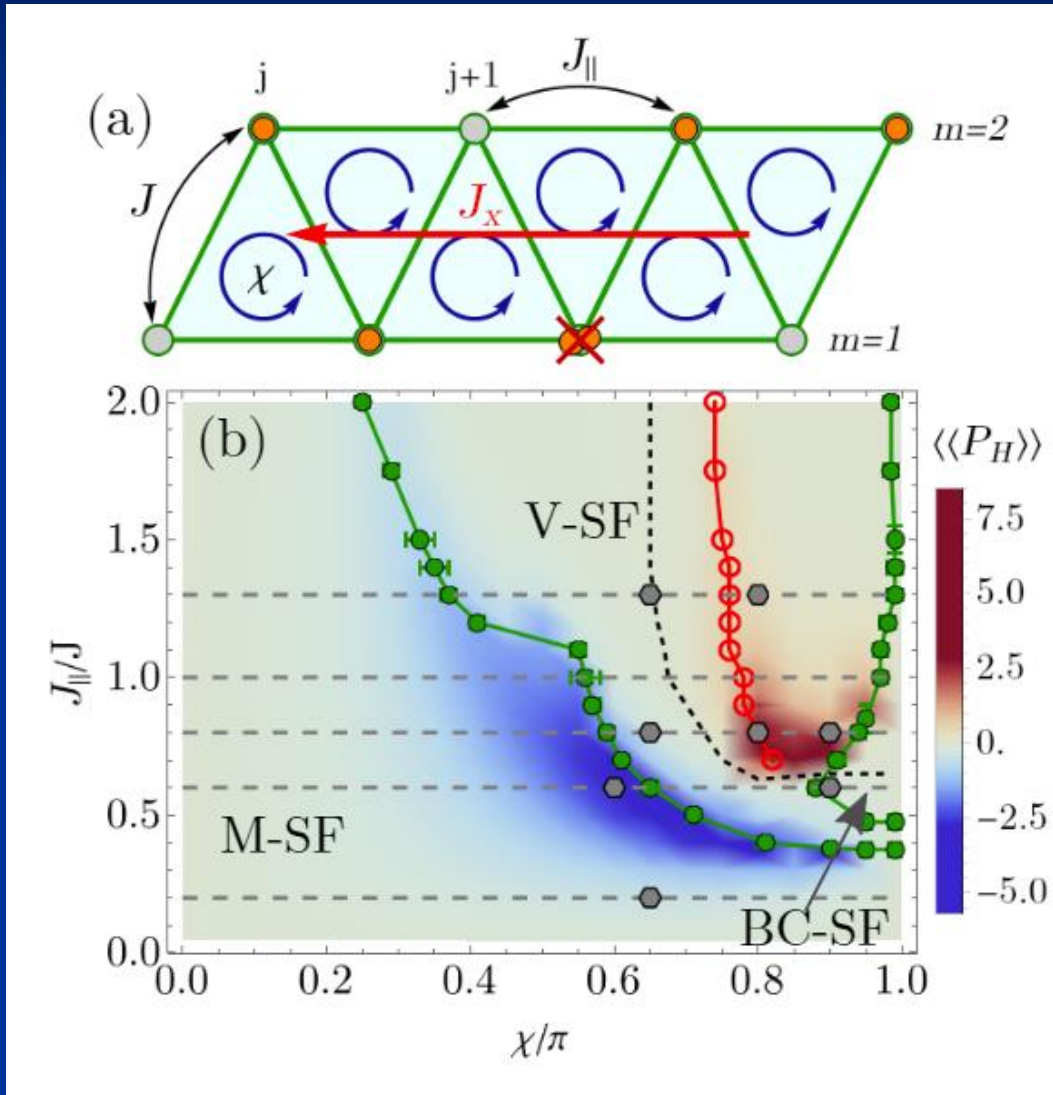


C. Halati and TG
PRR 5, 013126 (2023)



Hall effect

C. Halati, TG,
arxiv.org/abs/2405.19030



Harcore
bosons

Conclusions

- Remarkable properties of Hall effects for interacting quantum systems
- Low density: anomalous behavior of the Hall effect even for parabolic bands
- Ladders: Universal regime: $1/n$ for large tunnelling or large interactions.
- Confirmed in experiments in cold atomic gases
- Analytic expression for the Hall effect in a TLL
- Experiments in CM !!!!